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ENERGY BOND GRAPHS

The glass bead game of physics

A comprehensive thesis in three parts

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Part 1 — Physicizing
Part 2 — Generalizing
Part 3 — Quantizing.

Abstract

This thesis presents the formalism of energy bond graphs and various applications of it.

In the first part the scope of the formalism and its basic concepts are introduced as well as some applications, mainly in classical mechanics.

The second part introduces vector bonds and tensors in a general metric, and the formalism is applied to special relativistic dynamics and general relativistic kinematics.

The third part is an introduction to Quantum Semiotics, where energy bond graphs are combined with Quantum Mechanics in order to provide a philosophical interpretation of the latter.

I join with you
that sing bright and subtle
making shapes that my throat cannot tell
you that harden the horn and make quick the eye
you that run the fast fox
and the zigzag fly
you sizeles makers
of the mole and the whale
aid me, and I will aid you

(Robin Williamson: Invocation)

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PART III. Quantizing
The energy bond graph approach to Quantum Semiotics

Preface

When I now wish to present some of the results of more than thirty year's research with the Energy-Bond-Graph technique as a thesis in english, the purpose is not to further my academic career, which, anyway, is soon coming to an end, but I feel a certain obligation to present some results to the academic world, which has supported my research for all these years. Also, many colleagues and guest-researches have encouraged me to make the method of energy-bonds accessible in english, and therefore I finally decided to make an english translation as a self-contained version of parts of my earlier writings.

The purpose of this preface is to outline the structure of the thesis, to give an autobiographical account of my motivations and aims, and to acknowledge encouragements and help from friends, students and colleagues.

The beginning

About 1967 My career as a teacher and researcher in physics already seemed to be established. I had specialized in Statistical Mechanics and Solid State physics and was eagerly applying the new computer-technology for obtaining new results in the phenomenological theory of superconductivity. Having published several articles in established, international journals I thought I could see a straight road ahead of me. But I began having doubts about the ethical value of collaborating in the project of international physics, which I saw as providing new gadgets for industry and new weapons for the military, and I was concerned about the consequences of pollution and resource-depletion.

Internal critique

One of my main interests had been the concept of Entropy and the relation between Thermodynamics and Information Theory, and I began writing notes, connecting entropy and pollution, and, when finished in 1969, these notes were presented as material for a course, called "Information, Entropy, and Evolution", where the students were asked to present readings of newspaper-articles and political discussions about pollution, climatic changes, and consequences of nuclear power. Also, I tried to relate these discussions to cultural expressions of concern within music and art.

As one of the first attempts to raise an "internal critique" of physics at a time when the Marxist-inspired "external critique" began to dominate at the universities my course was met with interest from many students, but with severe reservation from some of my colleagues at Copenhagen University.

Ecophysics

After having continued political discussions for some time in an informal group ("KRISNA") I became convinced that there was a need for a regular course within the physics curriculum, and together with my long-time-collaborator Erland Brun Hansen I began writing notes for a course called "Ecophysics" (Ecological Physics). While Erland undertook the thermodynamic part, including some biological and ecological energetics in a book called "Energetics and Ecophysics", I was becoming more interested in computer-modelling and General System's Theory, and influenced by the Club of Rome (Meadows et al. "Limits to Growth"), Forrester's "World Dynamics", and H.T. Odum's "Environment, Power, and Society" I became interested in diagram-techniques for modelling. I tried to combine these new developments with my earlier studies of Norbert Wiener's "Cybernetics", and the result was my text for the Ecophysics course, called "Cybernetics and Ecophysics". This book also presented my first version of the Energy-Bond-Diagrams.

From energy flow to energy bonds

The Ecophysics course was accompanied by a study-group, that included both students and colleages (among the latter Erland and Karin Beyer). We were all interested in "Limits to Growth" and Forrester's "System Dynamics" and Odum's "Energy Flow Diagrams", but I was searching for a more fundamental approach to developing a suitable diagrammatic technique for dynamic modelling. One day Karin gave me a copy of an article from Nature telling about a new diagrammatic tool developed for engineering purposes by H.M. Paynter at M.I.T. At first, Erland took it up and presented the technique in a research-lecture at my department. I began studying Paynter's book, where I found the first reference to the philosopher C.S. Peirce and his theory of Semiotics. It struck me, that here was the basis for a scientific approach to development of diagrams. I could also see, that Paynter's method was able to remedy some of the vagueness and physical shortcomings of Odum's ecological diagrams, but I still preferred Odum's style of drawing.

The first result of my reflections thus arose as a combination of Paynter's formalism and Odum's attempt to make a "shape-value-notation" (see section I.2). It soon turned out, however, when I presented my combination in the Ecophysics course, that there were problems I had not realized. Two students, Poul Koch and Jesper Gundermann spotted these problems and tried to help me overcome them, but it took some time, before I found the solution by introducing two new icons (the symmetric transformer and the dotted x-junction). During that period I was also helped by Karin, who wrote an award-winning thesis about evolution-criteria in Energetics, combining Non-equilibrium thermodynamics with electrical network theory and energy bond graphs. Gradually, I was beginning to see, that the most important merits of the formalism were going to lie within the teaching of pure physics, rather than in ecology and engineering, and Jesper Gundermann made a real breakthrough for this line of thinking with his master's thesis "Energy Bond Graphs in General Physics" (1974)).

I can hardly overstate the importance of Jesper's thesis for the development of my project. Suddenly I had the theoretical basis for an EB-formulation of a general response-theory as well as Analytical Mechanics. Jesper also made a computer-implementation of a compiled programming language ENSILO (Energy Network SImulation Logic) that made it possible to generate the symbolic equations of a model based on a description of the topology of an energy Bond Graph on punched cards, as well as a solution of the equations. I never used the language, for at that time I had developed my own program CTS (Continuous Time Simulation) for numeric integration and graphical representation of dynamical systems. It was always very important for me to check out my ideas numerically, and using Forrester's System Dynamics and the associated simulation-language DYNAMO quickly proved insufficient. Later, after my transfer to RUC in 1985, I made in collaboration with Heine Larsen a PC-version of CTS, which has served me and many students well ever since. Today, after recent software-developments (Windows, etc.) the DOS-CTS-version has become obsolete, but the ideas behind it are still valid as a general philosophy of dynamical modelling, and I have therefore included a short description of CTS as an appendix to section I.3 of this thesis.

My colleagues at Copenhagen University were not very understanding of my efforts (except Erland and Karin). Some could simply not see anything of value to modern physics, and some tried to ridicule it as "birdhouse-physics"). However, Niels Boye Olsen, who at this time was still in Copenhagen, liked to discuss modelling-philosophy with me. Boye had a peculiar ability to make his ideas visible for me, and after many coffee-table-discussions with him, I became used to visualizing my theoretical ideas (e.g. of superconductivity) as diagrams and analog computer programs.

From Copenhagen to Roskilde

In 1978 I got the first possibility of a (non-permanent) position at RUC, and here I found a much more fertile field. The climate at RUC was much more inter-disciplinary and philosophical than in Copenhagen, and this suited me well. I had, together with Jesper, Erland, and Lars Josephsen, undertaken an english version of the lecture notes for the Ecophysics-course. We never got to the energy-bond formalism, but Lars paved the way, discussing dynamic modelling and diagrams and shortly introduced Peirce's semiotics. I began to introduce Peirce to many audiences, mainly in a philosophical study-group "the HH-circle", I had joined in 1975, and these ideas found a fruitful resonance with the biologists Søren Brier, Claus Emmeche, and Jesper Hoffmeyer and thus became the germ of a new international research-program, called "Biosemiotics".

When I first started at RUC in 1978, Boye was supervising the first group of students, preparing their master's thesis in experimental physics of viscous liquids. I was chosen as co-supervisor, and I undertook lecturing in two courses, the first an introduction to the EB-formalism and dynamic systems in general, the second about linear response and noise. For this purpose I made my first two texts in IMFUFA's text-series: no. 8 (1978) and no. 22 (1979). The students were quite interested, and the first group quickly found a way to design a method of measurement of frequency-dependent shear viscosity by using EB-modelling.

Modelling discussions

The verb "*to reticulate*" introduced by Paynter for the act of making network-models quickly became accepted in the frequent discussions of modelling at IMFUFA, but the EB-formalism was generally considered too complicated, compared to standard mathematical methods.

During my 25 years at RUC I have made four EB-texts, all used as lecture notes for courses:

Text no. 8, (1978) "Dynamics and Diagrams"
Text no. 22 (1979) "Linear response and noise"
Text no. 238 (1993) "Vector Bonds and Tensors"
Text no. 296 (1995) "Reticulate Classical Mechanics"

After having successfully developed the theory of linear response for 1-port systems in text 22, it was my intention to follow up with a generalization to multi-port-systems. I thought that it would be rather easy to generalize the scalar response-function to matrix-response-function, but the process of generalization turned out to be an enormous jigsaw-puzzle with a lot of surprises and obstacles. First, I had to realize the difference between *matrices* and *tensors* and to develop the formalism accordingly. Second, I discovered the *protensors*, that are neither matrices nor tensors. This was a hard struggle, but I had the unexpected gain of discovering that the new formalism satisfied the principle of relativity, and that both special relativity and parts of general relativity came tumbling out of it.

All these texts are written in danish, so when foreign guests sometimes were puzzled about how our experimentalists, Boye and Tage, were able to see through to the right solution to some problem, we could just tell them, that we had a "secret weapon" in the EB-formalism. This situation is now, hopefully, been remedied by this thesis, which contains english translations of large parts of the above mentioned four texts (except the first two, that were written before the age of electronic word-processing).

There is one fifth text, though, (no.419, (2003)), that was written in english, because it should present the formalism to an international audience at the third International Gathering in Biosemiotics in Copenhagen, july 2003. This final text "Energy Bond Graphs — a Semiotic Formalization of Modern Physics" gives a "bird's eye view" of the merits of the formalism and is therefore included as part I.2 in this thesis. Text 238 is fully included (in my own translation) as part II, and the first three chapters of text 296 is included as part I.3 together with an appendix about my CTS-program.

Quantum Semiotics

The next step, obviously, would be to incorporate Quantum Mechanics in the EB-formalism. Being brought up with Quantum Mechanics at the Niels Bohr Institute in Copenhagen I shared the common view among danish physicists, that the philosophical problems of Quantum Mechanics had all been solved by Niels Bohr in 1927-35, but working with EB-formulations of partial differential equations had given me a "local-realistic" philosophy, and I could not accept, that the locality-principle had been disproved by the violation of Bell's inequalities in experiments by Aspect et al. based on Einstein's Podolsky's, and Rosen's paper from 1935. Bohr's reply to EPR (also 1935) seemed to me vague and unconvincing. We had heavy discussions about this in the HH-circle from 1975 to

1982, until it finally became clear, that I had lost a wager about the outcome of Aspect's "switching-experiment" in 1982. This was the first time I experienced the benefits of having one's pet-theory unambiguously falsified. I quickly got the idea, that the seemingly non-local "entanglement" demonstrated by the violation of Bell's inequalities was due to the connectednes of the experimental set-up, as all registrations are made in one or more centrally placed coincidence-counters. I proposed a similar experiment without coincidence-counters and predicted that Bell's inequalities would be satisfied in such an experiment. I presented this idea in several danish articles and in discussions with the brillant science-journalist Tor Nørretranders, but I could not get any international journals to accept a paper about it. I then wrote an english article as IMFUFA text no. 93 (1985) "the Semiotics of Quantum-Non-Locality", where I, realizing that my intuition was due to my preoccupation with energy bonds, tried to put a semiotic perspective on the Quantum-measurement-process. Besides my original proposal of an Aspect-experiment without coincidence-counters I included two other proposals, because I thought they would be easier to perform. The simplest of these was to use an asymmetric set-up, such that one photon had a longer way to detection than the other, but shortly after Tor was able to show me in Aspect's unpublished thesis, that Aspect had already done it without observing anything but the same violation of Bell's inequalities. I had to stick to my first proposal of removing the coincidence-counters. At two international conferences in Joensuu, Finland, 1987 and 1990 about the philosophy of Quantum Mechanics I met the experimentalist Anton Zeilinger from Vienna and discussed the questions with him. At the second meeting in 1990 I finally convinced him, that the violations of Bell's inequalities did not necessarily indicate a violation of the locality-principle, but could be interpreted as a sign of *contextuality*, i.e. that the detection of the separated particles were made in a centrally placed coincidence-counter. Zeilinger could now see the point in making an experiment without coincidence counters, and in 1998 he announced to me, that such an experiment was now going to be performed by his current group of co-workers "the group of experimental metaphysics" in Innsbruck.

The angel over Innsbruck

I quickly went to Innsbruck to see the experiment with my own eyes, and I was presented with the *fait accomplie*, that Bell's inequalities had been shown to be violated once more, contrary to my expectations. I had no objections to the experimental procedure, so again I had to accept, that my idea had been falsified. Again I experienced immediately the benefits of such falsification. I had been invited to give a lecture about my ideas in Innsbruck, and after the lecture, when discussing with Zeilinger, I could suddenly see, that the non-locality, I had been

forced to accept, In fact existed as a possibility within the EB-formalism. This realization then gave the final spur to my development of, what I had come to call "Quantum Semiotics" and gradually developed in hand-written notes of "scientific diaries". Although this new conception of Quantum Mechanics is still far from being completed I have decided to present an outline of it as part III of this thesis. This part then summarizes some main results from text no. 93 and my diaries, and I hope it is sufficiently clear to see, that Quantum Semiotics is a reasonable alternative to current interpretations, like the Copenhagen interpretation and the "many-worlds-interpretation" of Quantum Mechanics. I have not yet developed a proposal for a falsification of these new ideas, but I hope that its ability to remove paradoxes and unfalsifiable propositions (like "many worlds") will prove convincing.

Structure of the thesis

Part I of this thesis has the headline "physicizing". Our modelling discussions were often centered about the process of "mathematification", a word that soon degenerated to "mathematizing". I thought that physical modelling ought to be considered as different from mathematical modelling, and that the EB-technique could be a paradigm of "physicizing". Now, application of energy bonds quickly becomes a mathematical exercise, especially when it is used for entering theoretical disciplines, like special relativity or analytical mechanics. However, I think, that a more direct, intuitive way of using the EB-icons for describing physical systems should not be obscured by heavy mathematics, and I have therefore emphasized this "direct" approach in part I, especially in section I.3 (the three first chapters from text 296 and in the appendix to part II (also from text 296)

The solution to the paradigmatic problem "a sphere rolling on a rotating disc" is presented in I.3 by the direct kinematical consideration of velocity-constraints. The surprising fact, that the sphere describes a closed trajectory, when seen from the laboratory-system, is in this way seen much more easily than by a dynamic treatment in the rotating system with its centrifugal and Coriolis-forces would be able to.

Part I.2 (is text no. 419) is also rather free from mathematics, as it was intended for biosemioticians, but it strikes a more philosophical note about reductionism and laws of nature.

Part II, "generalizing" consists of my translation of text no. 238, "vector bonds and tensors". This is, indeed, heavy mathematics, not because I intended it to be so, but because the process of generalizing the simple technique required it.

Part III, "quantizing" Is my first attempt to present the, as yet, incomplete theory of Quantum Semiotics (QS). Unlike part II, where disciplines like special relativity and analytical mechanics are really derived from the EB-formalism, in part III I take standard Quantum Mechanics for granted and use it, e.g. for deriving the Fluctuation-Dissipation (FD)- theorem — a theorem, that, in my opinion, deserves much more interest, than it usually gets from physicists. QS is here presented as a quantum *philosophy* and tries to address such questions as:

What is a quantum-process?
Is the wave-function real or fictive?
Is the "collapse of the wave-function" objective?
Is there a fundamental level of description?
How can we reticulate the measurement-process?
What is non-locality (EPR-experiments)?
Null-measurements?

Acknowledgements

I have never been able to make the mathematicians at IMFUFA interested in the EB-formalism, and this is a problem for me, as I easily get stuck in mathematical complexities (and probably have made some mathematical errors). However, I wish to thank my colleagues in physics, Jeppe Dyre, Tage Christensen, and Niels Boye Olsen for their never faltering interest and clever applications. my sons Morten and Jesper Voetmann Christiansen have helped me a lot by installing new computer hardware and adapting it to my old software. They also helped searching the internet.

Last, but not least, I wish to thank my wife Else Straarup for her loving patience with me and her constant encouragement.

Lejre, october 2003,
Peder Voetmann Christiansen.

I.2 Energy-bond-graphs - a semiotic formalization of modern physics.

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Biosemiotics, 3. International Gathering, Copenhagen, july 12th 2003

1. Scope of the formalism

An Energy Bond Graph (EBG) is a comprehensive diagrammatic representation of all the relations that constitute a dynamical system. As such, it is a mathematical model, but not just mathematical. The EBG formalism requires a total reduction to physical standard relations that exist both as hardware and as software. It thus resembles a program or a wiring diagram for an analog computer representing both a set of equations and a construction-recipe for a real system that acts out the equations. EBG modelling is a "glass-bead-game" - a "fundamentalist" way of doing physics (proceeding from real fundamentals), like Peirce's Existential Graphs are to logic. Peirce said in his second Lowell Lecture from 1903 that he had spent forty years of his life developing the existential graphs and he described its purpose in the following way that may just as well serve as an introduction to the EBG-formalism:

> "Before beginning, let us distinctly recognize the purpose which this system of expression is designed to fulfil. It is intended to enable us to separate reasoning into its smallest steps so that each one may be examined by itself. Observe, then, that it is *not* the purpose of this system of expression to facilitate reasoning and to enable one to reach his conclusion in the speediest manner. Were that our object, we should seek a system of expression which should reduce many steps to one; while our object is to subdivide one step into as many as possible. Our system is intended to facilitate the *study* of reasoning but not to facilitate reasoning itself. Its character is quite contrary to that purpose."

The EBG-technique is not an easy way to mathematical models in physics, because it forces the user through an initial phase of semiotic reflections on categorization of variables, etc. This initial work, however, will lead to insights that are not easily gathered from more direct modelling techniques.

2. Classification of dynamic variables.

There are four kinds of dynamic variables in the EBG-formalism, and these are conveniently displayed in a Greimasian semiotic square. This method needs two pairs of binary opposites. The two members of a pair of opposites are placed diagonally against each other in the corners of a square, and then the four sides of the square identify the possibilities.

The two pairs of opposites are:

1: Level/rate, where the level-variables are accumulated, i.e. they can only change by time-integration of associated rates, whereas the rates may change abruptly.

2: directed (x)/undirected (o). Directed variables have direction both in space and time, i.e. they require a spatial orientation-convention and they change sign by time-reversal. Thus, we get the semiotic square of figure 1:

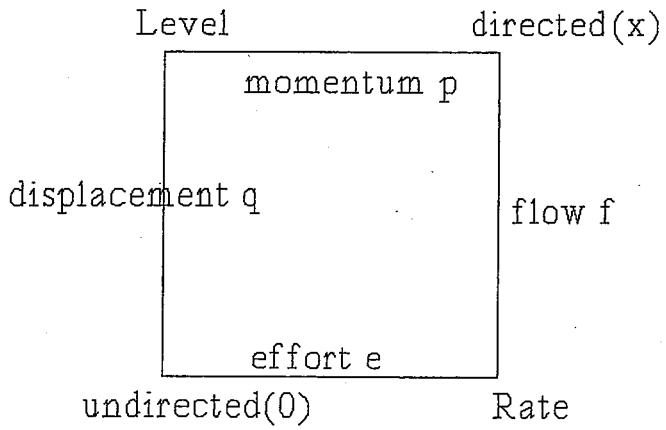


Figure 1. Semiotic square of dynamic variables.

In EBG diagrams the levels are associated with "bird-house" storage icons (inspired by H.T. Odum)[1] marked x for kinetic energy and o for potential energy, while the rates are marked on the line-icon of an energy-bond with an arrow for the directed flow and a stroke for the undirected effort. The consideration that efforts (forces) are undirected reflects a deep law of classical mechanics, namely Newton's law of action and reaction.
Figure 2 below shows how the same system - a harmonic oscillator - is depicted by the present author (a) and with the more austere drawing style of the EBG-formalism's inventor H.M. Paynter (b).[2]

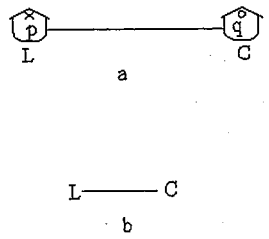


Figure 2. Harmonic oscillator.

3. Shape-value-notations

The difference between the two diagrams in figure 2 illustrates the concept of a *shape-value notation* (a) where the shape of the icons is chosen such as to convey an idea of their value or function and a *symbolic* notation where symbols like digits or letters are used to replace proper icons (b).
The present formulation was inspired by Odum's shape-value notation for Ecological Energetics[1], but it was felt that the lack of physical precision in Odum's definitions of his icons could be remedied by using Paynter's bond-graphs[2]. However, Paynter did not attempt to create suitable shapes for his icons and reverted to a symbolic notation, which lead to some ambiguities. By using, e.g., the letter T as an icon for a transformer/transducer one misses the fact that a proper definition of the parameter of the object requires a distinction between its *primary* and its *secondary* port, and this distinction cannot be brought out by a symmetric shape like T, but only by an asymmetric icon (except in special cases), like the triangular shape in figure 6.

Some icons have associated parameters, denoted by symbols. The shape-value notation demands that the meaning of these symbols shall be uniquely defined by the shape of the icon. This demand means that it is sometimes necessary to have two different icons for the same physical function, as with the gyrators and the sinks shown in figure 6.

Attempts to create shape-value-notations for mathematics and logic have been studiously ignored by mathematicians, who for the last hundred years have preferred to continue in the trend laid out by Hilbert's shapeless axiomatization of Geometry. Thus was neglected Peirce's construction of a shape-value-notation for the sixteen binary logical connectives[3] Peirce's notation was rediscovered by Shea Zellweger who has now developed it further to a complete "logic alphabet"

thereby stressing the merits of shape-value notations for educational purposes[4] Peirce's early version of his existential graphs (the alpha graphs) was also a shape-value notation of logic and a precursor of Spencer-Brown's "Laws of Form" from 1969.[5]

By developing the EBG-formalism as a shape-value notation the present author hopes to have helped strengthening an important historical strain that may prove valuable to Physics and Biosemiotics.

4. Basic response properties.

In a response experiment we act on a system through an energy bond, which, as we have seen, contain four variables, q, f, e, and p. These four variables are not independent, because the levels q and p are time-integrals of the rates f and e. In the experiment we leave the system undisturbed from time $-\infty$ to time 0. Then we choose one of the four variables as *stimulus s* and let it have a constant value from time 0 to ∞. As there are two causal groups of independent variables we can choose s from one group (e.g. (f,q)) and then observe the response r(t) as one of the variables from the other group (e,p). In the response-matrix shown in figure 2 there are thus two windows of possible response-properties, each containing four functions, but it can be shown that for *time-homogeneous systems* where the result does not depend on which instant we choose as time 0, two of the four functions in each window will be identical. Any physical system therefore has six different response-properties.

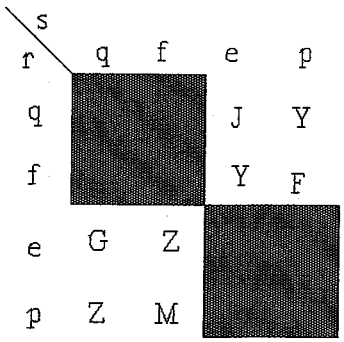


Figure 3. The matrix of response-functions.

Response-semantics, i.e. the choice of proper nouns to denote the response-

properties may then proceed from the *response-semiotics* of figure 3. In the following table we have combined usually used nouns with the six two-link Peircean sign classes: Read, e.g. (23) as "second of third".

(11) G: *Rigidity* or *elastic modulus.*
(12) Z: *Impedance* or *resistance,*
(22) M: *Inertance* or *Inductance.*
(13) F: *Lightness* or *Susceptibility.*
(23) Y: *Conductance* or *Mobility.*
(33) J: *Compliance* or *Capacitance.*

We have here defined the response-functions as functions of time from 0 to ∞, but by Laplace-Stieltjes-transformations they are defined as functions of a complex frequency s= -iw. These complex functions are usually just called *generalized susceptibilities.*
However, there are many advantages in distinguishing the different functions by different names because they are mathematically related by time-integration: G integrates to Z, Z integrates to M, and similarly for the set F, Y, J. In the frequency- (s-) domain time integration corresponds to division by s. Thus, by this semiotic/semantic approach it becomes possible to identify and name a response-property after a quick glance at the oscilloscope. And a careful distinguishing by names serves to develop the somewhat mysterious ability called "physical intuition".

5. The energy bond, causality and orientation.

The most basic icon of the EBG-formalism - *the energy bond* - is drawn as a simple line. It denotes an energetic *interaction* between two system-components. The interaction is mediated by the two rate-variables that belong to the bond: the directed *flow*, f, indexed as an arrow and the undirected *effort*, e, indexed as a stroke. If system A acts on system B with an effort, then system B acts back on system A with a flow. This causality is then indicated by placing the effort-index closest to B and the flow-index closest to A. By using an arrow as the flow-index we are forced to choose an *orientation* of the bond. Every bond may have either orientation, but when we want to view the same physical situation with the opposite orientation, we have to change the sign of the symbolic expression for the flow (f changes to -f). The interaction between A and B described above can thus be depicted in the two ways shown in figure 4:

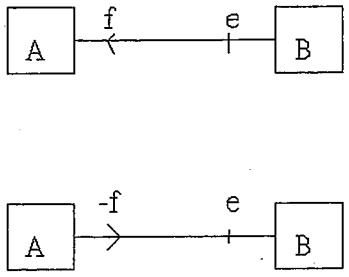


Figure 4. Interaction and orientation.

It must be stressed that the causality of an energy bond is not just a formal consideration made necessary by the algorithmic ordering. There exists a precise measurement prescription to decide the causal order of the rate-variables for each bond, based on the analysis of transients or noise.

Every energetic interaction is associated with a flow of energy. The energy flow in the direction of the orientation of the bond is the product of effort and flow.

In general effort and flow are defined as *vectors in a complex metric space of arbitrary dimensionality*. The energy flow is then defined as the *scalar* product of the effort- and the flow-vector. This generalization, however leads to the complication that the vectors must be represented as either *covariant* or *contravariant*. The flow-orientation rule shown in figure 4 then has to be modified such that the change of orientation is associated with a shift of variance of both the effort- and the flow-vector. By adopting the further rule that *the same physical situation may be described with either variance of the effort* (but not of the flow), the formalism becomes able to treat vectors of mixed kinds where some of the effort-components are flows and the corresponding flow-components are efforts. This feature allows for a very general treatment of response-experiments, where, e.g. the celebrated *Onsager symmetry relations* becomes a theorem of the EBG-formalism.

The general vector-formulation also demands that *each bond is associated with a metric tensor* that relates the covariant to the contravariant vectors. A unit metrical tensor is represented as a matrix with 1s in the diagonal and 0s outside. This is *Euclidean metric*. When coordinate vectors are defined by three spatial coordinates and one time-coordinate the flow-vector's first three components will be flows and the fourth will be an effort. This corresponds to a metric tensor with 1s in the three first places of the diagonal and -1 in the fourth place - *the*

Minkowsky-metric. The group of metric-preserving transformations for this metric are *the Lorentz-transformations*, and the whole formalism of special relativity follows from this. Likewise, by adopting the general tensor-formulation of *Riemannian metric spaces* the theory of general relativity also follows from the EBG-formalism.

6. Definitions of basic icons

EBG-modelling always proceeds through three semiotic stages: from *icons* through *indices* to *symbols*. Each icon denotes a basic physical relation between the input and output variables of the associated bonds (the *ports*). When indices - flow-arrows and effort-strokes are marked on the bonds, symbolic expressions for these variables are written near the indices, whereby the meaning of the symbols and the function of the icon is defined. Note that *nearness* is itself an indexical element inherent in the meaning of the symbols. Symbols near the icons are also used for defining certain parameters of the icons (like storage-capacity and transformer-ratio).
The basic icons are clearly divided in three classes corresponding to Peirce's three phenomenological categories:

1: *Active* systems (sources of flow or effort) have an output-variable that is independent of the input-variable.

2: *Passive* or *reactive* systems have an output that is determined by the present or the previous values of the input.

3: *Dissipative* systems (the sinks) mediate between active and passive behaviour. Their response is mainly passive, like the voltage over a resistor, given by Ohm's law, but the passive response is superposed with an active component - the *noise* - that depends both on the sink-parameter and the temperature.

In figure 5 we show definitions of the active and the reactive systems.

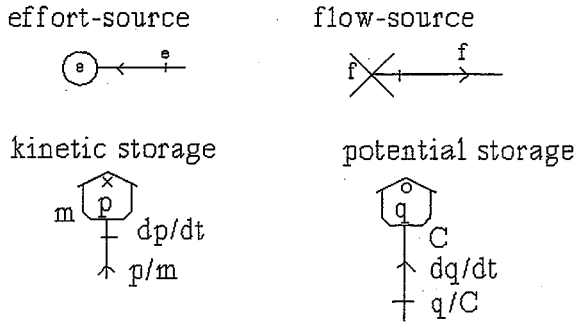


Figure 5. active (sources) and reactive systems (storage).

Figure 6 shows the sinks and two passive 2-ports, transducer/transformer and gyrator. The definition does not distinguish between transducer and transformer, but the name "transducer is used when the parameter t has a physical dimension, whereas transformers are dimensionless. Transformers and gyrators are very different although they look similar. The parameters t and g may be given as level-variables elsewhere in the system, but then t must be an undirected level, whereas g must be directed. Such *parametric feedback* is useful for modelling nonlinearities.

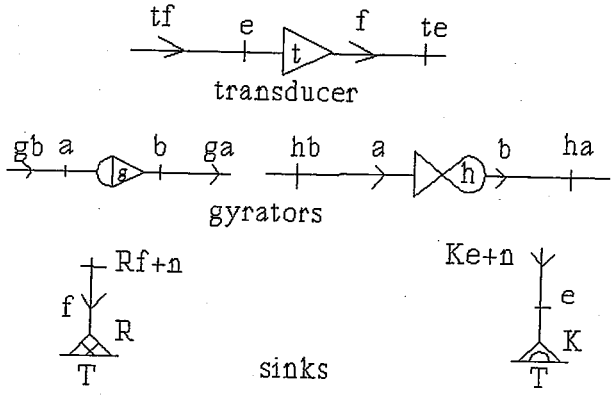


Figure 6. Passive 2-ports and sinks (n denotes noise).

Finally, figure 7 shows the *junctions* as 3-ports, thus representing the only *triadic* relations of the formalism. The junctions are *topological* constraints corresponding to Kirchhoff's two laws of electrical networks. The o-junction corresponds to a node in the network (parallel connection) and the x-junction to a mesh (series connection). The similarities between o- and x-junction reflect the nearly dual symmetry between efforts and flows, but the dual symmetry is broken by the fact that flows are directed, while efforts are not, and this leads to subtle differences between the junctions, which, however, we shall not discuss here.

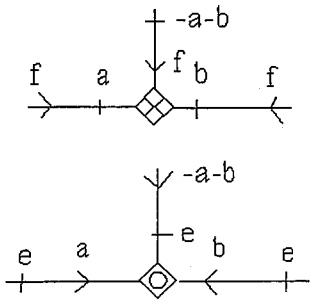


Figure 7. The junctions.

7. Beyond laws of nature

A strange consequence of the EBG-game is that what we usually regard as laws of nature seem to vanish out of scope. Instead we have the rules of the game, which ensure that the laws of nature are obeyed. The vanishing of laws is thus a result of mathematization and semiotization. Similarly, in elementary physics the pupils learn the law of nature that forces are combined by the construction known as the parallelogram of forces, but in more advanced teaching they are just taught that forces are *vectors*, so the former law of nature becomes the rule for adding vectors.

We have seen that Newton's law of action and reaction hides within the rule that efforts are undirected or the flow-orientation rule of figure 4. Similarly, we can find Newton's second law of motion (force equals mass times acceleration) in the definition of the x-storage icon of figure 5, where m is the mass, p the momentum, e the force, and f the velocity (*p/m*). It was also mentioned that Onsager's symmetry- or reciprocity-relations is a theorem of the EBG-formalism. It is therefore not necessary for users of the formalism to know this as a law of nature, because it is automatically satisfied by all EBG-models that are made according to the rules.

Certain combinations of icons are not allowed, because they lead to *causal conflicts*. It is absolutely forbidden to connect two effort-sources directly or through a transformer. The sources are absolutely rigid in their causality, while other systems may be forced to yield. By connecting an effort source to a o-storage a mild conflict arises where the storage element is forced to give in and accept "differential causality". In this way we are allowed to ascribe a conductivity to an electric, capacitor C, but the conflict still shows itself in the fact that the frequency-dependent conductivity *Cs* goes to infinity for large frequencies, which means that the causality will break down if the effort of the source changes very rapidly. Causal conflicts may often be resolved by introducing extra sinks. This happens, e.g. when a car tries to accelerate too fast;

the wheels will slide on the road - a sink has appeared to represent the friction between wheel and road. In the traditional way of describing physical systems - by equations - there is no formal treatment of causality, so the concept of causal conflict does not exist, but in reality it plays an important role, especially when systems break down.

Many laws of nature are formulated as partial differential equations, e.g. the diffusion- and wave-equations, Maxwell's electromagnetic equations, and the Schrödinger equation. Such equations are in EBG diagrams shown by combining icons, each representing an infinitesimal section of space in structures that are repeated in three dimensional space like the atoms in a crystal. As an example of this figure 8 shows the unit cell for representing Maxwell's equations in vacuum. Not shown in the diagram are storage elements connected to the junctions: x-storage to x-junctions representing the magnetic induction-vector , and o-storage to o-junctions representing the electric displacement-vector . The electric field and the magnetic field are represented, respectively, as the efforts and the flows in the bonds.

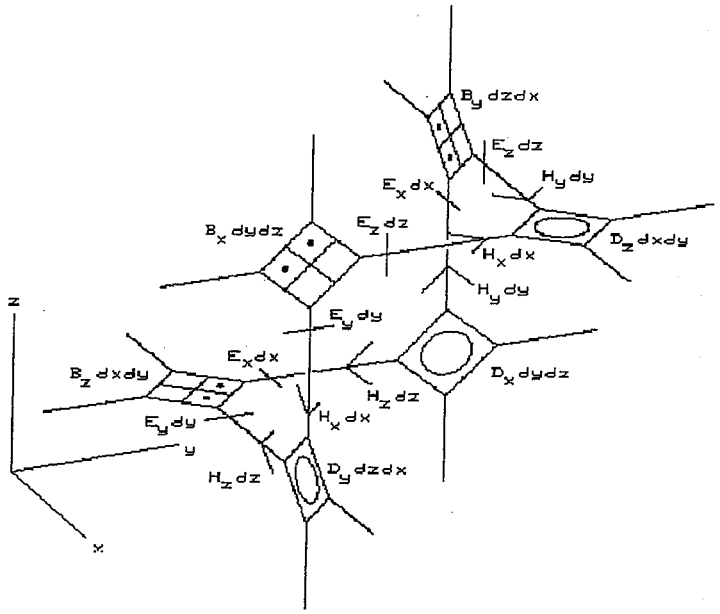
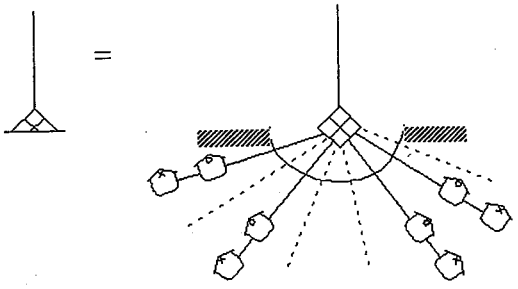


Figure 8. Unit cell for Maxwell's equations.

As mentioned, the sinks are sources of noise. This important insight follows from the *Fluctuation-Dissipation Theorem* by Callen and Welton (1951)[6]. As shown in figure 9 and the associated formula the complete complex impedance function $Z(\omega)$, $(w=is)$ may be spectrally resolved on the response-functions of harmonic oscillators. The real, or dissipative part of the impedance $R(\omega')$ is proportional to the density of oscillators on the real axis of their resonance frequencies ω'. The spectral resolution formula is then valid when ω has a positive imaginary part.

This formula is translated to icons in the figure and gives justification for the choice of the sink-icon. The noise is simply the output from all these uncorrelated oscillators in thermal equilibrium with the temperature *T*. For an ohmic resistance the oscillator-frequencies will be equally distributed over the real axis, and the noise-spectrum will be *white*.



$$Z(\omega) = \frac{i}{\pi}\int_0^\infty R(\omega')\left[\frac{1}{\omega-\omega'} + \frac{1}{\omega+\omega'}\right] d\omega'$$

Figure 9. Spectral resolution of a sink.

For a frequency-independent resistance *R* the prescription of the FD-theorem is simply that for each step *dt* of the numerical integration one has to add the effort-noise

$$n = N(\sqrt{(2RkT/dt)})$$

to the passive response of the sink. Here N(x) is a normally distributed random number with mean value 0 and standard deviation x. *k* is Boltzmann's constant, and *T* the absolute temperature. The formula is valid in *the classical limit*, when $dt >> \hbar/kT$.
It is noteworthy that the noise diverges to infinity when the step length *dt* goes to zero. The noise is, as Peirce said, "infinite in the here-and-nowness of immediate sensation, finite and relative in the recency of the past". (CP 6.135).

Every EBG-model containing dissipative elements thus becomes "animated with noise". It is easy to show that the level-variables of the system in this way get the exact fluctuation-moments that are prescribed by Statistical Mechanics, which in this way also becomes a result of the EBG-formalism.

The above expression for the effort-noise n demands that the sink-parameter R be positive. This is a further axiom of the formalism and it has the result that the energy flow to a sink, Rf^2 is always positive, thus ensuring that the laws of thermodynamics are always obeyed by EBG-models.

8. Completeness of the formalism

Within *Physics* and *Engineering* the EBG-formalism seems to be complete. Although new icons can be freely invented and added to the basic icons it seems always possible to make a full *reticulation*, i.e. to reduce these new functions to the basic icons. In practice it is easier to use a combination of basic and composite icons. The icons used by the author, shown in figure 10, are defined once and for all in an object-oriented drawing program (DrawPerfect 1.1)

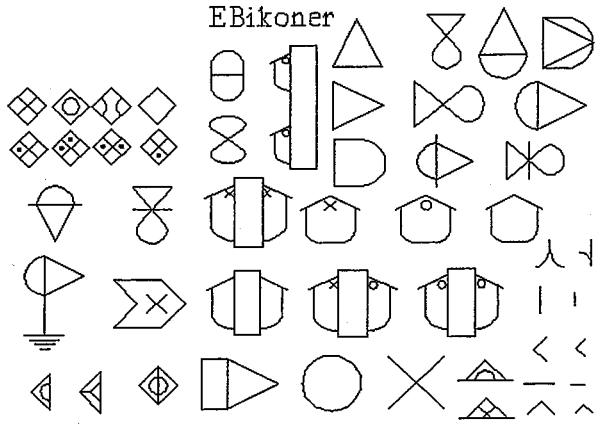


Figure 10. Iconic objects of the EBG-formalism.

A full proof of completeness is difficult to find (if not impossible), but many special cases have been considered[7] and algorithmic prescriptions for their translation to the EBG language have been found, of which I shall just mention a few:

1 *Chemical reactions* forwards and backwards were first described by a reaction-icon based on H.T. Odum's *work-gate-icon*, but it was later shown that the reaction-icon can be reticulated by sinks and gyrators. In this way it is possible to make EBG-descriptions of large biochemical reaction-networks, like photo-synthesis.

2 Topology of electrical and rheological networks are easily translated to EBG diagrams of junctions. It turns out, however, that the x-junction is a series connection in electricity but a parallel connection in rheology (and vice versa for

the o-junction).

3 Linear response can be reduced to electrical networks [8], and from there to EBG models. So in this case a full proof of completeness exists. On the other hand it is not always possible to translate EBG diagrams to electrical networks. (It may lead to short-circuits if one tries).

4 All systems that can be described with the Lagrangian and Hamiltonian methods of classical analytical mechanics can be translated to EBG-models by a relatively simple algorithm. So in this case the formalism is also proven complete, and it turns out that the EBG-models are generally more efficient for simulation than the Hamiltonian equations.

5 All relations exhibited by the basic icons are linear, but, as mentioned earlier, non-linearities may be introduced by parametric feedback from levels to transformers and gyrators. Such non-linearities will contain a time-delay, but many systems of classical mechanics require *simultaneous* non-linear relations. Such relations can also be given by an EBG diagram. In figure 11 it is shown how a simultaneous rendering of the relations s=sinè and c=cosè is reticulated:

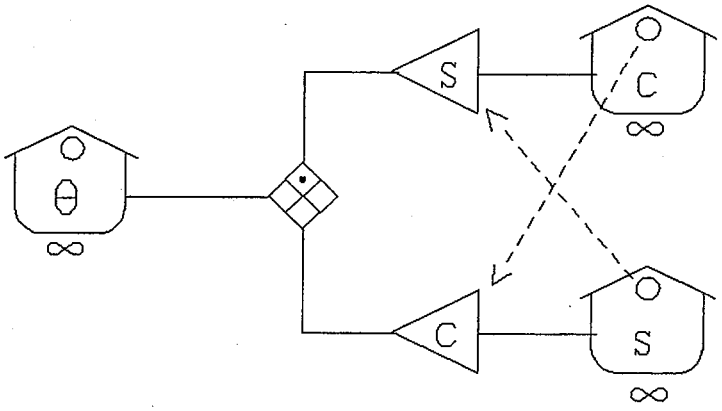


Figure 11. Simultaneous reticulation of c=cosè and s=sinè.

Finally, figure 12 shows how a complicated mechanical system - a double-pendulum - is fully reticulated with all its cosines and square-roots:

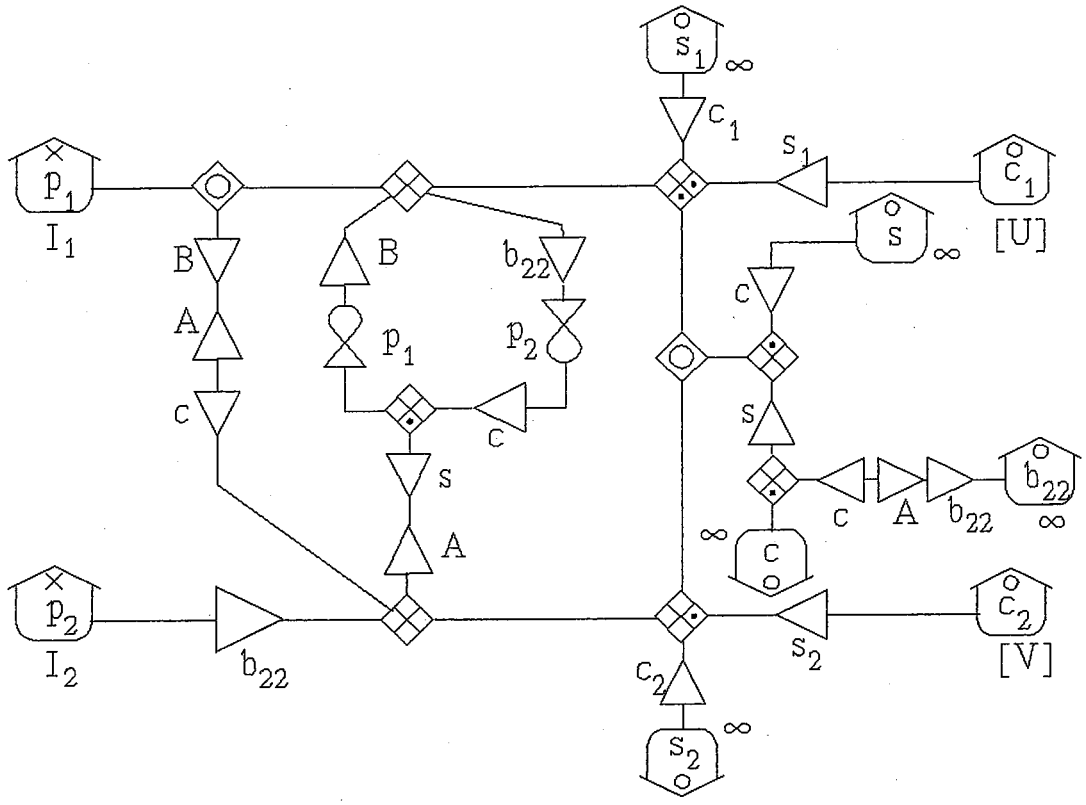


Figure 12. Full reticulation of double-pendulum.

9. Perspectives

Some biosemioticians are apprehensive about the dangers of physical reductionism. It must be admitted that EBG-formalism is an example of such reductionism, and even that it tries to carry physical reductionism to an extreme (by reducing basic physical concepts to even more basic semiotic concepts). Against such warnings I have several comments:

1) Kurt Gödel carried logical positivism to an extreme and found something on the other side of great importance to mathematics and philosophy.

2) An EBG-model is always open to expansion into the environment, except when the underlying theory demands a closure.

3) Regarding Biosemiotics: An EBG model is a sign of a general idea and thus, as Peirce said (CP 6.270) a living entity analogous to a person. The variables in an EBG-model are always superposed, both with thermal noise and quantum-fluctuations, that may be regarded as "living feeling" and spontaneity. If it is reductionistic to use such models in biology, it is, at least, not a simplistic kind of reductionism.

4) Regarding Quantum Mechanics: The EBG-formalism has a strong affinity to Quantum Mechanics, as it has to Relativity. If it becomes possible to construct an analog computer that can act out all the simultaneous feedbacks of EBG-models, it will be a quantum computer. The non-local constraints exhibited by the junctions give a simple explanation of Quantum-Non-Locality (the Einstein- Podolsky-Rosen experiments). Many other seemingly counter-intuitive quantum mechanical effects also appear as natural and understandable when viewed in the light of EBG-reticulation of the physical sign-relations. A full investigation of this research program, called *Quantum Semiotic*, is in progress, but may take many years to complete.

Lejre, july 1. 2003

Peder Voetmann Christiansen.

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I.3 Basics
1: Semiotics of the energy bond

1.1. The three sign-classes

A *sign* is something that means something for somebody. The sign is, according to Peirce a *triadic relation* between 1. a *sign-vehicle*, that mediates between 2. *an object* and 3. an *interpretant*. According to the character of this mediation we distinguish between three main classes of signs:

1. An **icon** mediates a *potential* relation between object and interpretant, i.e. a relation, that "is not really there". It may, e.g. be, that the object is something purely speculative, as an angel, or a dragon, or a mathematical straight line. We have an iconic conception about, how these objects "look like", but we only know them as pictures, not as material things.
   We may therefore also say, that an icon is a "picture-like sign", but this must be understood in the way, that the object (if it exists at all) is "put into the shadow" of the icon. Icons are the main subject of mathematics. Physics employs icons as diagrams, curves, graphs, that form the basis of mathematization.

2. **An index** mediates an *actual* relation of meaning. An index, though, has no general meaning. It just shows, that the object exists, and it serves to distinguish an object from other objects. A pointing finger is an index (in fact the proper translation of the latin word "index") for the object, it points out. A CPR-number is an index for a specific citizen in society. A footprint in the sand is an index for a person or an animal, that went there. A portrait-photograph can be regarded as an index for a certain person, because we can account for an actual causal relation between the person, the light, the lens, the emulsion, and the development, that has produced exactly this picture. A painted portrait will be less indexical and more iconic than the photograph.

3. A symbol mediates a *general* meaning from the object to the interpretant. I.e. the sign-relation for the symbol is *habitual*, based on learning a rule. Words are symbolic, because we only understand them by having learnt the language. If, e.g., we see the sign "O", we may conceive it iconically ("a circle") or indexically ("probally made by a laser-printer"), but it is only a symbol, when we see it as a member of a general class of signs that exist in many variants and qualities, but where the conventional belonging to the class is the most important.

The three categories of sign-mediation are not on equal footing: A higher category

must necessarily be based on and contain the lower. A symbol, that is a general sign, must necessarily be built up via something iconical and something indexical. First, the connection between object and interpretant must be potentially present, i.e. there must be an icon, before the connection can be actualized, so that a specific, indexical pointing out of the object can take place. And a general mediation, the symbol, referring to a class of objects, must necessarily build on an indexical reference to something individual.

Consider, e.g. the diagram below of an electric network (a "Brune-transformer" from an examination in electrodynamics, january 1995). The figure shows icons for wires (lines), a voltage-source (the circle), an inductive transformer with two coils and an iron-core, a capacitor, and a resistance. The symbols V, L_1, L_2, M, C, and R refer to physical quantities (numerical values and units of measurements) for, respectively, voltage, self-inductances, mutual induction, capacity, and resistance, but if they were just arbitrarily placed on the paper, they would not have the symbolic meaning. The symbols get their meaning by their *proximity to the icons*. The proximity (nearness) is the indexical element in these symbols. The voltage-symbol V also requires an index for *orientation*, as it denotes a difference in electric potential between the two poles of the voltage-source, and here the up-down-dimension of the paper serves as an orientation-index, as it is understood that the upper terminal is "the plus-pole". For the two current-symbols I_1 and I_2 there are, besides the nearness-index to wire-icons, also denoted orientation-indices in the shape of two arrows. That, which the two arrows point out, is *a choice between two possible directions on the wire*, and without this indication of direction the meaning of the current-symbol would be unclear.

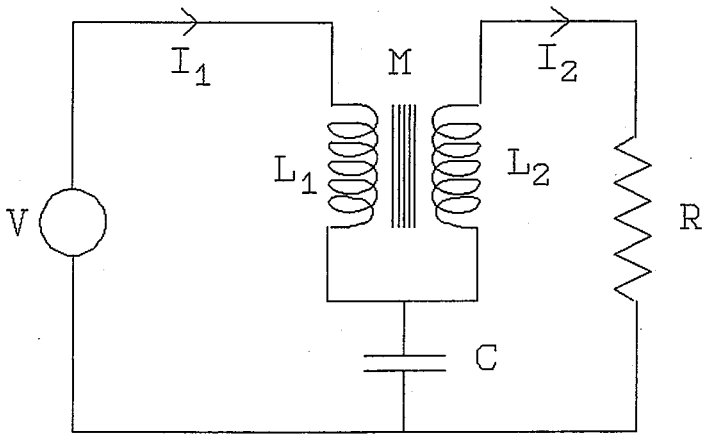


Figure 1. Electric diagram with icons, indices, and symbols.

1.2. The flow-orientation rule.

The symbol I_1 in figure 1 stands for that current (flow), an amperemeter would show, if it were inserted in the wire, such that the direction through it from the plus-pole to the minus-pole is in the direction of the arrow. If we reverse the arrow, but stick to the measurement-prescription, we shall also reverse the amperemeter, and it will then show the value $-I_1$, but it is still the same physical situation, we are describing. We can therefore formulate the following *flow-orientation-rule for electric wires*:

The same physical situation can be described with two different choices of orientation along the wire. If the orientation is reversed, one must simultaneously change the sign of the symbolic expression for the flow. We shall mostly use the word "flow" as a more general expression for the type of variables that electric currents belong to.

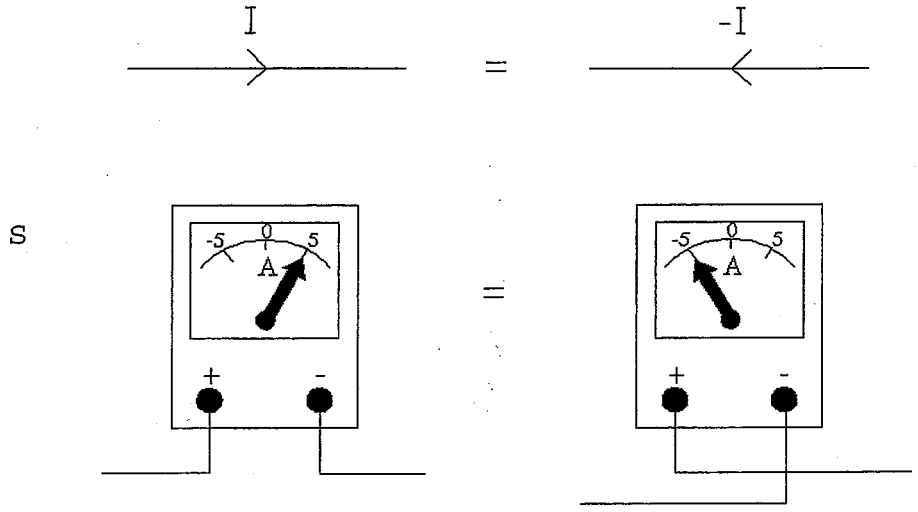


Figure 2. Flow-orientation-rule and its justification in the measurement-prescription for electric current.

Semiotics makes possible a more precise formulation than normal language concerning orientation. Strictly speaking, it is meaningless to say "the current changes sign, when we reverse the orientation", for, as the current is a physical quantity, and, as it is the same physical situation, we are considering with two different orientation, the current is precisely the same. It is exactly *the symbolic expression for the current* that changes sign. The two wire-icons on figure 2, that are both "dressed" with orientation-index and current-symbol are, thus, two different signs that refer to the same object, but with two different symbol-descriptions. Semiotics asserts, that *the symbol is arbitrary*, i.e. it is dependent on chosen conventions that have nothing to do with the object, but this must be understood in

the way, that there are certain "rules of game" for, how the symbolic appearance must be changed, when conventions are changed, and these rules must be strictly adhered to, if we shall know which object, the symbol refers to.

The flow-orientation-rule is found again in other contexts, e.g. by representing linear velocities. If we push at an object with a rigid rod, the rod perhaps will be displaced and slide along its length-direction with the same velocity as the object. The velocity of the rod is then a variable that is *analogous* to the current in an electric wire in the sense, that its measurement-prescription requires an orientation in one or the other direction along the rod. If the orientation is indicated with an arrow, a velocity-symbol "v" in connection with this index means "the velocity in the direction of the arrow", and if the same physical situation is described with the opposite orientation, the symbol "v" must be changed to "-v".

Electric wires and rods to push with have more things in common: Both serve to transfer energy from one place to another. In both cases the energy-transfer is described with two variables, voltage and current for the wire, pushing force and velocity for the rod. Both for the current in the wire and the velocity of the rod is valid that their measurement-prescriptions require an orientation, and that a shift of orientation can describe an unchanged physical situation, if we simultaneously change the symbolic expression for, respectively, the current and the velocity. In this sense we may assert, that *currents and velocities are analogous*, as we consider them as attached to an index (the arrow), that denotes orientation along an icon (the line). The line thus becomes an icon for both the wire and the rod, or, more abstactly, for an *energetic interaction*, that involves two variables "effort" and "flow", understood generally, so that it may also mean "pushing force" and "sliding velocity", "pumping pressure" and "pumping velocity", "chemical potential" and "reaction-rate", or whatever is relevant in a given context. This basic icon is called *the energy bond.*

1.3. Energy-bond and construction-kits.

An energy bond is an icon for an *interaction with possible energy-transfer*. Strictly speaking, we do not know of other forms of interaction in physics, the energy is the universal "currency" of physics, that may be exchanged between different parts of a system, if anything at all is going to happen, but the sign-system of energy bonds may also be applied in other contexts, e.g. economical, where the currency is money. The variables effort and flow for the energy bond, whose product gives the energy transfer per unit of time, may for the economical interaction bond be the price of an item and the number of items transferred per time-unit, such that their product gives the amount of money transferred per time-unit.

When we speak of *interaction* between two system-components, A and B, it must be conceived quite literally: A acts on B, and B acts on A. We can say, that the action of A on B is mediated by a physical quantity r_B, and B's action on A is mediated by a physical quantity r_A. These two quantities will in classical mechanics be described with symbolic variables, so called *rate variables*, but at first, we shall not presume, that they are directly measurable, but just regard them as indices. When we draw the energy bond icon as a line, that connects the two system-components A and B, we can denote the causality of the two rate-variables by indicating r_B (input to B, output from A) closest to B and r_A (input to A, output from B) closest to A. Besides, the energy bond shall be equipped with an arrow for denoting the orientation of the energy flow.

Generally, we may regard r_A and r_B as *indexical vectors*, i.e. as vectors that are not written out in coordinates. The symbolic denotation of numerical values for vector-coordinates requires a coordinate system and some measurement-prescriptions, but we wish to be able to speak about these quantities as something, that exist independently of these conventions, and therefore, provisionally, we shall only regard "r_A" and "r_B" as symbolic indices, attached to a pair of iconic indices, that are a couple of marks on the energy bond icon, whose placement denotes the causality.

Although we do not have numerical coordinates for the two indexical vectors in the energy bond, we shall still assume, that there is defined an *inner product* or *scalar product* for them. A scalar product is, as the name says, a *scalar*, i.e. a quantity, that is independent of the chosen coordinate system. The energy flow, w, i.e. the energy transferred per unit time in the direction of the chosen orientation, must therefore be defined as the scalar product of the two indexical vectors r_A and r_B.

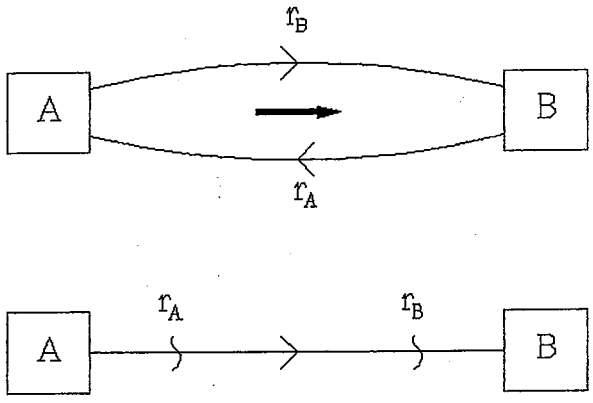


Figure 3. Above: Signal flow diagram for interaction. Below: Energy bond with causality and orientation.

For an energy bond we shall claim that its two variables belong to two different classes, that we call *flows* and *efforts*. These two classes are defined by, their different behaviour with respect to *time reversal*: flows change their sign, efforts do not. We shall use two indices, x and o, to distinguish the two kinds of behaviour by time-reversal: x-variables change sign, o-variables do not. We may therefore characterize *the flow as an x-rate, the effort as an o-rate*. Besides, we have seen, that flows and efforts (their symbolic expressions) behave differently by reversal of the energy bond's orientation: flows change sign, efforts do not. It is in no way evident, that sign-shift by time-reversal is connected with sign-shift by reversal of the spatial orientation, and there can be given counter-examples. But for the applications, we are interested in this context, we may safely assume, that the flow-orientation-rule is valid.

We shall therefore make the indication of the energy bond somewhat simpler by letting the orientation-arrow indicate the flow. The effort is then indicated with a straight line across the bond. In this way the flow-orientation-rule gets a naturally looking appearance, as shown below.

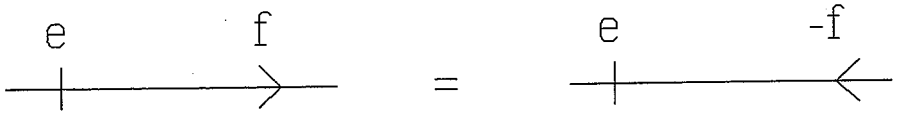


Figure 4. Indices of the energy bond and the flow-orientation rule.

The two symbols "e" and "f" have an index-function like the stroke and the arrow, as they refer to the english words "effort" and "flow".

The difference of flows and efforts, as represented by the flow-orientation-rule, is often expressed by saying, that flows are *through-variables*, while efforts are *across-variables*. This is also very reasonable, when we think of electric networks, where the measurement-prescription for effort in every point can be given "globally", e.g. the effort relative to "earth", whereas the flow requires a local orientation of every separate wire. For other types of energy bonds, however, the description seems not quite as appropriate. E.g. for the rigid rod, where the effort is a pushing force and the flow a sliding velocity, both effort and flow go along the energy bond.

Let us consider the energy-bond-description of the situation "A pushes B" in order to see, what the flow-orientation-rule properly expresses in this case. It is clear that there is a description of direction from A to B, when we say "A pushes B", although

nothing is said about B moving. It may be, that B slides towards A, but still it is A that pushes B. If we orient the energy bond in the direction from A to B and call the pushing force F, then the velocity v shall be measured in the direction from A to B, but the orientation is also connected with the force, that we shall say goes from A to B, and the product Fv denotes the energy transfer per second from A to B. The pushing force F can only be positive, for the energy transfer is the work A does on B, and this will be positive, when B slides away from A ($v>0$) and negative, when B slides towards A ($v<0$).

If we reverse the orientation, so it goes from B to A, then the force shall be unchanged, but the velocity shall be changed to -v. At the same time we must change the verbal description of F; now it is the force, that B pushes A with. The flow-orientation-rule, saying that it is the same F, when it is the same situation, we consider with the opposite orientation, is in this case equivalent with *the law of action and reaction*: When A pushes B with a certain force F, then B pushes back on A with the same force F. Although the force is a quantity with a direction, when we speak about a system acting on another, then it is still the same, when the direction is reversed. This is a consequence of the flow-orientation-rule, so we may say, that the law of action and reaction is not a proper law of nature, but a semiotic law. One of the purposes of the energy-bond-formalism is to incorporate semiotic laws in the applied system of signs, like, e.g. the law of the parallelogram of forces is incorporated in the sign for the force as a vector.

A prescription for, how the energy bond and the other eb-icons can be realized materially, such that it is connected with unambiguous measurement prescriptions for eb-variables, is called a *construction kit* (like a box of toy bricks). We shall, provisionally, only discuss the mere energy bond in some different kits.

In *the electric kit* the energy bond consists of an electric double wire. The effort, that is an *across-variable*, is the voltage-difference between thje two wires, so to ensure unambiguity it must be possible to see a difference of the two wires. We can, e.g. let one wire be equipped with "round terminals" and the other with "flat terminals" and then define the effort of the bond as the difference between the round terminal's and the flat terminal's electric potential. The flow is defined as the current in "the round wire". in the direction of the chosen orientation. The figure below shows how the voltmeter and the ampèremeter shall be inserted in order to give the symbolic expressions for the effort and the flow.

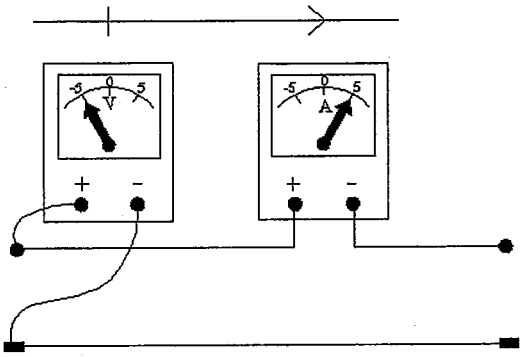


Figure 5. Energy bond and measurement prescriptions in the electric kit.

A construction kit, where the energy bond is a rigid rod, one can push with, as described previously, can be called a *longitudinal mechanical kit*, because both the flow and the effort (sliding velocity and pushing force) are measured along the rod. The effort can be measured, e.g., by the compression of a spring, situated at one of the ends of the rod. If the rod can only push, the kit has the limitation, that efforts can only be positive. This limitation can be remedied, if the rods are equipped with hooks in their ends, so one also can pull with them. A pull-situation then corresponds to the effort being negative. We may also think of a mechanical kit, where the energy bonds are threads, one can only pull with. With the same measuring prescriptions as for the rods, the threads will have the limitation, that efforts can only be negative.

In a *transversal mechanical kit* the energy bonds consist of rigid shafts, that can rotate around their length-axis. The shafts can be equipped with cog-wheels of a definite size in their ends. For this kit both flows and efforts are *across-variables*. With a given orientation of the energy bond from A to B the flow may be defined as the angular velocity in the direction, that together with the orientation forms a right-screw. The effort will then be the torque, that system A acts on system B with. One can easily convince oneself, that the flow-orientation rule is valid, and that the unchanged effort-sign by orientation-shift expresses the law of action and reaction for torques.

The transversal mechanical kit does not suffer the limitations with respect to the possibilities of the eb-formalism, that both the longitudinal mechanical and the electric kit have. In fact, all the basic icons can be realized transversal-mechanically, but some of them (e.g. the o-junction, that is a kind of differential) require quite complicated "fine mechanics", whereas one in the electric kit has the possibility of "cheating" around some limitations by using active electronic components, like, e.g. operational amplifiers without requiring extraordinary skill.
A *hydraulic construction kit* has energy bonds consisting of rigid tubes, wherein an incompressible fluid may stream. If the flow is defined as the linear velocity of the

fluid in the direction of orientation, the effort will be the force, whereby the fluid presses on a plate at the end of the tube. With this kit it is very simple to make a *dc-transformer* by just joining two tubes with different areas of cross-section. As the pressure of the fluid is the same in the two tubes, the efforts will have the same ratio as the cross-sectional areas.

The *EB construction kit* (the basic kit) is the diagrammatic language, we shall use for developing the formalism in the following. It does not consist of material components, but of icons, that can be dressed with indices and symbols after certain rules (like, e.g. the flow-orientation rule). The icons can, to a higher or lower degree, be realised within a meterial construction kit like one of the previously mentioned. In the basic kit can be used *basic icons* or *higher icons*. There is no limit to, how many higher icons, one may define, but one must, as much as possible, take care that the shape of the icon shall express the symmetry in that function, the icon shall describe, and that indices and symbols can be added to the icon, such that the meaning of the symbols becomes precise. The basic icons is a well-delimited set of icons, that together make up a *complete set* for description of all dynamical systems in physics. The collection of basic icons make up *the basic kit*, that is thus a partial set of the eb-construction kit. Every higher icon in the kit can in principle be reduced to basic icons. The carrying out of this (often quite difficult) exercise of reduction is called *reticulation*, i.e. network-structuring.

No guarantee can be given, that the basic kit really forms a complete system for physics, but neither we have examples of physical systems, that certainly can *not* be reticulated down to basic icons. The claim of completeness must, thus, be taken with a grain of salt. For *linear* systems it can be proved, but for non-linear systems a reticulation may prove so difficult, that it is preferable to describe parts of the function with higher icons. In analytical mechanics we shall be satisfied with a partial reticulation, that allows one higher icon, viz. a multi-port-storage for potential energy. Besides, we shall allow non-linear *parametric governing* of certain basic icons, transformers, transducers, and gyrators.

1.4 Apology for change of notation.

As earlier mentioned, the Energy-Bond-Graph (EBG) formalism was developed by H.M. Paynter at M.I.T. around 1960. Paynter's formalism still prevails in the international literature [1], and this is a problem that may be disturbing for the readers of this exposition, where a different notation is used. I shall, however, try to explain the main differences between Paynter's original notation and the present — and why I have found it necessary to make some changes.

In doing this I have — to some extent — violated C.S. Peirce's "Ethics of Terminology" (CP 2.219) which forbids authors to make unnecessary changes of well-established notation and nomenclature, but Peirce himself did not prevent himself from creating new words and diagrams, when it could lead to new insights in logic and mathematics.

I have earlier mentioned (ch. I.2.3) Peirce's construction of a *shape-value-notation* for the 16 binary logical connectives.
Although Paynter acknowledged inspiration from Peirce's semiotic in his invention of the EBG-formalism he did clearly not attempt to create it as a shape-value-notation. He mentions early attempts to represent the EBG-elements by squares and circles, but also that he rejected such attempts and deliberately chose a more austere style of non-drawing, using only letters (like C for a capacitance and S for a source) such that his diagrams look more like chemical constitution-diagrams than like energy-flow-diagrams, used, e.g., in ecology. This style was not in accordance with my intentions for several reasons:

1. The use of symbolic letters to denote physical functions obscures Peirce's clear distinction between icons, indices, and symbols. I wanted to make this distinction clearly visible as three separate layers of the formalism.

2. The aesthetic challenge in *drawing* a model appeals to fantasy and creativity, and this appeal is restricted by Paynter's austerity. I wanted to make it possible for users of the formalism to invent new icons to represent their ideas.

3. Indications of orientation and causality are somewhat unclear in Paynter's diagrams, as I shall now demonstrate.

Figure 6 shows how Paynter represents an energy bond connected to a flow-source S_f. Orientation of the bond is indicated by a *half*-arrow pointing away from the source. effort-symbol (e) is indicated on the same side of the bond as the half-arrow and flow-symbol (f) on the other side. Finally, causality is indicated by a stroke, showing that the effort is input to the source:

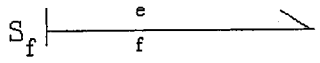


Figure 6. flow-source and energy bond in Paynter's notation.

As there are no separate indications of flows and efforts in this notation it is difficult to see how their symbols are related to the orientation. *The flow-orientation-rule does not exist in Paynter's formalism.* On the contrary, the rule of placing the effort-symbol on the side of the half-arrow lead some authors[1] to claim that orientation-shift should be accompanied by a change of sign for the effort-symbol. This is clearly wrong for most applications in physics and engineering, and I see it as a major drawback in Paynter's diagrams. For these reasons I shall continue to use the style of drawing, I have developed, but apologize to the reader for the inconvenience this departure from the original notation may cause.

2.The basic construction kit.

2.1 Sources.

The most basic icon of the basic kit is *the simple energy bond*, where flow and effort are one-dimensional vectors or scalars. As it is classical mechanics that we are here interested in, we shall without further discussion assume, that flow and effort can be expressed symbolically with *real numbers and measurement-units*, expressing results of *potential measurements*. So we assume that measurement of flow and effort can be done after well-defined prescriptions, and that such measurement will not cause notable disturbance of the measured quantity. We further assume, that the flow and the effort can be distinguished by the time-reversal-criterion, and that the flow-orientation rule is valid, as described in the preceding section.

Every icon has a certain number of *ports*, where energy bonds can be attached. The mere energy bond has two ends that are also two ports. So the energy bond is a two-port icon, that can be called an *ideal transmitter* (for energy).

The icons of the basic kit can be divided in three categories, called *active*, *passive*, and *dissipative*.

An *active* system determines sovereignly the output-energy bond variables in all its ports without regard to the input-variables. For a *passive* system the output variables are uniquely determined by the present and the past values of the input-variables. Sometimes a distinction is made between *purely passive* systems, where the outputs only depend on the present inputs and *reactive* systems, where also the past values of the inputs are determining the present outputs. *Dissipative* systems have a predominantly passive output, determined by present and past inputs, such that the system always consumes (dissipates) energy. However, there is for dissipative

systems an active component in the output, that is independent of the input. Normally, this active component will be small compared to the passive component, so we may discern *the signal*, i.e. the passive response from *the noise*, that is the active component.

In the basic construction-kit *sources* (x- and o-source) represent the active systems, *transformers, transducers, gyrators, and junctions* are purely passive (like the pure energy bond, that is a *transmitter*, corresponding to a transformer with the ratio 1), *the storages* (x- and o-storage) are reactive, and *the sinks are dissipative.*

the sources are active 1-ports. As there are only two kinds of energy-bond-variables, flow and effort, in the one port, there are only two kinds of sources: *the x-source or the flow-source* and *the o-source* or *the effort-source.*

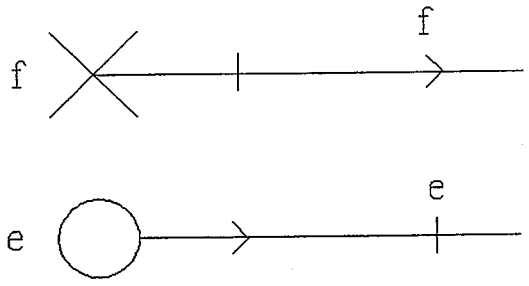


Figure 7. The sources.

It is seen from figure 7, that a source not necessarily gives away energy.
If the product of flow and effort with the given orientations is negative, the source will receive energy. If, e.g. a wind-mill drives an asynchronous generator, attached to the electric power-net, then the net will be described with three effort-sources, corresponding to the three 50 Hz phases. These phases deliver to the generator a magnetic field revolving with 50 revolutions per second, and if the wind-mill drives the anchor round a bit faster than that, the input-flow will have that sign (dependent on orientation), that corresponds to the mill delivering energy to the three o-sources.

It is further seen, that the causality of the energy bond is very important in the definition of the sources. If we try to connect to sources of the same type through a single energy bond, a *causal conflict* will arise, that cannot be solved without adding more icons. On the contrary, there is nothing that prevents connecting two sources of different types.

2.2 Storages

Storages are reactive 1-ports. As for the sources there are two kinds: *the x-storage*

or *the kinetic energy storage* and *the o-storage* or *the potential energy storage*. The reason why, that there are only two kinds of storage is, again, that there are only two kinds of variables in the one port. For the x-storage the output is a flow, determined by previous values of the input-effort, and for the o-storage the output is an effort, determined by the previous values of the input-flow. The reactive functioning appears for both variables by the input-variable (effort, or o-rate for the x-storage and flow or x-rate for the o-storage) being integrated over time to a *level-variable*, that is attached to the storage and determines its *energy-function*. As an o-rate is integrated to an x-level, indicated with the index x in the icon of the storage, and the o-storage gets an o-level, indicated with an o in the storage-icon. The icon is a "bird-house". If we call the rate-variable R, the level-variable L, and the energy-function E(L), we thus have:

$$L(t)=\int_{-\infty}^{t} R(t')\,d't + L(-\infty) \qquad \textbf{(1)}$$

The constant L(-∞) properly represents an active contribution to the storage's, otherwise reactive, response. We may often, though, be allowed to disregard the system's memory of the infinitely distant past; if we can't, we say that the system is *non-ergodic*. If there are sinks in the system, they will often make the system ergodic, so we may disregard the constant in (1), but the pure storage is, thus, non-ergodic. If we write instead

$$\frac{dL}{dt} = R \qquad \textbf{(2)}$$

we don't need to be concerned with the ergodicity problem, but the algorithmic causality still goes from rate to level: Changes in the level-variable are calculated by integration of the rate-variable over the passed time. We say, that the rate- and the level-variable are connected by *kinematic causality*.

When the level-variable L and the energy-function E(L) are known, we can calculate the output-variable in the attached energy bond. If this bond is oriented *in towards the storage*, the rate of increase of the energy-function shall be given by the product of the two variables in the bond, so if we call the output-variable U, we have:

$$\frac{dE(L)}{dt} = \frac{dE(L)}{dL} \cdot \frac{dL}{dt} = U \cdot R \qquad (3)$$

Comparing with (2) we then get

$$U = \frac{dE}{dL} \qquad (4)$$

We see from this, that if E is a linear function of L, $E=U.L+E_0$, then U is independent of L and therefore also of R. So in this case the storage is not reactive, but active, and we therefore call it an *active storage*, although its activity is restricted to keeping the output-variable constant, irrespective of what happens.

If, on the contrary, U is a linear function of L, we shall call it a *linear storage*. So in this case the energy-function i *quadratic* in L.

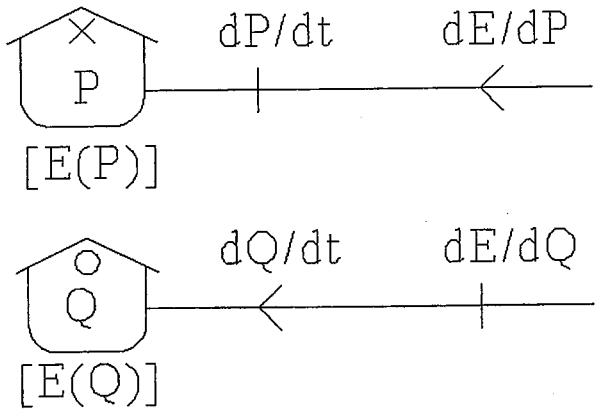


Figure 8. The general definition of storages.

The level-variables P and Q for, respectively, the x- and the o-storage are in mechanics called *generalized momentum* and *generalized coordinate*. Correspondingly, the rate-variables are called *generalized velocity* (the flow) and *generalized force.*

For *linear storages* we introduce a constant, *the storage-capacity* for expressing the quadratic energy-function and thereby also the linear connection between the level-variable and the output-rate-variable:

$$E(P)=\frac{P^2}{2M}\ ;\ E(Q)=\frac{Q^2}{2C} \qquad (5)$$

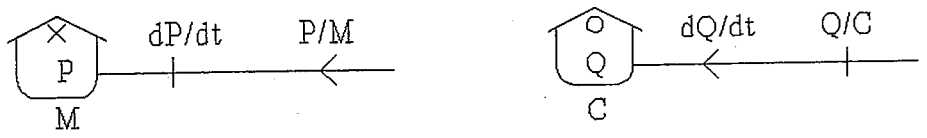


Figure 9. Linear storages.

The capacity M of the linear x-storage is called the *inertance*, and the capacity C of the linear o-storage is called the *compliance*. For electric systems the linear x-storage corresponds to a magnetic coil, whose capacity is called the *inductance*. Here the word "capacity" is reserved for the capacity of the linear o-storage, that corresponds to a condenser. The reciprocal property to the capacity (compliance) C, G=1/C is called the *rigidity*.

The linear x-storage may be, e.g., a particle with mass M, momentum P and velocity v=P/M, or it may be a flywheel with moment of inertia M, angular momentum P, and angular velocity ω=P/M. The linear o-storage can, as mentioned, be a condenser with capacity C, charge Q, and voltage V=Q/C, or it can be a spring with rigidity (modulus) k=1/C, compression Q, and force kQ. The expansion of the spring is then x=-Q: the spring-force in the direction away from the spring is F=-k.x (Hook's law).

The port of a storage has a causal preference: the x-storage "prefers" effort-input, the o-storage "prefers" flow-input. This id the so-called *integral* (or kinematic) causality, where changes in the level-variable are calculated by integration of the rate-variable over time. If one connects an o-source with an o-storage or an x-source with an x-storage, there is a causal conflict, but it is not insolvable. The source is regarded as more "stubborn" in its causality than the storage and therefore forces its causality upon the storage. The storage must thus accept to get *differential* causality. For an effort-source connected to a condenser it happens that the voltage V determines the charge on the condenser: Q=C.V, and this then determines the current by differentiation: J=dQ/dt.

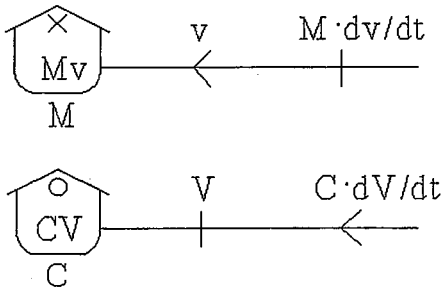


Figure 10. Linear storages with differential causality.

There are calculational reasons for preferring the integral causality for the differential, but also physical reasons. If we want to force upon a particle a much too rapidly varying velocity v(t), the "inertia-force" Mdv/dt will be too great for the system to sustain (just think about the particle being the head of a hammer, and the flow-source that will force the hammerhead to stop instantaneously, is a finger).

2.3 Transformers

We shall now consider **the transformer**, that is a passive two-port. The transformer of the basic kit is *ideal* in the sense, that it neither stores nor dissipates energy. I.e. the energy flow in through the primary port must be the same as the energy flow out from the secondary port. We draw the transformer-icon as an isosceles triangle with the primary port in the middle of the base-line and the secondary port in the apex above. The transformer-ratio t is defined as the ratio between the secondary effort and the primary effort. When the energy flows in the two ports shall be the same, t must also be the ratio between the primary flow and the secondary flow. That is, if the effort is input to the transformer on the primary side, it must be output on the secondary side, and vice versa. Thus, the single port has no causal preference, but when causality in the one port is fixed, the causality in the other port must comply with that. If we connect effort-sources to both sides of a transformer, there will be an insolvable causal conflict.

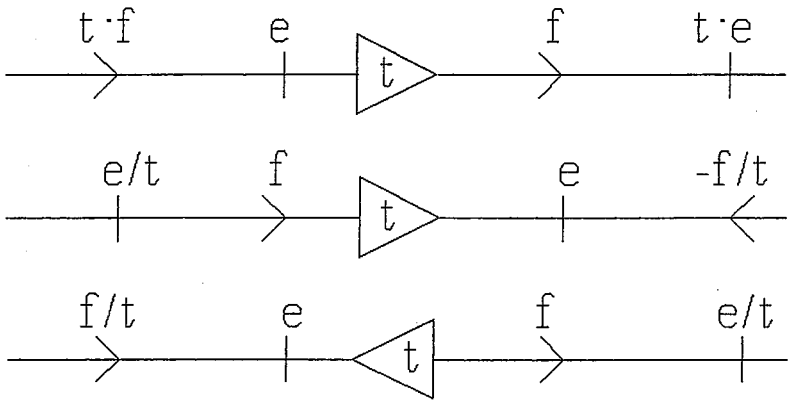


Figure 11. Three ways of showing the transformer-definition.

By comparing the upper and the lower exposition of the transformer-definition in figure 11 we find the following rule for *reversal of the icon*:

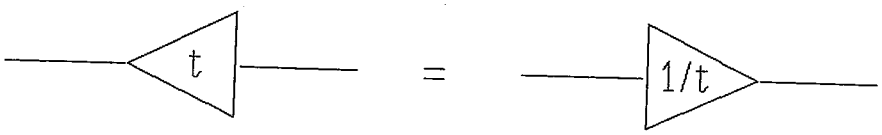


Figure 12. Reversal of the transformer-icon.

In the special case, where t=1/t, we cannot distinguish between the primary and the secondary side, and the we ought to use a *symmetric* icon. There are only the two possibilities t=1 and t=-1, and the first possibility is the ideal transmitter, represented by the pure energy bond. The other possibility, t=-1, however, deserves a special icon, *the symmetric transformer*:

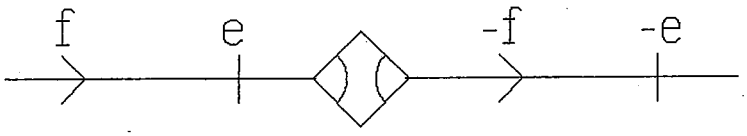


Figure 13. The symmetric transformer.

As an example of the use of the symmetric transformer we look at a *harmonic oscillator*, a particle with mass m, bound to a spring with modulus k. The oscillator-behaviour is essentially produced by connecting a linear x-storage with a linear o-storage, but then there is a problem with the orientation, for when the velocity v=p/m is oriented in towards the x-storage, it is oriented away from the o-storage, so if we want the o-level to be the x-position of the particle, the velocity will be oriented after the negative x-axis, which appears unnatural. The problem is solved by inserting a symmetric transformer in the connecting bond, as shown below.

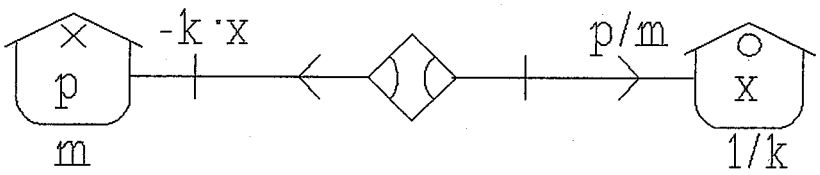


Figure 14. Harmonic oscillator.

2.4 Gyrators

Another passive two-port is **the gyrator**. Here there is either effort-input in both ports or flow-input in both ports. The conversion-ratio of the gyrator is defined as the ratio between secondary flow and primary effort or as the ratio between secondary effort and primary flow. As there thus are two possibilities for the meaning of the symbol for the conversion-ratio, we must have two different icons, that are called the o-gyrator and the x-gyrator. So, it is not the case, that these two functions are different in a physical sense; an x-gyrator can be replaced with an o-gyrator, while at the same time the symbol for the conversion-ratio is changed to its reciprocal value.

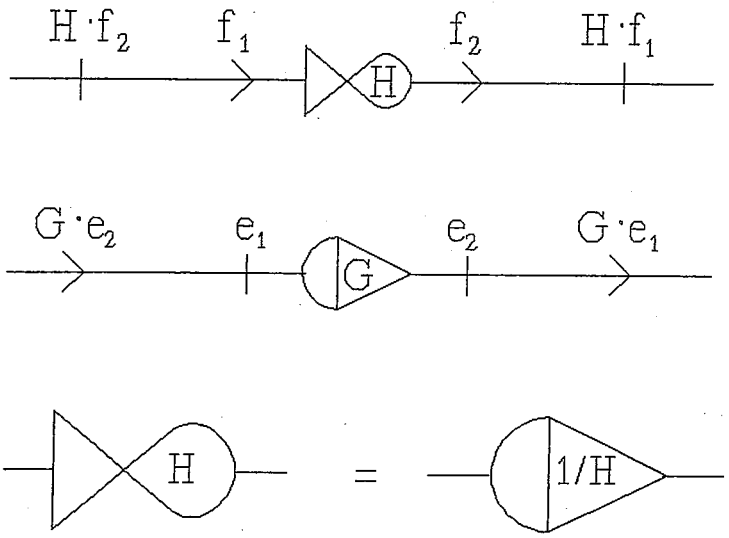


Figure 15. Gyrator-definitions: above: x-gyrator, middle: o-gyrator, below: equivalence, primary sides to the left.

BY use of the flow-orientation-rule in both ports is seen, that icon-reversal of both gyrators may be done by simultaneous changing sign of the conversion-ratio.

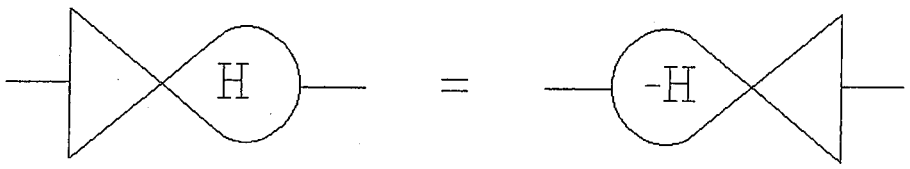


Figure 16. Icon-reversal of gyrator.

The equation H=-H has only the solution H=0, a value that is not allowed as conversion-ratio of a gyrator. I.e. an icon-symmetric gyrator does not exist in the basic kit.

2.5 Junctions.

Until now we have only defined 1-ports and 2-ports, so we have no possibility of describing branching of energy-flows. This possibility is introduced with **the**

junctions, that are passive n-ports with n>2. We shall first look at junctions as 3-ports. Like the transformers and gyrators the junctions are ideal elements, that neither store nor dissipate energy. If all ports are oriented towards the junction, the sum of the products of flow and efforts in the ports must be zero. There are two simple ways in which this can be achieved: either by letting the flow be the same in all ports; then the sum of the efforts must be zero. This is the **x-junction**. We say that the x-junction entails a *distributive flow-relation* and a *conservative effort-relation*. The other possibility is to let the effort be the same in all ports; then the sum of the flows must be zero. This is the **o-junction**, that has a *distributive effort-relation* and a *conservative flow-relation*. That variable, which has a distributive relation, must be input to the junction in just one bond. This bond is called *the strong bond* and is said to have *strong causality*, while the other bonds are called *weak* and have *weak causality*.

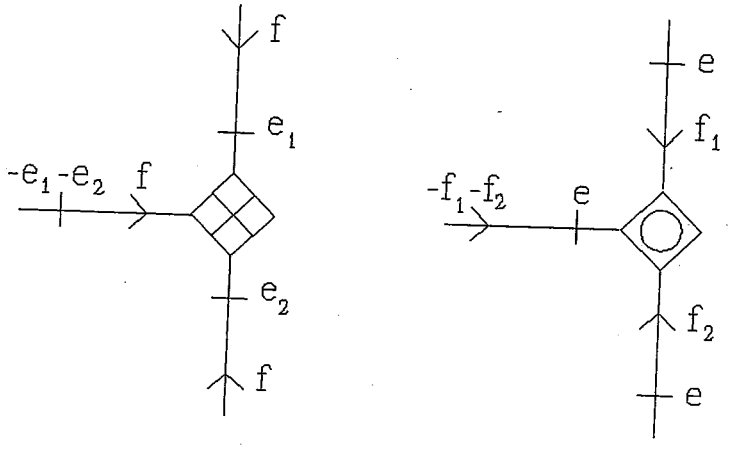


Figure 17. Three-port junctions.

Although the two junction-definitions appear completely symmetric with respect to flows and efforts in figure 17, there is still a difference, because the flow-orientation-rule treats flows and efforts differently. If we for an o-junction reverse the orientation in one or more ports it has no effect on the distributive relation of the junction, which only refers to the efforts. The signs of the flows in the conservative will change, but this will be intuitively clear, how they reflect the orientation, so instead of saying "the sum of the inward-going flows is zero" one may say "the sum of the inward-going flows is equal to the outward-going flow". The x-junction is more problematic, for here the sign of the distributive flow-relation must be changed, where the orientation is changed, while there are unchanged signs in the conservative effort-relation. It is often more convenient to have a fixed sign in the distributive relation. If a three-port x-junction has ingoing orientation in one of the bonds and outgoing in the other two, we may insert a symmetric transformer in that port where the orientation deviates,

then we shall have the same flow in all three ports. We shall therefore introduce a special icon: **the dotted x-junction**, where the inserted symmetric transformer is "swallowed" by the x-junction-icon and marked with a dot in the relevant port. There may well be more dots, but in a physical sense a 3-port x-junction with dots in port 1 and 2, but none in port 3 is the same as one, that has a dot in port 3, but none in ports 1 and 2.

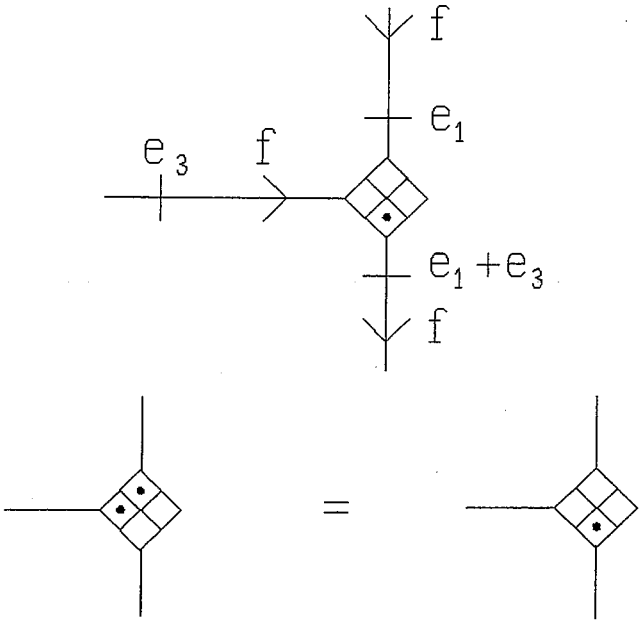


Figure 18. the dotted x-junction.

For *the electric kit* the x-junction corresponds to a series connection and the o-junction to a parallel connection, and the two conservative relations correspond to *Kirchhoff's laws*, the mesh-law for the x-junction and the node-law for the o-junction. For the longitudinal-mechanical kit, also called *the rheological kit* it is, however, vice versa: A 3-port x-junction with one dot corresponds in rheology to a two-branched fork, where the handle is the dotted port and the two branches the undotted ports. In the transversal mechanical kit the x-junction corresponds to a single cog-wheel, that all the attached shafts' cog-wheels can roll upon, whereas the o-junction is a more complicated *differential.*

By the introduction of the 3-port junctions the basic kit is able to build relations with arbitrarily many ports. Peirce introduced in his logic of relatives the basic forms *monadic*, *dyadic*, and *triadic* relations and then showed in his *valence-theorem*, that all relations with n>3 can be built up by means of the three basic forms. In the basic kit we have, correspondingly, introduced 1-ports, 2-ports, and 3-ports. Here the sources correspond to monadic relations. Storages and sinks (not yet formally defined) correspond to dyadic relations between flow and effort in the same bond,

transformers and gyrators have a pair of dyadic relations between variables in two different ports, and the three-port junctions have a pair of triadic relations, the one between the flows and the other between the efforts in three different ports. Corresponding to the valence-theorem we can now show that n-port junctions with n>3 (multiport junctions) can be made of 3-port junctions. The figure below only shows the construction for n=4. Note, that in the construction of the x-junction is used a dyadic element, viz. the dot (the symmetric transformer).

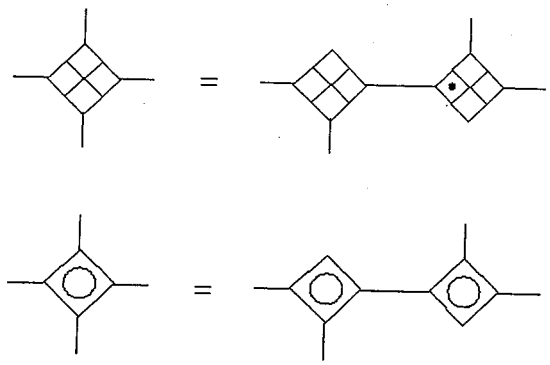


Figure 19. construction of multiport junctions.

With multiport junctions we may now also define "multiport storages". Here we do not speak about genuine multiport storages that have just as many level-variables as they have ports (these do not belong to the basic kit, but are higher icons).

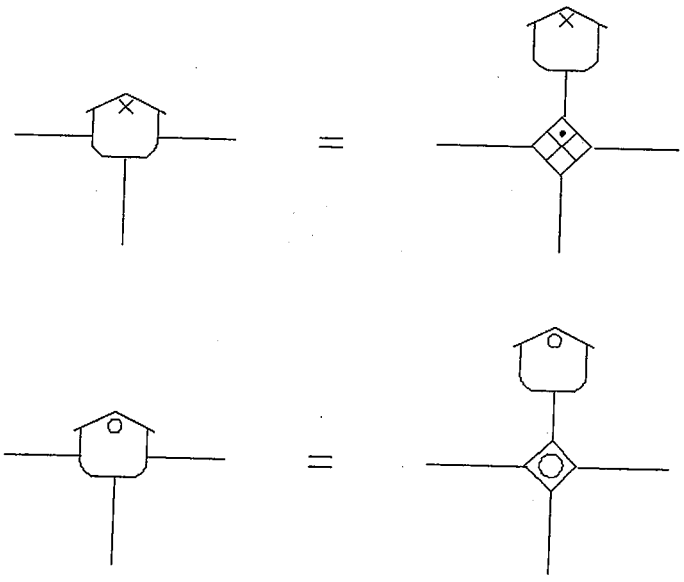


Figure 20. Non-genuine multiport storages.

We can also go the other way and reduce the number of ports on the junctions. With two ports they become the two well-known symmetric transformers.

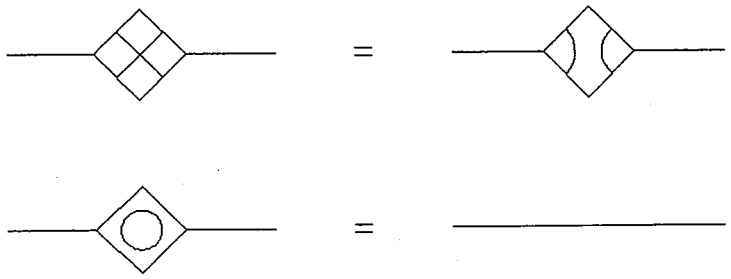


Figure 21. Two-port junctions.

So, one may do without two-port junctions in the basic kit, but in the generalized version of the eb-technique that uses complex vector-bonds in arbitrary metrics (IMFUFA text 238, or part II of this thesis) they are non-trivial, as the identities in figure 21 are not always valid.

On the other hand, the **1-port junctions** have an important function in the basic kit. As a junction must have one strong bond, the one and only bond of the 1-port junction must be strong. So the 1-port x-junction must have flow-input, and the output effort must be zero, due to the conservative effort-relation, that now only contains one term in the sum. Correspondingly, the 1-port o-junction must have effort-input, and the output-flow must be zero. WE see from this, that the 1-port junctions are not passive, but active, as the *1-port x-junction is an effort-source with the effort 0, and* the *1-port o-junction is a flow-source with the flow 0.* Such elements are useful for signifying *boundary-conditions* or *catastrophes.* If one connects a storage with a 1-port junction of opposite type (i.e. an x-storage with an o-junction or vice versa), the junction at once will "kill" the storage, because it forces on the storage differential causality and annihilates the level-variable become zero. In the electric kit the 1-port x-junction corresponds to a *short circuit and the 1-port o-junction to a disconnection.*

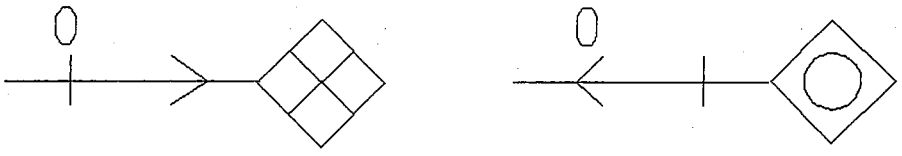


Figure 22. 1-port junctions.

2.6 Network-topology.

The limitations of the electric kit are exhibited, when we try to translate certain eb-systems to electric networks. The other way is quite easy: Every *linear graph*, i.e. a structure of nodes, connected with branches, can be translated to an eb-structure of junctions after the following simple prescription:

1) In the linear graph we choose a "reference-node ("earth"), that is cut away.

2) The remaining nodes are translated to o-junctions.

3) Every branch in the graph, that is not connected to the reference-node, is translated to an energy bond with an inserted 3-port dotted x-junction, connected to those o-junctions that correspond to the nodes, that are connected with the branch in the electric graph. The dot shall be situated at one of those ports that connects to one of these nodes. The third port (the external) is connected to the system, that is inserted in the branch of the electric graph.

4) The EB-graph may, possibly, be simplified by moving the dots around: A dot in a certain port can be moved over to the two other ports (comp. figure 18). FRom here it may move out in the bonds. When it meets a junction, it divides and continues out into the other bonds from the junction. When it meets another dot, both are cancelled. Dots in external ports vanish.

The figure below shows the method applied to a tetrahedral graph (a Wheatstone-bridge).

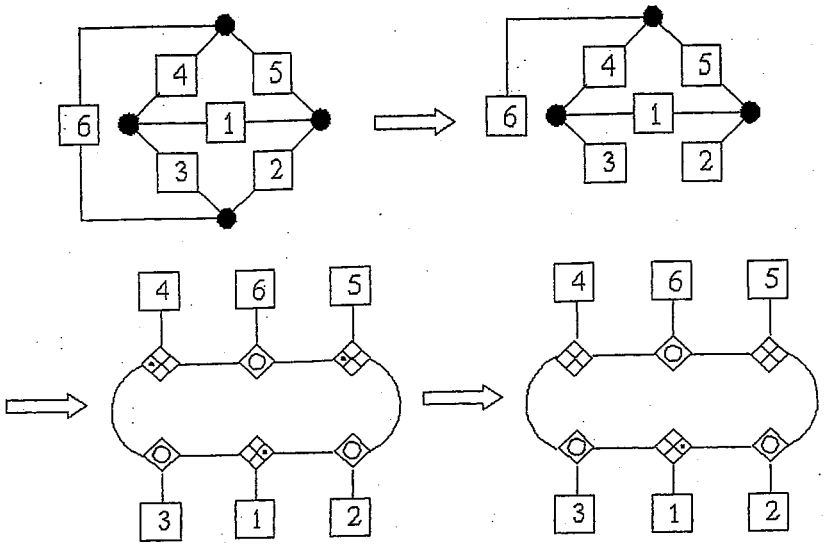


Figure 23. Translation of electric network to junction-structure.

If we stop after p. 3) in this method, it is also easy to see, that the translation can be performed the opposite way: An EB-junction-structure, that only consists of o-junctions and 3-port dotted x-junctions with the dot in an internal port, can always be translated back to an electric network. In order to do this, one must just apply the prescriptions given below for the junctions in the electric kit. Note, that is *no transformer* in the dotted x-.junction. I.e., if one in the electric kit has to use *undotted* x-junctions, one has to use symmetric transformers, and one may say, that the difficulty in constructing dc-transformers is a serious limitation of the electric kit.

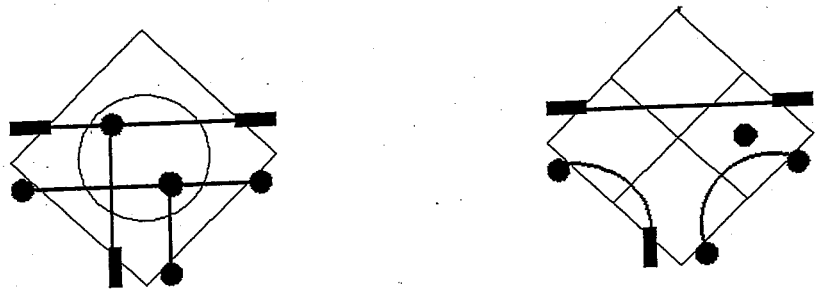


Figure 24. Junctions in the electric kit.

In the basic kit it is, though, quite easy to make a transformer, when one has junctions. Below is shown a prescription, but also, how disastrous it may go, if one tries to realize it by using the junctions on figure 23.

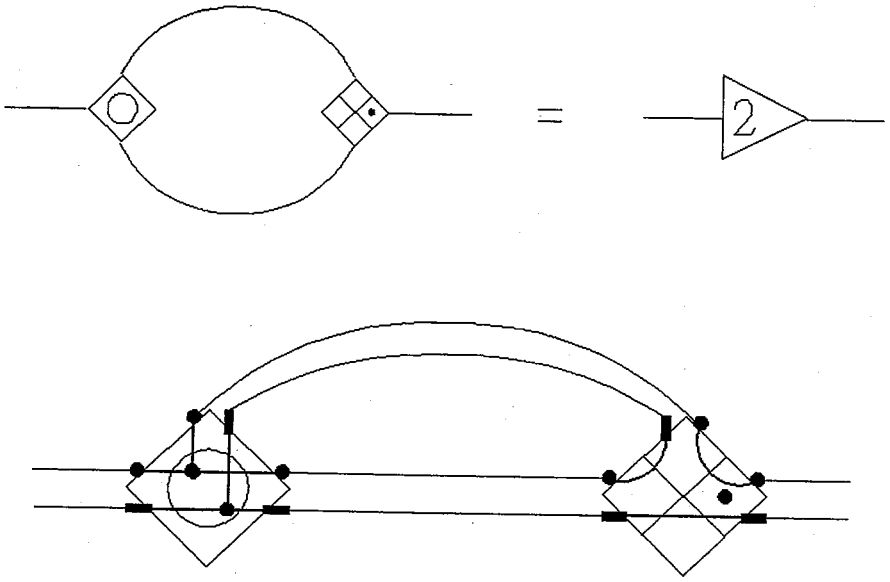


Figure 25. Failed attempt to make a dc-transformer.

2.7 Transducers.

We have now, by and large completed the account of the active and passive elements of the basic kit, but we just lack mentioning **the transducer**, a passive two-port, that acts as an "interface" between two different energetic media (e.g. two different construction kits). We shall define the transducer simply as *a transformer with dimension*. A gyrator may also serve as a transducer (e.g. an electro-motor), whose dimension of its conversion-ratio deviates from the ratio between flow and effort in the primary port. The formal EB-definitions can in all cases be traced back to figure 11 and figure 15.

2.8 Sinks.

The dissipative elements are represented in the basic kit by **the sinks**. A dissipative system has both a passive *response* to the applied stimulus-signals, but also an *active noise*. For an Ohmic resistance R with current-input J the voltage-output is given by

$$V = R \cdot J + \delta V \qquad \textbf{(6)}$$

where δV is the noise, that in most contexts is small compared to the passive response R.J. For *linear sinks* like the Ohmic resistance the magnitude of the noise is determined by the resistance R and the absolute temperature T in *the fluctuation-dissipation theorem.* If the time in a numerical integration is divided into steps with the length δt, The noise-contribution for each single step *in the classical limit* be expressed as

$$\delta V = N\left(\sqrt{\frac{2 \cdot R \cdot kT}{\delta t}}\right) \tag{7}$$

where k is Boltzmann's constant, and where N(b) is a normally distributed random number with mean value 0 and standard deviation b. As the noise goes inversely proportional to the square root of the steplength δt, it has, strictly speaking, no sense to talk about *the instantaneous value* of the output-variable from a sink. We are talking about *white noise,* that is uniformly distributed over all frequencies and therefore diverges over an infinitely small time-interval. One speaks in this connection about an *ultraviolet catastrophe.* The classical limit, where(7) is valid, is defined by

$$\delta t >> \frac{\hbar}{kT} \tag{8}$$

But for smaller time-intervals, where quantum mechanics come into play, the ultraviolet catastrophe just becomes still worse (due to zero-point-fluctuations).

We shall here confine ourselves to *linear sinks.* As for gyrators we use two different icons, an x-sink and an o-sink, not because these are physically different functions, but because the icon must be able to define the meaning of the symbol, that for the x-sink is a *resistance* and for the o-sink a *conductance* (admittance) or *mobility.*

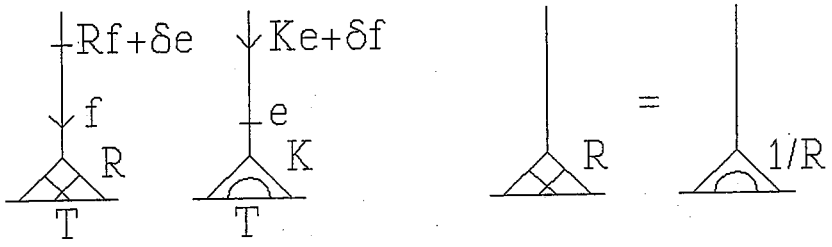


Figure 26. Linear sinks.

This completes the account of the basic construction kit.

3. Velocity-constraints.

3.1 Rigid rod.

In Analytical Mechanics one tries to formulate the equations of motion of the system in terms of a set of *independent generalized coordinates*. However, it is not always that those *constraints* that are imposed on the motion are *holonomous*, such that they allow elimination of dependent variables. All constraints in an energy-bond-model must be expressible as relations between the variables of the energy bonds, thus, they are *velocity constraints*, and it is not certain, that they are *integrable* to relations between o-levels, that is the class, the generalized coordinates belong to. In an EB-diagram the constraint-relations are contained in the *skeleton-diagram*, i.e. the topological part of the diagram, that expresses the connectedness of the system by means of junctions and transformers.

As an example we shall consider two particles with masses m_1 and m_2, that can move in three dimensions, but are connected with a rigid, massless rod. On the diagram below are shown 6 x-storages for the 2*3 momentum-components and 3 o-storages for the relative coordinates $x_{12} = x_1 - x_2$ etc.

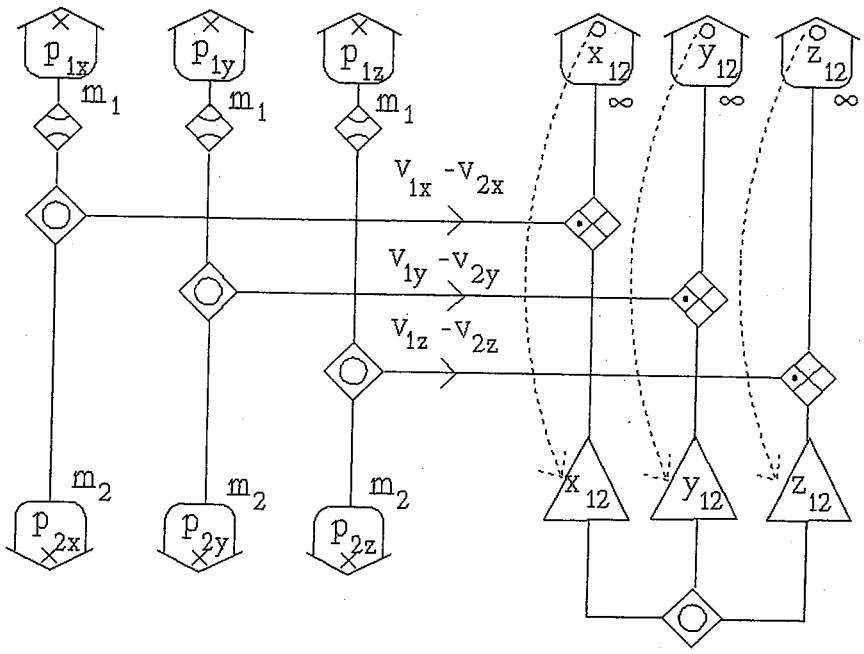


Figure 27. Motion of two particles connected with a rigid rod.

The three o-storages are so-called *cyclic storages*, that are shown as linear storages with the capacity ∞, i.e. the output-effort from them is zero. So, we don't reckon, that there are proper forces connected with the relative coordinates (they are cyclical), but, as we shall see, the structure of the skeleton diagram causes the appearance of *constraint forces*, that describe the action of the rigid rod. In the diagram appears three *governed transducers*, whose conversion ratios are given by the three relative coordinates. The governing is shown with a dotted arrow, departing from the iconic index (o) for the level-variable in the storage and ending at the governed icon. Such governings may in principle denote arbitrary functional connections, but it is in the spirit of the EB-game to avoid too complicated connections and (as here) stick to *linear governings*. As the flows on the secondary sides of the transformers are the three relative velocities v_{1x}-v_{2x}, etc., the primary flows are equal to the products of the relative velocities and the relative coordinates. The three primary bonds are connected to the same o-junction, whose conservative flow-relation thus expresses,

$$\left.\begin{aligned}(x_1-x_2)\cdot(v_{1x}-v_{2x})+(y_1-y_2)\cdot(v_{1y}-v_{2y})+(z_1-z_2)\cdot(v_{1z}-v_{2z})=0\\ \text{or}\ \frac{d}{dt}\left((x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2\right)=0\end{aligned}\right\} \qquad (9)$$

i.e. that the distance between the two particles is constant, which exactly is, what the

rigid rod serves to make sure.

Because the o-junction must have one strong bond and two weak, we must choose one of the three bonds to have the opposite causality of the two others. Let us choose the y-bond as the strong one. Then there must also be chosen a strong bond at the o-junction, that sits between the two p_y-storages to the left in the figure, and this means, that one of these two x-storages must be subjected to differential causality, e.g. the p_{2y}-storage. There will then as output-effort from this storage appear an *inertia-force*

$$f^{(i)}_{2y} = m_2 \cdot \frac{d\, v_{2y}}{dt} \qquad \textbf{(10)}$$

and this effort-signal will be distributed out from the o-junction to the left and on to the o-junction to the right, from where it again will be distributed and cause effort-input to the other x-storages in the diagram. In this way the EB-diagram thus describes the constraint-forces as coming from causal constraints in the skeleton-diagram, that lead to differential causality in one or more x-storages. The number of storages with differential causality corresponds to the number of topological constraints from junctions. In this case there is only one constraint, that is expressed by the rigid rod and the symbolic condition (9).

3.2 Signal-rails.

In order to get a quick survey over the causality in an EB-diagram one may use a graphical technique, called *signal-rails*. The idea in this sort of presentation is, that a junction corresponds to a railway-junction. For an o-junction the effort-signal going in to the junction will be distributed out to the weak bonds, and a flow-signal coming from one of the weak bonds will proceed to the strong bond, but there *no signal-way* from a weak bond to another weak bond. The signal-rail-structure for the example considered , where we choose to impose differential causality on the p_{2y}-storage then gets as shown below.

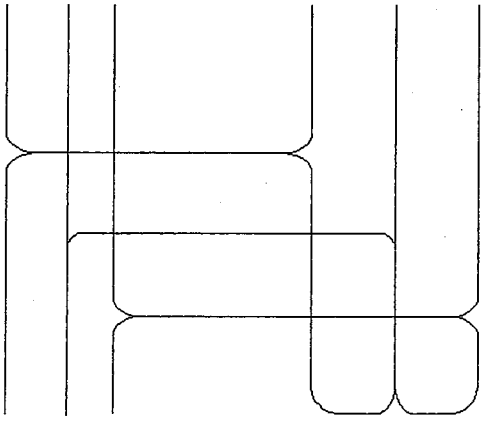


Figure 28. Signal-rails for the EB-diagram figure 27.

In this case the constraint is *holonomic*, i.e. it can be formulated as a reduction in the number of independent coordinates. The original 6 coordinates x_1, y_1, z_1, x_2, y_2, z_2 (not shown in figure 26) can be expressed as functions of 5 *independent generalized coordinates*, that may be the three coordinates of the center of mass plus two angular coordinates that gives the direction-vector between the two particles. We shall not in this section enter the discussion of, how an EB-diagram for the generalized coordinates and momenta can be constructed, but return to this question in a later section.

3.3 Rolling on a rail.

Another example concerns *rolling*. We consider a rigid, massless wheel, that rolls on a rail. On the rim of the wheel is fixed a particle of mass m, that as influenced by a conservative force-field with the potential U(x.y), where x and y are the cartesian coordinates of the particle (the rail has y=0).

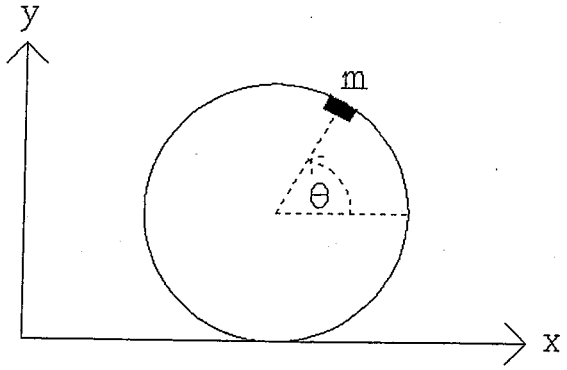


Figure 29. Wheel on a rail.

In this case there is only one generalized coordinate, that can be the angle θ. However, to calculate the force on the particle we need to know its x- and y-coordinates, so we need the non-linear functions cosθ and sinθ. For showing the principle in the construction of non-linear functions by linear governings of transformers from level-variables, we shall just see, how it is done in this case. We introduce two levels *c* and *s* (to be, respectively cosθ and sinψ) in addition to θ. We then have:

$$c^{\bullet} = -s\theta^{\bullet} \;;\; s^{\bullet} = c\theta^{\bullet} \qquad (11)$$

where the dot over the symbols denotes differentiation with respect to time. These relations are expressed by the following diagram:

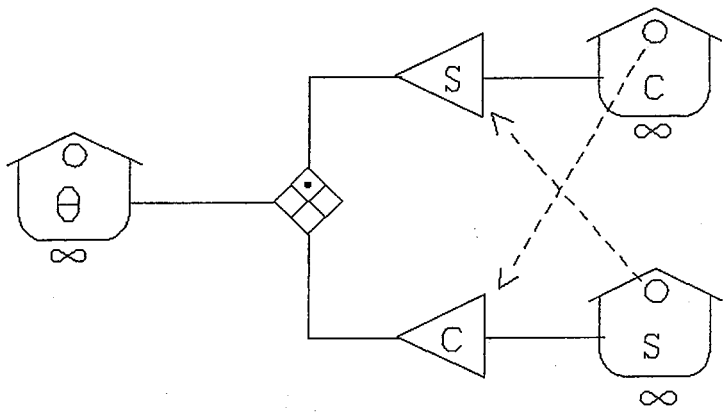


Figure 30. Construction of c=cosθ og s=sinθ by linear governings.

To get the right functions one has to choose initial values, so c=1 and s=0 for θ=0. One of the storages in figure 30 must have differential causality, and that is not generally possible for active storages, but when they, as here, are all cyclical the problem disappears, for when all output-efforts are zero, the conservative effort-relation for the x-junction is automatically satisfied.

The proper (external) forces that act on the particle, we shall represent generally by a potential function U(x,y), and this must then in the diagram be shown by a higher icon: *the multiport storage for potential energy*, here with two ports for x and y.

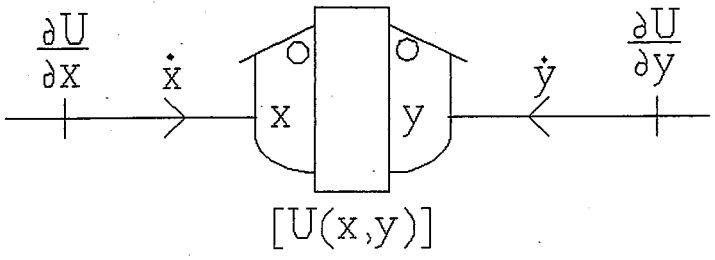


Figure 31. Two-port storage for potential energy.

The condition, that the wheel is rolling on the rail is the following constraint between the angular velocity and the particle's two velocity-components:

$$x^{\bullet} = r \cdot (c^{\bullet} - \theta^{\bullet}) \; ; \; y^{\bullet} = r \cdot s. \qquad \textbf{(12)}$$

where r is the wheel's radius. We can then draw the diagram, where the rolling-condition (12) enters with the parts on figure 30 and 31:

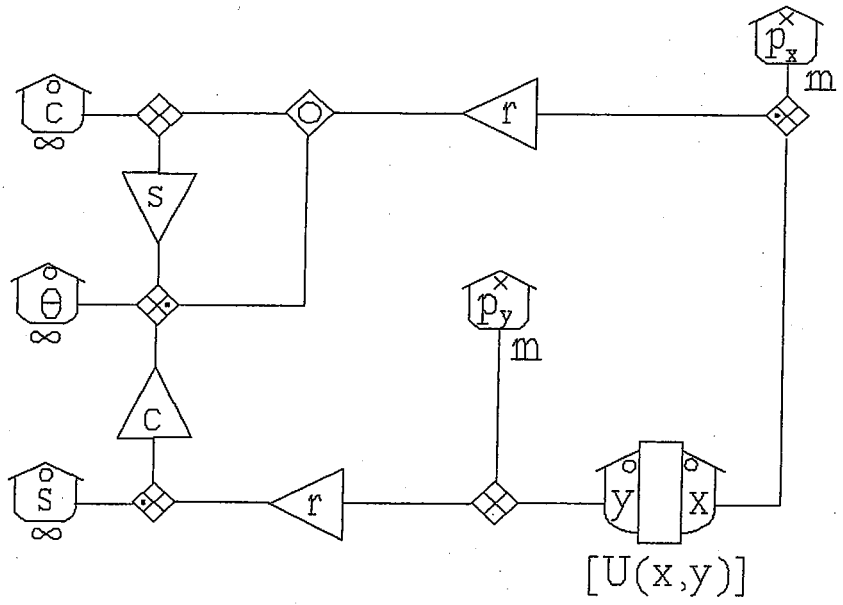


Figure 32. EB-diagram for wheel on rail (figure 29).

We have here omitted governing arrows from the s- and c- storages to the two transformers, because the governings are apparent by the transformer- symbols.

3.4 Loops.

A causal analysis of the diagram figure 32 shows that one of the two x-storages must be subjected to differential causality, corresponding to the existence of one causality-constraint, represented by the o-junction in top of the figure. Again, this is a holonomic constant allowing a reduction of the number of coordinates from two (x and y) to one (τ). If we choose differential causality of the p_y-storage, we get a signal-rail-structure as shown below.

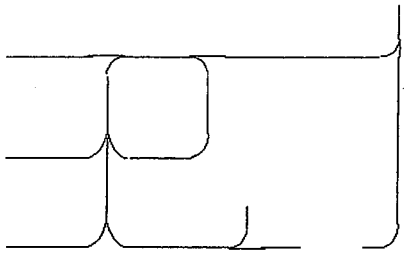


Figure 33. Signal rails for figure 32.

In this diagram appears a *loop*, i.e. a signal-route, that turns back in itself. The loop in the energy-bond signal-rails corresponds to two signal-loops, because signals run both ways in the energy bonds. The *gain factor* for an effort-signal, that runs once around clockwise in this loop, is calculated in the following way: Every x gives a factor -1, the dot also gives -1, so two xes and a dot gives -1. The transformer gives 1/s, all in all -1/s. A flow-signal running counterclockwise has the same gain-factor, and it will always be true, that a loop only containing junctions and transformers (or -ducers, but no gyrators) has the same gain-factor for efforts and flows. Such a loop is a *simultaneous feedback* and leads to an equation, that must be solved "by hand". A loop with gain-factor G of the type, we have in figure 30 and 31, can be reduced to the following structure:

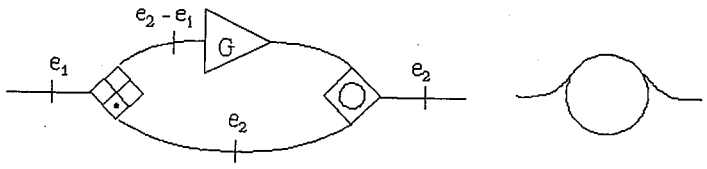


Figure 34. Loop with gain-factor G.

When we here have weak causality, i.e. we wish to calculate e_2, expressed by e_1, the loop leads to an equation, we then must solve:

$$e_2 = G \cdot (e_2 - e_1) \Rightarrow e_2 = -\frac{G}{1-G} \cdot e_1 \qquad \textbf{(13)}$$

We could, however, have acted as if there were strong causality, i.e. that we knew e_2 in advance and would calculate e_1. Then there would be no loop, and the diagram gives directly:

$$e_1 = (1-1/G)e_2 \qquad \textbf{(14)}$$

Which we also might have found using the icon-formula:

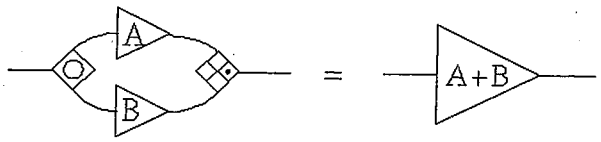


Figure 35. Addition of transformers.

We see from (13) that a loop becomes critical, when its gain-factor is 1. In our example with the wheel this happens for s=-1, i.e. when the mass m is down in the touching point between the wheel and the rail. In that situation it will not work to assign differential causality to the p_y-storage, so one must do it to the p_x-storage instead. If we had chosen this solution from the beginning, there would have been no loop in the signal-routes, and we would have avoided the problem with the gain-factor, that may become 1. Although the energy-bond-structure does allow different types of causality, one cannot be certain, that they are equally good for the calculations. Drawing the signal-rails is a good method for finding the best form of causality, which generally will be the one with the fewest number of loops.

3.5 Forces of inertia.

Differential causality of x-storages is an unavoidable consequence of velocity-constraints, and the forces of inertia following from this (m.dv/dt) is a memento to the model-builder, that these constraints may break down in the high-frequency-limit. If a car tries to accelerate very quickly from start, when the light turns green, the tires will scream against the road, signifying, that the rolling-constraint breaks down under the inertial stress, and a sink appears, that accounts for the friction between the tires and the road.

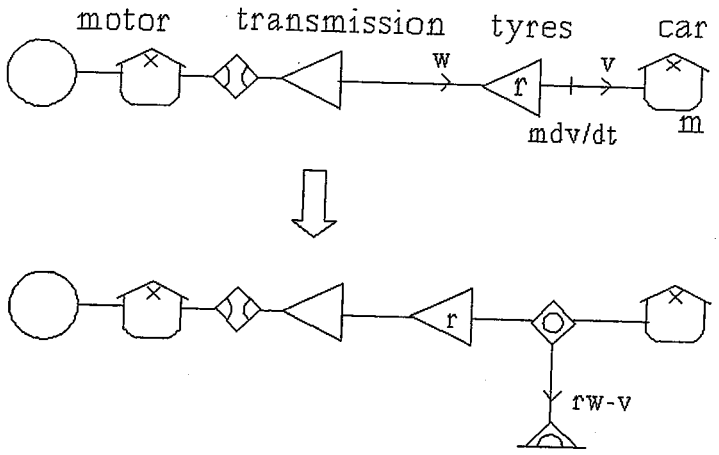


Figure 36. Breakdown of rolling-constraint by fast start of car.

3.6 Rolling sphere.

As a last example we shall look at a non-holonomic constraint: A sphere rolling on a plane. It requires 5 generalized coordinates to define the position of the sphere: 2 rectangular coordinates to give the place of the center over the plane and 3 angles (e.g. the Euler angles) to give the orientation of a coordinate system fixed to the sphere relative to the laboratory-system. The rolling condition gives in this case two velocity-constraints, but it is not possible to reduce the number of coordinates, because the two constraints are not integrable. The reason for this is that the sphere can rotate about a vertical axis without moving in the plane, so for every position in the plane all orientations are possible.

The usual methods of analytical mechanics, Lagrange- and Hamilton-formalism have a shortcoming against non-holonomic constraints, but, as we shall see, they do not present a serious problem for an energy-bond-reticulation.

We shall assume, that the mass-distribution of the sphere is symmetric, such that the mass density only depends on the distance from the center, for this means, that the

sphere's moment of inertia is a constant, independent of the actual axis of rotation. If the sphere is massive with a constant mass-density (billiard-ball), is $I=(2/5)MR^2$, where M is the sphere's mass and R its radius. If all the mass lies out at the surface (pin-pong-ball), the moment of inertia is $I=(2/3)MR^2$.

That the moment of inertia thus is a scalar (and not a 3*3 symmetric matrix, as we shall consider later) means that the angular momentum vector always has the same direction as the instantaneous rotation-vector:

$$\vec{L} = I \cdot \vec{\omega} \qquad \textbf{(14)}$$

The two rolling-constraints then only concern the x- and y-components of the rotation-vector: ω_z describes just a "neutral spin" that does not make the sphere move in the plane, but ω_x makes it roll in the negative y-direction, and ω_y rolls it in the positive x-direction. If the plane itself moves relative to the laboratory-system, such that the touching point has the velocity components v_x and v_y, the center-coordinates x and y will change with the velocities

$$x^{\bullet} = V_x + R\omega_y \; ; \; y^{\bullet} = V_y - R\omega_x \qquad \textbf{(15)}$$

With the introduction of linear x-storages for the angular momentum and the linear momentum we get an EB-diagram that is divided in two separate halves:

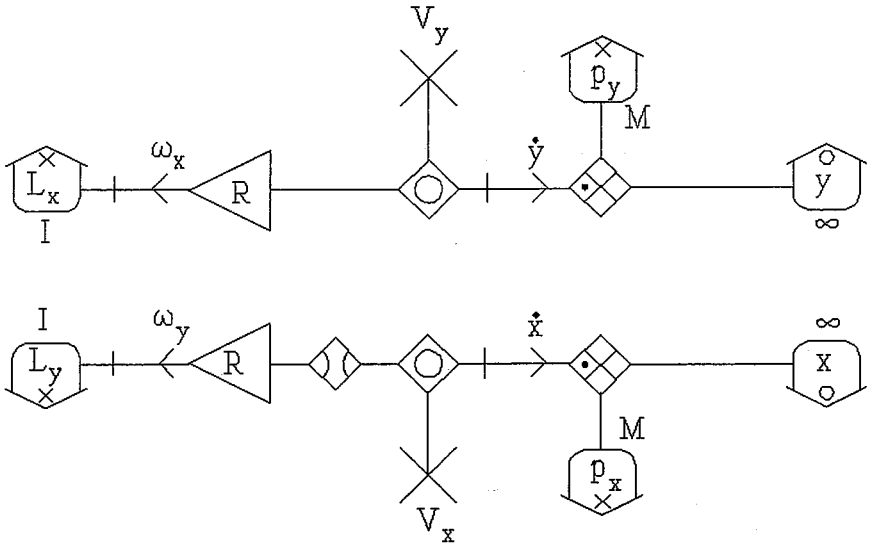


Figure 37. sphere rolling on a moving plane

Here is denoted differential causality on the two p-storages. The force of inertia from these storages then gives the following torque, that determines the rate of change of the angular momentum:

$$M_x = L_x^{\bullet} = RM\, y^{\bullet\bullet} \;;\; M_y = L_y^{\bullet} = -RM\, x^{\bullet\bullet} \qquad \textbf{(16)}$$

The two halves of the diagram can be brought together, if the velocities v_x and v_y depend on the position of the touching point (x,y). This is the case for the famous example **rolling sphere on a rotating disc**. If the disc rotates with the constant angular velocity **Ω**, we have:

$$V_y = \Omega x \;;\; V_x = -\Omega y \qquad \textbf{(16)}$$

The diagram for this motion can then be drawn with a fixed flow-source **Ω**, that acts through two governed transducers x and y. The velocity of the sphere is now obtained from (16) and (18):

$$x^{\bullet} = R\frac{L_y}{I} - \Omega y \;;\; y^{\bullet} = \Omega x - R\frac{L_x}{I} \qquad \textbf{(17)}$$

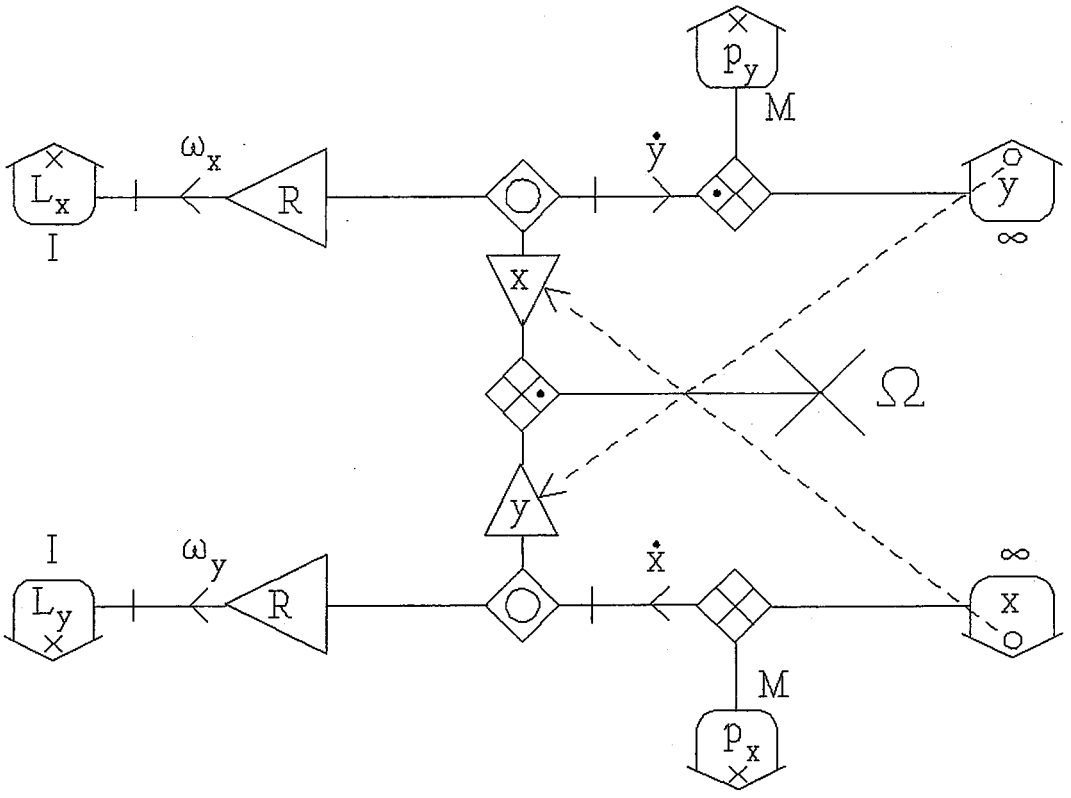


Figure 38. Sphere rolling on a rotating disc.

By one further differentiation and use of the angular momentum theorem (17) we then get:

$$x^{\bullet\bullet} = -\frac{\Omega y^{\bullet}}{1+\frac{MR^2}{I}} \; ; \; y^{\bullet\bullet} = \frac{\Omega x^{\bullet}}{1+\frac{MR^2}{I}} \qquad \textbf{(18)}$$

which can be integrated to

$$x^{\bullet} = -\alpha\Omega y + v_{0x} \; ; \; y^{\bullet} = \alpha\Omega x + v_{0y} \qquad \textbf{(19)}$$

where we have introduced the symbol α for the dimension-less constant in (20):

$$\alpha = \left(1+\frac{MR^2}{I}\right)^{-1} \qquad \textbf{(20)}$$

that will be 2/7 for a billiard-ball and 2/5 for a ping-pong-ball. Insertion of (21) in (20) then gives:

$$x^{\bullet\bullet} = -(\alpha\Omega)^2\left(x + \frac{v_{0y}}{\alpha\Omega}\right);\ y^{\bullet\bullet} = -(\alpha\Omega)^2\left(y - \frac{v_{0x}}{\alpha\Omega}\right) \qquad (21)$$

We see from this, that the sphere will move in a *circular orbit* with center in the point $(v_{0y}/(\alpha\Omega), v_{0x}/(\alpha\Omega))$. The angular velocity of this motion will be $\alpha\Omega$.

The example shows, that velocity-constraints can lead to a behaviour, that immediately seems strange. If one considers the case from a rotating system, that follows the disc, one would think, that the centrifugal force would pull the sphere further and further away from the center. This would actually happen, if the sphere begins to slide instead of rolling. Seen from the rotating system the orbit will be a rosetta, that is closed if and only iff α is a rational number.

We have with this example seen, that non-holonomic constraints can be treated with the EB-technique, but in the following we shall assume, though, that they are holonomic, so that it is possible to introduce independent generalized coordinates.

Appendix

Model-treatment with CTS

A1. Generally about the program.

CTS — the name means **Continuous Time Simulation** — is a program for numerical integration, graphical exposition, and file-administration of dynamical models, that are described with a set of connected first order differential equations. The program is for DOS-PCs and works under the Turbo Basic compiler from Borland and is accessible from RUC's local net under IMFUFA's menu. From here one can also get a thorough documentation and user-manual of the program. CTS is worked out by the author in collaboration with Heine Larsen.

In CTS the model is defined by the set of *level-variables*, LE(1),----,LE(n), that are defined by the user, and LE(0), that is *the time*, t, that is administered by CTS. The user must write his model-definition as a set of *rate-equations* in Basic in a frame-program. Herein is also inscribed output- and other equations. Both rate- and output-expressions must be defined by the set of level-variables and with user-defined external constants

The method of integration in CTS is a fourth-order Runge-Kutta-method with variable step-length. Thus, the user shall not denote the step-length, but gives instead a number, DG, that is the number of significant digits, the results shall be valid to and be shown with. Besides, the user fixes an output-interval, OI, that, firstly, is the maximal step-length, and, secondly, a fixed interval for output, and, finally, governs the program's first guess at the step-length, that is put to 1/8 of OI. A third CTS-constant, the reference-level LR has also significance for the accuracy, as the error, by and large, is calculated as a relative error to LR.

CTS operates with four types of numbered data-files:

a) *Model-files* contain information about the number of levels-variables as well as their values and meaning. Besides, there are names and comments to the output-variables, and names, values, and comments to external constants.

b) *Result-files* contain values of output-variables from an entire run.

c) *Scale-files* put values on the axes for plots of output-variables, including time. CTS also contains possibility for automatic fixation of scales.

d) *Graphics-files* can be made both in an internal format for use in CTS and in an external format (PCX) for use in other programs, e.g. WordPerfect.

A2. Briefly about the integration-method.

Although the CTS-user does not need to know about the method of integration, we shall still briefly sketch, how it works. Let us for simplicity assume, that we just have a single differential equation

$$\frac{dx}{dt} = f\,(x, t) \qquad \textbf{(A22)}$$

Let us further assume, that we know the value of x, $x=x_0$ for the time $t=t_0$, and we want to calculate the value $x=x_3$ to the time $t_3= t_0+\Delta t$, where Δt is the step-length. This interval is now divided into three sub-intervals, by defining for i= 0,1,2,3

$$t_i = t_0 + i\frac{\Delta t}{3} \qquad \textbf{(A23)}$$

Then we define coefficients u_i as

$$u_0 = u_3 = 1\ ;\ u_1 = u_2 = 3 \qquad \textbf{(A24)}$$

Now is made three projections x_1, x_2, x_3, of x to the times t_i on the basis of the increments

$$k_i = \frac{\Delta t}{3} f\,(\,x_i, t_i\,) \text{ for i} = 0,1,2,3 \qquad \textbf{(A25)}$$

The three projections are made with the formula

$$\left.\begin{aligned} x_i &= x_{i-1} + g_i \text{ for } i = 1,2,3 \\ \text{where } g_0 = 0 \wedge g_i &= u_{i-1} k_{i-1} - 2 g_{i-1} \text{ for } i = 1,2,3 \end{aligned}\right\} \quad \textbf{(A26)}$$

Finally, the last projection is corrected by the equation

$$x_3 = x_0 + \frac{3}{8} \sum_{i=0}^{3} u_i k_i \quad \textbf{(A27)}$$

And after this the integration can proceed with the next step, where the old t_3 is the new t_0 and where the calculated x_3 becomes the new x_0.

Based on the first two projections CTS estimates, if it is necessary to reduce the step-length. The first projection gives

$$x_1 = x_0 + \frac{\Delta t}{3} f(x_0, t_0) \quad \textbf{(A28)}$$

That is a simple Euler-projection, where one just "goes out of the tangent" in the beginning point of the interval. The second projection is:

$$x_2 = x_0 + \frac{2\Delta t}{3} f(x_1, t_1) + \frac{\Delta t}{3} [f(x_1, t_1) - f(x_0, t_0)] \quad \textbf{(A29)}$$

The last term gives information about the error of a simple Euler-integration. As this error is proportional to Δt^2, and as the error of the completed fourth order Runge Kutta calculation is proportional to Δt^5, CTS makes a comparison of the from the second projection estimated error with the of the user accepted error to the power 2/5. If the error is too big, the step-length is halved, and the calculation of the step is started again from the beginning. If the error is too small, the calculation of the step is finished, but before calculation of the next step, the step-length is doubled. In this way time-intervals become infinitely divisible, which justifies the claim, that CTS-time is continuous.

The user-chosen output interval serves as an upper limit for the step-length. By the start of an integration the step-length is put to 1/8 of OI, so if OI is very small compared to the characteristic times of the model, CTS will after three doublings of the step-length use OI as a constant step-length. Every time a whole number of

output intervals from the starting time, CTs must produce output, so if the distance to an output-time is less than the actual value of the step-length, the step-length is adjusted to hit the output-time. After output the integration is resumed with the previously used step-length.

A3. Model-example

We look at *the elastic pendulum*, figure 42 (here reproduced as figure A1)

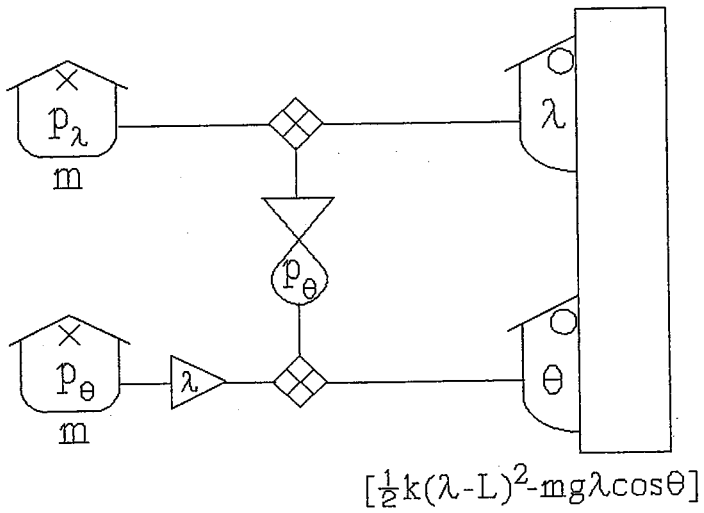


Figure A1. Elastic pendulum.

As level-variables are used:

LE(1) = λ, length of the pendulum (m). Initial value 1.
LE(2) = θ, angle from bottom (rad). Initial value 0.
LE(3) = p_λ, radial momentum (k*m/s). Initial value 0.
LE(4) = p_θ, tangential momentum (k*m/s). Initial value 7.

As output variables are used:

$x = \lambda \sin\theta$,the particle's horizontal coordinate (m).
$y = -\lambda \cos\theta$, the particle's vertical coordinate (m).
$E = (p_\lambda^2 + p_\theta^2)/(2m) + \frac{1}{2}k(\lambda - L)^2 - mg\lambda\cos\theta$, the energy (J).

The calculation of the energy serves as a check on the accuracy of the integration. The energy shall be constant, but errors of the Runge-Kutta-integration cause a slightly decreasing tendency. By the run over 1000 steps (t from 0 to 20) is the

relative decrease less than 10^{-3}.

As *external constants* are used:

m = 1. The Particle's mass (k).
k = 100. Elastic modulus of spring (k/s^2).
L = 1. Equilibrium length of the spring (m).
g = 9.82. acceleration of gravity (m/s^2).

Besides, the following CTS-constants are used:

DG = 4. Number of significant digits.
LR = 1. Reference level.
OI = 0.02. Output interval.

The *rate-equations* of the model can now be formulated:

RA(1) = LE(3)/m
RA(2) = LE(4)/(m*LE(1))
RA(3) = k*(L-LE(1)) + m*g*cos(LE(2)) + LE(4)^2/(m*LE(1))
RA(4) = - m*g*sin(LE(2)) - LE(3)*LE(4)/(m*LE(1))

As the rate-equations shall be run through many times during the integration (at least 4 times per step), it may be an advantage to shorten calculations by using *internal constants* that are only calculated once at the start of integration.
One may, e.g. in the rate-equations above introduce the internal constant mgg=m*g and thereby save a couple of multiplications in the rate-equations.

When the integration is finished, the output-variables are stored in the machine's memory and may be saved from there to a result-file. The results make possible among other things two-dimensional plots, where every output-variable (including time)can be assigned to each of the two axes. As scale on the axes one may let CTS use the during integration determined minimum- and maximum-values of the output-variables, or one may oneself determine a scale.

On the plot below of x and y for the elastic pendulum is fixed a scale from -2 to 2 for x and from -1.7 to 1.3 for y. As the dimensions of the screen by VGA-graphics is ca ¤/3, one hereby obtains a plot, that resembles the motion of the particle in real space.

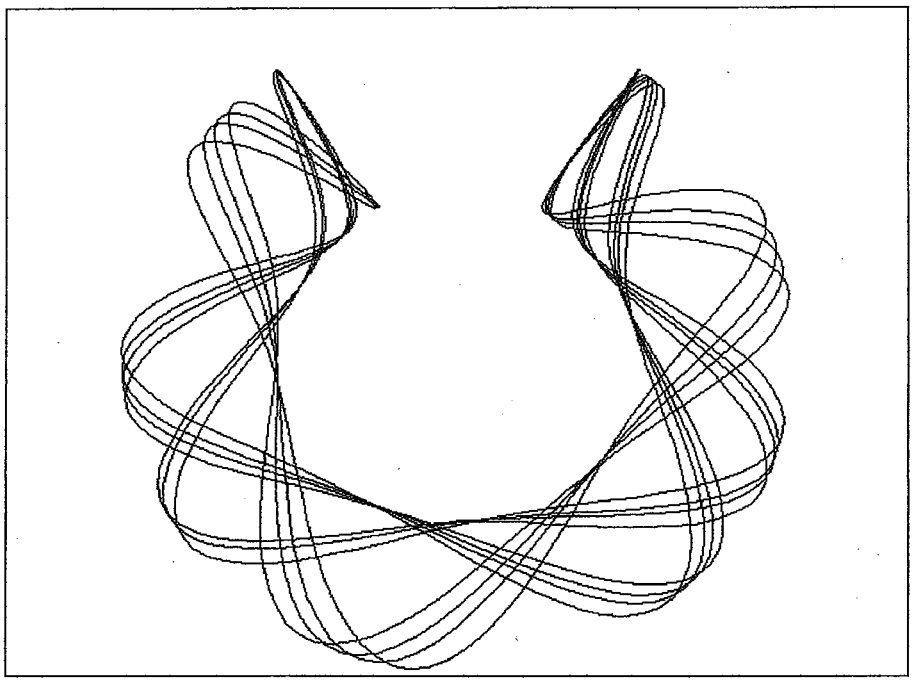


Figure A2. Motion of the particle in the elastic pendulum.

This motion seems to be *almost periodic*, which is understandable, considering the interplay between the two inherent periods for the pendulum's and the spring's oscillations. For systems with three or more level-variables it is normal to find almost periodic behaviour, that in special cases can be periodic.

A third possible form of motion, that requires at least three level-variable, is *chaos*. The elastic pendulum is not chaotic, but the double pendulum shows both almost periodic and chaotic motions. It is usually easy to decide by visual inspection of the solution curves, if there is chaos or not, but in some cases there may be a need of quantitative methods, such as calculation of *Liapounov-exponents, to decide the question. CTS* offers various possibilities for investigation of chaotic motions, but it will take us too far to discuss them here.

References.

1. Lennart Ljung and Torkel Glad, **Modelbygge och Simulering, ch. 6,** Studentlitteratur, Lund, Sweden, 2004.

PART II, GENERALIZING

Vector Bonds and Tensors

1. General semiotics and model-building.

When physicists talk about the use of mathematical models they often overlook the fact that mathematics in itself cannot give a model any credibility. A mathematical symbol, like a real number does not have any physical significance, unless it refers to definite measuring-prescriptions, and measuring-prescriptions don't have physical significance, either, unless there are signs outside any linguistic context. A physical *semiotics* (doctrine of signs), intending to explain from where signs in models get their meaning, must necessarily be *realistic*, i.e. presume that there exist signs outside human consciousness. Such a Semiotic is founded by the american philosopher Charles Sanders Peirce (1839-1914). In a manuscript from 1897 (CP 2.227-9, Buchler p. 99) he says:

> "A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the *ground* of the representamen."

So, a sign is, according to Peirce a relation between three factors:

1. A *representamen*, or *primary sign, that is the vehicle, or carrier of the sign, apart from any reference and meaning.*

2. A so-called *object*, that is what the sign refers to, or "speaks about". The sign-object is of course not "das Ding an sich"; it is not a physical object, but rather a certain feature or abstract quality of such an object, called the *ground* of the sign.

3. An *interpretant*, that comprises or interprets the relation between 1. and 2. The interpretant can be said to contain the *meaning* of the sign, because it, as Peirce says, "contains the conception that it *itself* enters in the same triadic relation with the object as the primary sign".

Peirce gives a more abstract, relation-logical description of the sign-relation in a manuscript from 1902 (CP 2.274):

> "A *Sign* or *Representamen*, is a First which stands in such a genuine triadic relation to a Second, called its *Object*, as to be capable of determining a Third, called its *Interpretant*, to assume the same triadic relation to its object in which it stands itself to the same object."

The sign-relation is irreducibly triadic, it cannot be reduced to dyadic relations, but it contains the three dyadic relations R-O, O-I, and I-R, and each of these three dyads is mediated by the third factor; i.e. the relation of reference from representamen to object (R-O) is mediated by the interpretant. The interpretant can be regarded as representamen in a new sign-relation, and this semiosis (O-I relation), that the object, mediated by the representamen, becomes a new representamen to be interpreted further in a new sign-relation is clearly brought out, if the single sign-relation is depicted as O-R-I, or I-R-O.

The two diagrams O-R-I and I-R-O are called, respectively, the *progressive* and the *retrograde* representation of the sign-relation. We can imagine that the sign-vehicle R originates as a *signal* from the object O, which later becomes transformed to the interpretant I, and thus, the progressive diagram O-R-I, where we proceed in the reading-direction from left to right, appears most natural. However, in a mathematical context, like the present, There are strong reasons for preferring the retrograde diagram I-R-O. This is because transformation of some "signal" x by some "operator" F and later by some other operator G is usually described by applying operators from the left, subsequently as they are applied, thus GFx, so that the chain of transformations proceeds from right to left. For this reason (that will appear more pronounced in later chapters) we prefer the retrograde representation I-R-O.

This diagrammatic representation, where the factors of the sign-relation (R,O, and I) are connected with "sign-links" (—) is invented by the biochemist Mogens Kilstrup, inspired by the notion of energy-bonds (Kilstrup 1997).
As remarked, the interpretant I can be regarded as a new sign, leading to a new interpretant J. This is clearly brought out by considering the sign-link diagram for a "cascade-coupling of two sign-relations J-I-R-O, consisting of the two elementary sign-relations 1: I-R-O. and 2: J-I-R (remember, we proceed from right to left). We see, that the interpretant I of the first relation appears as the representamen of the second relation, and that the representamen R of the first relation appears as the object of the second relation.
These connections are brought out by the following Peirce-quote, continuing the previous (CP 2.274) where he speaks of I as "the Third" and of J as "the Third's Third":

> "The Third must indeed stand in such a relation, and thus must be capable of determining a Third of its own; but besides that, it must have a second triadic relation in which the Representamen, or rather the relation thereof to its Object shall be its own (the Third's) Object, and must be capable of determining a Third to this relation. All this must be equally true of the Third's Thirds and so on endlessly; and this, and more, is involved in the familiar idea of a sign."

What Peirce refers to as "the familiar idea of a sign" seems to be the idea of *unlimited semiosis*, that a sign can be interpreted and re-interpreted indefinitely, so that chains like J-I-R-O can be endlessly prolonged to the left.

Although the *physical* conception of the sign relation I-R-O places the Object first, as the place where the signal originates and the Representamen second we must remember that Peirce's *logical* conception of the sign describes the Representamen as a First and the Object as the Second; the Object only comes into being as Object of our attention after our attention has been drawn to it by the Representamen. And the Interpretant is Third, because it mediates and presupposes the relation between First and Second.

The logical sequence of the factors of the sign-relation (1R, 2O, 3I) is the prototype of Peirce's phenomenological (Phaneroscopic) distinction between three *modes of being*, which he simply calls First, Second, and Third

Peirce describes the three categories in the following quote from 1903 (CP 1.23, Buchler p. 75):

> "My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future."

Let us begin by considering actuality, and try to make out just what it consists in. If I ask you what the actuality of an event consists in, you will tell me that it consists in its happening *then* and *there*. The specifications *then* and *there* involve all its relations to other existents. The actuality of the event seems to lie in its relations to the universe of existents. A court may issue *injunctions* and *judgments* against me and I not care a snap of my fingers for them. I may think them idle vapour. But when I feel the sheriff's hand on my shoulder, I shall begin to have a sense of actuality. Actuality is something *brute*. There is no reason in it. I instance putting your shoulder against a door and trying to force it open against an unseen, silent, and unknown resistance. We have a two-sided consciousness of effort and resistance, which seems to me to come tolerably near to a pure sense of actuality. On the whole, I think we have here a mode of being of one thing which consists in how a second object is. I call that Secondness.
Besides this, there are two modes of being that I call Firstness and Thirdness. Firstness is the mode of being which consists in its subject's being positively such as it is regardless of aught else. That can only be a possibility. For as long as things do

not act upon one another there is no sense or meaning in saying that they have any being, unless it be that they are such in themselves that they may perhaps come into relation with others. The mode of being a *redness*, before anything in the universe was yet red, was nevertheless a positive qualitative possibility. And redness in itself, even if it be embodied, is something positive and *sui generis*. That I call Firstness. We naturally attribute Firstness to outward objects, that is we suppose they have capacities in themselves which may or may not be already actualized, which may or may not ever be actualized, although we can know nothing of such possibilities, except so far as they are actualized.

Now for Thirdness. Five minutes of our waking life will hardly pass without our making some kind of prediction; and in the majority of cases these predictions are fulfilled in the event. Yet a prediction is essentially of a general nature, and cannot ever be completely fulfilled. To say that a prediction has a decided tendency to be fulfilled, is to say that the future events are in a measure really governed by a law. If a pair of dice turns up sixes five times running, that is a mere uniformity. The dice might happen fortuitously to turn up sixes a thousand times running. But that would not afford the slightest security for a prediction that they would turn up sixes the next time. If the prediction has a tendency to be fulfilled, it must be that future events have a tendency to conform to a general rule. "Oh," but say the nominalists, "this general rule is nothing but a mere word or couple of words!" I reply, "Nobody ever dreamed of denying that what is general is of the nature of a general sign; but the question is whether future events will conform to it or not. If they will, your adjective 'mere' seems to be ill placed." A rule to which future events have a tendency to conform is *ipso facto* an important thing, an important element in the happening of those events. This mode of being which *consists*, mind my word if you please, the mode of being which *consists* in the fact that future facts of Secondnes will take on a determinate general character, I call a Thirdness."

In short, we can describe the three categories, or modes of being as 1: potentiality (being), 2: actuality (existence), and 3: generality (reality). As Peirce did, we shall use these categories for creating a classification system for signs in general. However, when Peirce applies his categories to each of the *factors* in the relation I-R-O, thus creating $3^3 = 27$ possible sign classes (of which some are excluded by what I call a "selection rule"), we shall use a slightly different procedure, and, in accordance with Kilstrup, apply the categories to the *sign links* instead.
Kilstrup proposes the following graphic representations of the three categories of sign links

1:	B-/-A	potential
2:	B:-:A	actual
3:	B<—:A	general

In the simplest description of the sign, as a link from the Object to the Interpretant (I — O) we thus get Peirce's most basic division into three types of signs:

1: *Icons*
2: *Indices*
3: *Symbols*

So, *Icons* have a *potential* reference to their Object (like a geometrical sketch, an abstract painting, or a piece of music), *indices* have an *actual* (causal) connection to their Object (like a photograph or a footprint in the sand), and *symbols* have a *general* connection to their Object (habitual or conventional rule of interpretation). When considering chains of sign-links, like J-I-R-O the link-categories cannot be combined arbitrarily, but a *selection rule* has to be respected. If the R-O-link is only potential (category 1: R-/-O) then we cannot have an actual I-R link (category 2: I:-:R) for the Representamen, as a mere possibility, cannot have an actual link to an Interpretant. If we proceed to the left in a long chain of sign links and encounter a category 1 link, then the chain is cut off at that point and cannot continue further to the left. Likewise, Generality of a link (category 3) cannot be obtained, unless all the previous links to the right of it are of category 3. Generalizing these observations we get the following *selection rule*:

The link-categories cannot increase when we move from right to left in the chain.
The appearance of the sign-link-diagram I-R-O may seem at variance with Peice's insistence on the sign relation being genuinely or irreducibly triadic, i.e. not reducible to dyadic relations. The linear diagram I-R-O *looks* as if it might be decomposed into the two dyadic relations I-R and R-O. However, the selection rule ensures that these two relations are not independent, because the I-R category cannot exceed the R-O category. So, the relation is irreducibly triadic, after all.

For a chain of n links it is easily shown that the number of sign classes, respecting the selection rule, is:

$$S(n) = \tfrac{1}{2}(n+1)(n+2) \qquad (1)$$

Thus, for a three-link chain (J-I-R-O) there are not 3^3=27 classes of signs, but only ½.4.5 =10 classes.

The link-categories can be connected to Peirce's categorization of the factors R,O, and I. The category of each factor is then identified with the category of the link leading to the next factor (thus: R-category (r) = R-O category, O-category (o)=O-I category, i.e. the smallest of the R-O and the I-R categories, and I-category (i)= J-I category. In this way the sign classes are identified with the number-triplets (ior)

where each number r, o, and i can be 1,2,or 3 and the selection rule $i \leq o \leq r$ is obeyed. So, by arranging the categories in the logical sequence as (ior) we have the same selection rule for the number-triplets as for the link diagram I-R-O, that the categories cannot increase when we move from right to left.

Peirce placed the ten classes, so obtained, in a so-called *tetraktys* — a number-magical figure that played a big role for the Pythagoreans. In the figure below the sign-classes are placed in the tetraktys with their coordinate-sets (ior) and their link-graphs I-R-O:

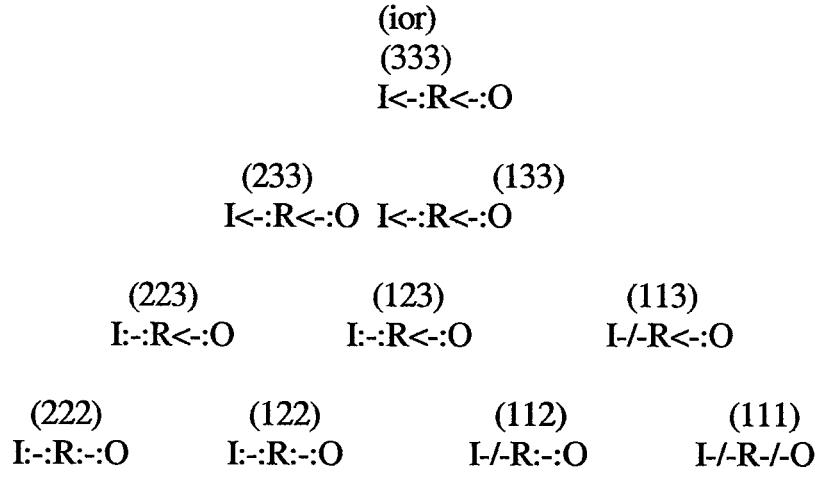


Figure 1. Tetraktys of sign classes with (ior) coordinates and I-R-O sign link graphs.

Peirce also introduced different names for the categories on the three places in the (ior) triplets. For r they are called 1. qualisign, 2. sinsign, and 3. legisign. Qualisigns are pure qualities, as the whiteness of the paper. Sinsigns are individual signs, as a random doodle you drew, when you spoke in telephone. Legisigns are signs that can be recognized as representatives for a general class, like a letter. With another terminology qualisign, sinsign, and legisign is called, respectively tone, token, and type. The tetraktys contains 1 qualisign, 3 sinsigns, and 6 legisigns.

For o the categories are the previously mentioned 1. icon, 2. index, and 3. symbol. The icon is a sign where the I-R link is potential. One may say that the icon puts the object in the shadow or replaces the object, which not necessarily needs to have physical existence. A drawn line may be an icon for the mathematical abstraction "a straight line", an energy bond is an icon for an idealized physical interaction. For the index the R-I link is actual. It may be a concrete physical cause-effect-relationship as between a meteor and a crater on the Moon, or it may be an automatic, uninterpreted reference, like proper names or cpr-numbers that only serve to distinguish one individual from another. The symbol has a general reference to its object, like a convention for interpretation of a word. Symbolic reference may also

be a habitual natural process, e.g. the action of an enzyme, that through a long evolutionary process has become suited for occupying a certain place in a certain molecule or a nucleic base-triplet coding for a certain amino-acid. The symbol demands that the tepresentamen R is a legisign, the index demands a legisign or a sinsign, while the icon does not demand anything of R. The tetraktys contains 3 icons, 4 indices, and 3 symbols.

For the interpretant the three categories are called 1. Rheme (or term), 2. dicisign (proposition), and 3. argument. A rheme is a singular term, like a word that doesn't in itself say anything. The meaning of the word is potential, and first when it is combined with other words to a sentence there appears an actual proposition, or dicisign. An argument consists of meaningfully connected propositions. In the tetraktys there are 6 rhemes, 3 dicisigns, and 1 argument. It may seem strange that the most complicated type of sign, the argument, only gets one place in the scheme, while simple terms have 6. <The explanation is that that an argument not can be fully described with a single sign relation. When we here describe it as (333) or <-:I<-:R<-:O we are just saying that it demands the highest degree of generality in the beginning of the sign-chain, which is a necessary vondition for, that the chain may continue with general sign links. These higher terms we can then leave to Rhetorics. In Physics, on the contrary, we are especially interested in simpler sign relations that may describe transformations of signs that come to us from the non-linguistic world to terms in theories of physics and its experimental praxis.

When we verbally name a given sign-class, e.g. (123), i.e. a legisign, an index, and a rheme,we usually read the coordinates from left to right, so we say: a rhematic, indexical legisign.

While Firstness in Peirce's doctrine of categories appears as completely non-composite, every higher category must contain the lower. The categories are *inclusive*, as the french philosopher Gérard Deledalle said (Deledalle 2000).
In order for a representamen to be a legisign (3), e.g. a representative of the letter-class "A", it must be an individually distinguishable sign, i.e. a sinsign (2), and its distinguishability builds upon a contrast between two sensory qualities, the whiteness of the paper and the blackness of the ink, i.e. qualisigns (1). The same feature is valid for the other places in the sign relation, O and I, because it rests in the definition of the categories.
A representative of a given sign-class (ior) can therefore be said to include all the sign-classes (i'o'r') for which

$$r' \leq r,\ o' \leq o \text{ og } i' \leq i \qquad (2)$$

In this way we get an ordering of the classes, although it is only partial, as the class

(223) neither contains nor is contained in the class (133).

An *analysis* of a sign, therefore, consists in an enumeration of all signs that are contained in the given sign. A *synthesis* of a sign is a process that starts in the lower right corner (111) of the tetraktys and gradually moves to the left and upwards, until it stops at the intended sign.

The purpose of the bond graph techniques and a wealth of similar formalisms is to deliver a tool for systematic synthesis of signs.

A sign-synthesis proceeds gradually upwards in the tetraktys without jumping over any class. Therefore, there are two fundamental steps in the synthesis: A category 1 can be raised to category 2, which we call *discrimination (D), and a category 2 can be raised to category 3, which we call* Abstraction (A). In figure 2 is illustrated how the synthesis of physical models, experiments, and theories proceeds. In the following chapters we shall see how the energy-bond-semiotics in its formalistic construction systematically proceeds through the two lowest layers of the tetraktys and how the applications of the formalism leads further towards general theoretical frames of concepts.

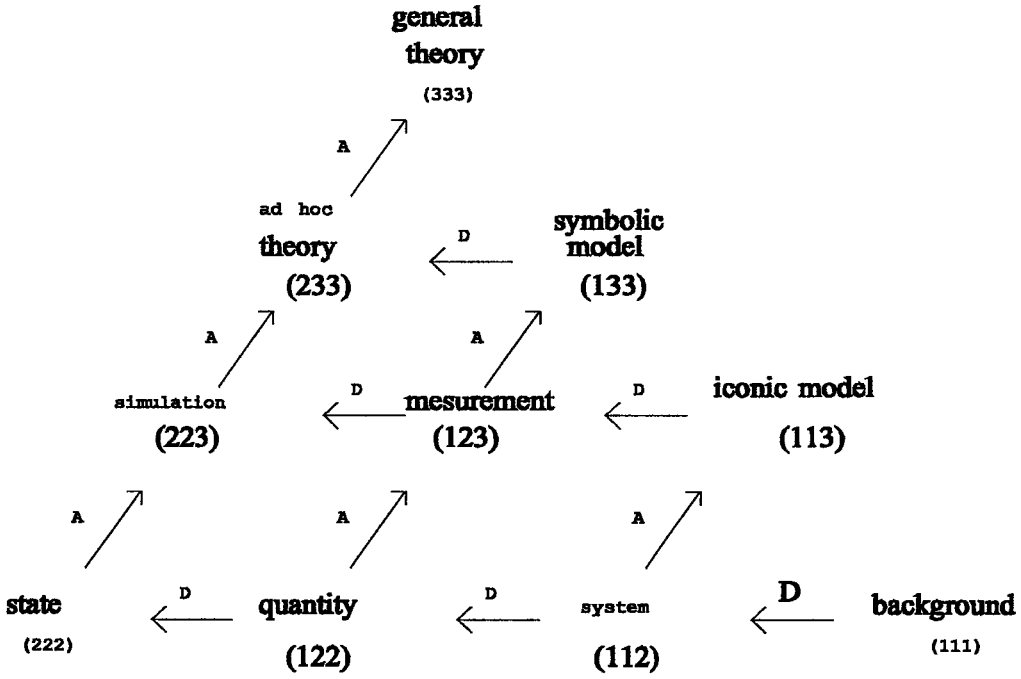


Figure 2. Synthesis of model, experiment and theory.

The bottom-layer of the general tetraktys, figure 1, containing the sign-classes (222), (122, (112), and (111) can be called *the concrete layer*. The signs in this layer have a non-linguistic character, because they are devoid of the abstract category 3. These are the signs that physics is all about. As there are four kinds of concrete signs, they can be grouped in two pairs of opposites and put together in a so-called *semiotic square*, an often used schematic form within the european semiotic tradition (Saussure, Greimas). When we here use such squares it must not be confused with that school's use of the same figure, for in the european tradition all signs are regarded as linguistic, but here it is exactly the non-linguistic signs that the scheme is concerned with.

The two pairs of opposites, or *dichotomies* for the concrete signs can be described as external/internal and even/odd. If the object-reference is actual or indexical we talk about external signs, if it is potential or iconic, we talk about internal signs. If the sum of the categories i+r+o is odd, the sign is odd, otherwise it is even. This division of the concrete signs is found again in the division of dynamical variables of the energy bond graph formalism. External signs are called *levels* and the internal are called rates. Even signs are called o-signs and odd signs are called x-signs.

Relations between these four classes can be referred to the second layer from the bottom of the tetraktys. Here we distinguish between *active* relations, that are *monadic* and corresponds to the sign-class (113), *reactive* relations that are (at least) *dyadic* and correspond to the sign-class (123), and *dissipative* relations that are *triadic* and correspond to the sign-class (223). In the energy bond formalism these three types of relations are represented, respectively, by the sources, the storages, and the sinks.

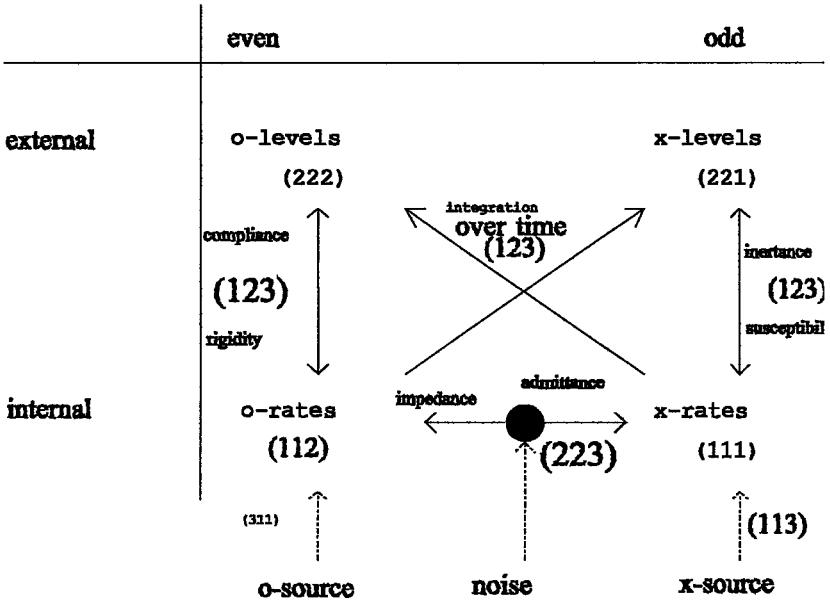


Figure 3. Concrete signs and relations in the energy bond formalism

In addition to the abstract names for the dynamical variables in figure 3 we also use some more common nouns, but the use of these must be suited to the context. For an electric system o-levels are *charges*, x-levels *magnetic fluxes, o-rates are* voltages, and x-rates *currents*.

In a less specified context we use the word *displacement* and the letter q for o-levels, the word *momentum* and the letter p for x-levels, the word *effort* and the letter e for o-rates, and the word *flow* and the letter f for x-rates.

2. Simple energy bonds and dynamical variables

The energy bond-technique is invented by an associate professor in Mechanical Engineering at M. I. T., Henry M. Paynter, and first described in a duplicated set of Class Notes for M. I. T. course 2.751 with the title "Analysis and Design of Engineering Systems" from 1960. The most important sources of inspiration for Paynter are Peirce's Semiotics and bond-graph-techniques, the general systems theory by Bertalanffy, Norbert Wiener, et al., the network-theory of Gabriel Kron, and the Energetics of the danish chemist J. N. Brønsted. The version of energy bond technique that is described and applied in previous texts by this author "Dynamik og diagrammer" (IMFUFA tekst nr. 8, 1978) and "Semiotik og systemegenskaber - 1" (IMFUFA tekst nr. 22, 1979) are tightly connected with Paynter's formalism, but deviates at certain points, mainly by the more systematic use of icons (shape-value notation) and in the indexing system for the bonds. The more general point of view, introduced by the present text, shall treat the energy bond variables as vectors in a general Riemannian metric. Hereby is obtained, firstly, a semiotic introduction to tensor-calculus in network-analysis, and, secondly, a stronger coupling of this traditional engineering topic to basic disciplines in modern physics, like general relativistic and classical analytical mechanics and Quantum Field Theories. The latter ambition is also found in Gabriel Kron's work (Tensor Analysis of Networks, 1939) and Léon Brillouin's (Wave Propagation in Periodic Structures, 1946), but somewhat attenuated by their limitation to traditional electrical network-diagrams that lack the semiotic completeness, provided by the energy bond technique.

In the beginning Paynter describes in "Analysis and Design-" how a model- or theory-building starts with delimiting a system, which corresponds to the discrimination from (111) to (112) in figure 2. The next phase is an abstraction from (112) to (113) by the use of general icons. This process Paynter calls *reticulation*, i.e. structuring in the likeness of a network

> "The rational process of endowing a system with structure we call *reticulation*. - - Since the structural attribute of a system which interests us most is the functional connectedness of its elements, the final step in the reticulation process is the sketching of the important relations and bonds of interaction among the elements and between each of the elements and the environment".

So we need general icons to describe "elements" and "bonds of interaction". Reticulation leads to a network-diagram where the elements appear as nodes and the bonds as branches.

Alongside with the reticulation discrimination goes on from (112) to (122), identifying and indicating (with indices) variables in the system, such that measuring prescriptions are attached to these variables and to the iconical model. Hereby the model builder becomes able to reach to a symbolic model (133), a set of equations between the so-defined quantities, such that the mathematization of the model is completed.

The *energy bond* or *interaction bond* is a general icon, that is only indirectly quantifiable, because the interaction is described in terms of internal variables, flows and efforts (comp. figure 3). The product of these is an energy flow, also an internal variable and, as such, not directly measurable. Paynter writes:

1. An *energy bond* may be conceived as an interaction; associated with each bond are two variables, the first pertaining to an *effort* and the second to a *flow*, their product yielding the *power* or energy flow rate.
2. Alternatively, an interaction may be conceived as a *bilateral signal flow* between two elements, thus attributing a direction of causality to the interaction.

An energy bond is a connection between two elements A and B. It has two *ports* to its environment and defines itself the ports of the elements to their environment.

Loosely speaking, the connection of the icon to its object is "picture-like", not as a photograph that is more like an imprint of its object, and therefore indexical, but rather like a sketch or a diagram making a base for further development of the sign-system. An icon must emphasize the features that the object needs to be more specified, and, preferably, nothing more. That is, what Peirce calls *the ground* of the object. For an energy bond it is essential that it connects exactly two system-components, so a suitable icon will be a piece of line or curve with two endpoints.

The energy bond icon forms the basis for *indication* of the inner variables flow and effort, f and e), whose product is the transferred energy flow. As the energy flow is a quantity with direction in space (from A to B or the other way, indication demands an *orientation*. For this an arrow is used: The product of flow and effort gives the energy flow in the direction of the arrow.

There is one further aspect that the indication is required to describe, namely the *causality*, as emphasized in the above Paynter-quote. An interaction bond is really describing an interaction in the full sense of the word, i.e. that the two system-components, connected by the bond, affect one another mutually via the bond. A acts on B via one of the two internal bond-variables, and B acts on A via the other. We shall use the following convention to denote the causality: First, we place a couple of "marks" on the bond, a couple of indexical sinsigns that are to stand for flow and effort, so one must be nearest to A and the other nearest to B. The sinsign that is closest to A indicates the inner variable that B acts on A with, i.e. it is input to A and output from B, and the other sinsign indicates the other internal variable, that A acts on B with, that is output from A and input to B. When these sinsigns are placed we can with indexical legisigns, f and e, show which of the two sinsigns that indicates the flow, and which indicates the effort. We can also provide the sinsigns with a shape that makes them legisigns, e.g. by using an x for the flow and an o for the effort. In this way the need of using fs and es is reduced; We can point to x and o and say "this flow and this effort we would like to have measured", whereby we are entering the way from the iconic model-type into the measurement-situation (123). However, we are still missing something before we can proceed so far.

In figure 4a is seen the premature indication-system, that is now introduced, and figure 4b shows with a *signal flow graph* how it refers to the causality (comp. the last Paynter quote)

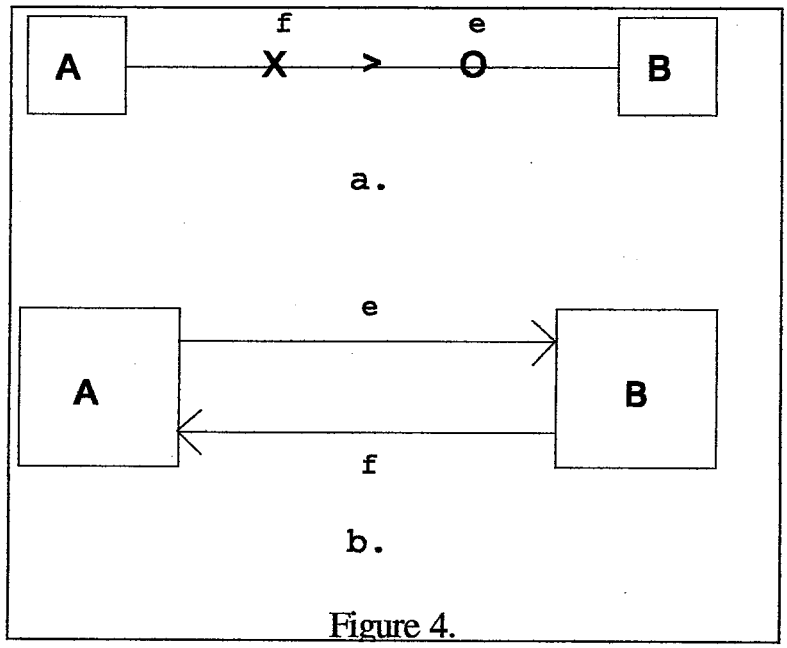


Figure 4.

Until now we have only loosely described flow and effort as "inner variables" belonging to the sign-classes (111) and (112), but said nothing about how we know the difference between flows and efforts, how they are characterized mathematically, and how their "product", the energy flow, is formed.

When we characterize the energy bond variables as "inner", we mean that they are not directly measurable. What is needed in order for them to give measurable or "external" expressions is that their action becomes integrated over time, thereby creating a change in some external variable in one of the systems that the bond connects.

Think of a cistern with water inside. The *level* of the water is a directly measurable *level-variable* (hence the name). This level can only be changed if we send a *flow* of water in or out through a tube on the cistern, and if we want to measure the flow we need to let it run a certain time and regards how big a change of the level it causes.

Think of the Gulf Stream. An eel in the middle of it does not feel the current. The current is a quality of an extended spatial area and has the character of a qualisign (111). Only where it collides with or glances other currents sensible effects occur. Think then of a water surface, separating two regions, each having their characteristic flow. In the proper surface there is a tension, that is an internal property of this layer of separation. The surface tension is also an internal, or iconic property, only

potentially measurable, and regarded as a sign it has, like the flow, a 1 in the place of the object-reference. But as representamen it is attached to a clearly discernible thing, a sinsign, and must therefore be attached to the sign-class (112). Inside the bulk of an elastic substance there is also effort (stress), but only as something connected to real or virtual cuts in the material.

If we have a lazy eel floating with the Gulf Stream and above a balloon drifting with the wind, then we may from a suitable point of view record a movie of the sequence of events. The finished movie can show what was seen, but it can also, shown backwards, show the *time reversed* action. If the flow is described as the velocity of the eel and the balloon, we can see that it changes sign by the time reversal. The tension, on the contrary, is unchanged. The surface tension tries to diminish the area of the surface. It is this that makes a soap-bubble assume spherical shape, and this phenomenon is brought out equally well in a movie whether it is shown forwards or backwards

The energy flow is a flow and changes sign by time reversal, and as it shall be the product of the internal variables flow and effort, it is consistent to assume that the flow changes sign, while the effort is unchanged by time reversal.

In this way we get a definition of what is flow and what is effort, at lest for a *simple* bond, where flow and effort each can be denoted by a single real number after measurement. In the more general situation, where flow and effort are described as vectors of more than one dimension, the definition has to be modified somewhat, but we postpone that discussion a while.

The time-reversal operator $\boldsymbol{T}$ is an example of a *mirroring* operator, i.e. an operator that, when applied twice, gives an identity

$$\mathrm{T}^2 = I \qquad (3)$$

(we use fat, cursivated types for operators)
A mirroring operator, therefore can only have the eigenvalues +1 and -1. If a quantity has eigenvalue +1 to a given mirror-operation we say that it has *even parity*, or just, that it is even, and if the eigenvalue is -1, that it is odd. So the flow in a simple bond is odd with respect to time reversal, while the effort is even.

To the simple bond we can formally attach an o-level q, that is the integral over time of the flow f from an unspecified instant in the past (e.g. $t=-\infty$) and an x-level p that similarly is the time-integral of the effort e. The attachment is shown iconically with a pair of junctions and a couple of "cyclical storage-elements" that are characterized by having output 0 and therefore do not change flow and effort of the bond.

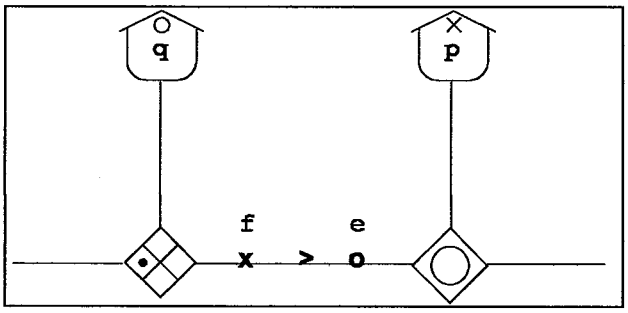


Figure 5. Attachment of cyclical level variables to a bond.

By differentiation with respect to time the parity shifts, such that o-levels are even and x-levels are odd to time reversal. If time is frozen as in a photographic snapshot, x-levels become invisible, so, even though they are measurable and therefore has the object-category 2, they are not permanently registrable, so they must be assigned the interpretant-category 1. Therefore we assign x-levels to the sign-class (122) and o-levels to (222), and we get the rule that the sum of their sign-categories are even or odd, as we earlier defined the even/odd dichotomy for the concrete layer of the tetraktys (comp. figure 3).

The indication of a bond demands an orientation, so we have to discuss one more mirroring-operation, viz. *shift of orientation*. For a simple bond it must be possible to characterize flow and effort with respect to their parity by orientation-shift. The common point of view, advocated by Paynter is that flows are "through-variables", whereas efforts are "across-variables", so that it is the flow that has odd parity by orientation-shift. It is easy to accept this if one thinks of an energy bond as an electrical double-wire, where the flow (the current) is measured with the ampèremeter inserted in the wire, while the effort (the voltage) is measured with the voltmeter inserted across between the two wires. In other cases, though, we cannot use this consideration; e.g. for a longitudinal mechanical energy bond, a rod, where the flow is the velocity and the effort is the force both flow and effort are along the bond, and for a transverse mechanical energy bond — a rotating shaft, where the flow is the angular velocity and the effort is the torque, both variables are across. Still, it is

difficult to find counter-examples to the rule that the flow has odd parity and the effort even by orientation-shift, so we shall assume this rule, somewhat weakened by the modification that when it is fulfilled for a simple bond we call it a *scalar* bond, but if it is the effort that is odd by orientation-shift, we call it a *pseudoscalar* bond. At the same time we shall introduce a change of the indexical sinsigns for the internal variables that we let the flow-index fuse together with the orientation-index to an arrow and we let the effort be indicated with a stroke across the bond-icon. This will look the more naturally for scalar bonds, but we shall use it in all cases. In figure 6 the indexical legisigns f and e for flow and effort are also shown.

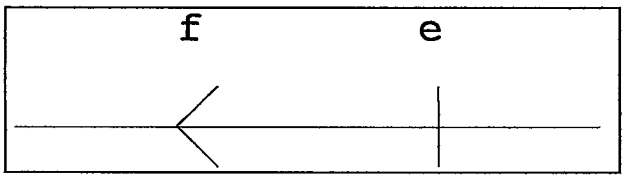


Figure 6. Indication of simple bond.

The orientation of the bond cannot be regarded as belonging to the general icon — the energy bond, but is an index that emerges when we begin the transition from the iconic stage of modeling (113) to (123), where the measuring prescriptions appear. In *the ground* of the energy bond there is no kind of distinguishing one direction from the other. This distinguishing is connected with the measuring prescriptions, not with the energy bond proper. It follows from this, that a given interaction can be described just as well with the one orientation as with the other. If we for an energy bond have chosen orientation from right to left and used the legisign f for the flow, the *same* situation must be describable with orientation from left to right, but, because the flow has odd parity for a scalar bond, measurement with the opposite orientation will result in a change of sign for the flow, which we, therefore, with the new orientation signify with -f. This *flow-orientation rule* is shown on figure 7a, while 7b shows the corresponding rule for pseudoscalar simple bonds.

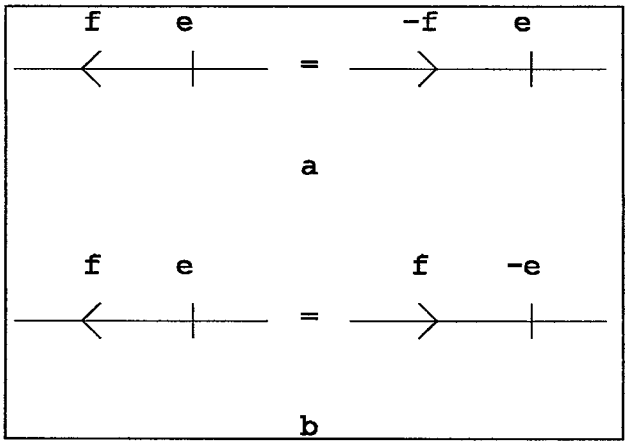


Figure 7. Orientation rules: a scalar, b pseudoscalar bonds.

When we begin to operate with shifts of signs for f and e, they are no longer just indexical, but must be regarded as symbols. For simple bonds they are real numbers. This transition to symbols can, according to figure 2, only take place when the measuring situation is defined. We cannot here enter a discussion of the different measuring prescriptions for all the media that the energy bond technique shall be able to describe. We have briefly mentioned electrical double wires, where the flow is measure alongside and the effort across the bond, so let us consider briefly another example: the longitudinal mechanical bond:

If a system A interacts with a system B via a rigid rod, so that A can push on or pull in B in the rod's direction, we may regard the rod as a longitudinal mechanical bond. We can choose orientation from A to B. The flow f is then the velocity of a fixed point on the rod, regarded positive in this direction. The effort e can then be defined as the force, whereby A pushes on B, i.e. if A pulls instead of pushing, e will be negative. The product of e and f will be the energy flow from A to B. The measurement-prescriptions are thus defined from the orientation of the bond, independent of A and B. If we change orientation, so that it now goes from B to A and describe the *same physical situation as before,* then the symbolic expression for the flow change from f to -f. The symbolic expression for the effort must then be unchanged, for the product of flow and effort must change its sign. As the orientation now is from B to A, the effort is now defined as the force B pushes on A with. We see, therefore, that the flow-orientation rule (figure 7a) for the longitudinal bond implies the law of *action and reaction* B pushes just as much on A as A pushes on B.

For other media one will also find that the flow-orientation rule corresponds to a

"deep" law of Nature, the law of action and reaction for the particular medium, so this law is not really a law of Nature, but a semiotic law. A string can be described with the same measurement prescriptions as a rod, but the "pushing force" can then only be negative. For a transversal mechanical energy bond, a rotating shaft, the measurement prescriptions demand a "handedness", traditionally a right-screw. The flow can then be defined as the angular velocity that way around that together with the orientation gives a right-screw, and the effort as the torque that the system behind the arrow exerts on the system in front of the arrow (that A exerts on B, if the orientation is chosen from A to B). In this way the flow-orientation rule becomes a law of action and reaction speaking of torques instead of forces. One may say that forces and torques, efforts in general for scalar bonds strictly speaking are not quantities with a direction, acting "from" one system "on" another. They rather act "between" the two systems. However, it is not easy to free oneself from normal language, and perhaps not desirable, either, for the crucial point is that both flow and effort are defined from the energy bond icon and its indexing, and this indexing *demands* an orientation that is the only feature that gives a direction for both variables. How this choice of direction is reflected in the measurung prescriptions for different media can be widely differing like the specific formulation of "the law of action and reaction". The flow-orientation rule is the general law comprising all these special cases. Bertrand Russell asserted, though in an essay from 1953 that the concept of causality in physics is deceased and only appears to survive because we, erroneously, believe that it," like monarchy, doesn't do any harm". Russell's viewpoint is based on the conception that everything in physics can be described with deterministic equations, and as one always can move a quantity from one side of the equal-sign to the other it makes no sense to distinguish between cause and effect. If this were true it wouldn't do any good to discuss measuring prescriptions for causality, but, as shown in text#8, it is possible to operationalize the concept of causality, although only for *transient*s, i.e. transitory relaxation phenomena. Thransients is exactly what properly expresses the fact that *not everything* in physics can be described deterministically with equations. They can be initiated in a way that is unpredictable in principle, by active interference from outside or by the noise, that is always present in dissipative systems (comp. figure 3). The semiotical discussion conducted here should also make clear that equations between mathematical symbols first appears at a relatively high level in the hierarchy of signs. They cannot float freely and govern anything with greater exactness than that inherent in the measuring situation.

A *simulation* of a system's behaviour, ascribed here to the sign-class (223) will, if performed with a digital computer, also introduce elements of the symbolic description. The program for such a simulation, however, will not use equations, but algorithms including a *causality of computation.* If we abolish causality in physical theories it is bound to appear elsewhere. One of the ambitions of the energy bond technique is to formulate algorithms for models where the computational causality is based on physical-semiotical cosiderations. In this way we'll obtain a stronger coupling between the numerical stability of the algorithm and the physical stability of the simulated system.

A direct translation of an energy bond model to an algorithm that can be carried out by a serial digital computer, however, is not always possible. The signal-flow of an energy bond model may contain *simultaneous feedbacks* where the output from an operation is determined by the input *at the same time.*

Such a structure cannot be handled directly by a serial computer, which instead has to put up the equations symbolically and then solve them- An parallel computer, e.g. an old-fashioned analog computer can, under certain circumstances handle simultaneous feedbacks, but not always, and it is not a simple matter
to formulate conditions for the possibility of it. The problem of the simultaneous feedbacks is fundamental and, properly, of a semiotic nature, This is due to the fact that the digital computers operate with *symbolic* representations of the *indexical* energy bond variables. One runs into the same difficulty when trying to simulate a quantum mechanical measuring process. Until further development we have to resign to the condition that an implementation of the energy bond formalism on a digital computer can be performed by genrating the simultaneous equations and then solving them formally or numerically.

3. Aggregation of simple bonds.

We are now going to generalize the concept of an interaction- or energy-bond. The generalization will proceed in two directions: Firstly, we shall expand the class of mathematical symbols that can be assigned to the dynamical variables to encompass *complex numbers*. Secondly, we shall regard these quantities as *vectors with a general metric*.

The use of complex flows and efforts in scalar bonds, where the flow-orientation rule (figure 7a) is valid, is well known from the theory of linear response, thoroughly described in text#22. Now, it is not linear response in particular we should have in mind, but also other disciplines, like quantum mechanics, where the use of complex numbers is necessary. However, we shall use the linear response theory as a jumping-off ground for generalization, because it has proved consistent and because the same rules can be used for quantum mechanics.

For every type of interaction bond there must be defined a *scalar* or *inner* product of flow and effort. This product wee shall normally interpret as the energy flow (in the direction of the orientation), but it will turn out, even for physical applications of the technique it is not always energy in the traditional sense that is flowing in the bonds. The decisive feature is that this quantity is defined as the scalar product of flow and effort. For a complex scalar bond flow and effort are each scalar quantities, so here we may just define the energy flow as the product. In response-theory, however, it turns out that one of the factors in this product shall be a complex conjugate, so we shall define the energy flow in the direction of the orientation for a complex scalar bond in the following way:

$$w = \mathrm{Re}(e^{*} f) = \mathrm{Re}(e f^{*}) \qquad (4)$$

where the asterisk denotes complex conjugation and Re means "real part of". If e and f are signal-amplitudes in response theory, the energy flow is only half of the expression above, but we omit the factor ½ in order to use the same expression for real and complex bonds (later, we may regard the factor ½ as expressing *the metric tensor* of a complex amplitude bond). Without the factor ½ the expression makes

sense in response theory if we regard f and e no longer as amplitude for flow and effort, but as *effective* flow and effort that are $\sqrt{2}$ times smaller than the amplitudes.

By introducing the real and the imaginary part of effort and flow:

$$e = e_1 + ie_2 \; ; \; f = f_1 + if_2 \tag{5}$$

we get from (4)

$$w = e_1 f_1 + e_2 f_2 \tag{6}$$

which suggests that the complex scalar bond can be reticulated out on two real scalar bonds, where the one has the effort e_1 and the flow f_1 and the other has the effort e_2 and the flow f_2.

However, we must remember that what is the flow and what is the effort is defined by the time-reversal-criterion: The flow must change its sign, but not the effort, by time reversal in a real scalar bond, and, besides, that the flow-orientation rule shall be valid for scalar bonds.

In the linear response-theory signal amplitudes for the frequency ω are associated with a "pointer", turning clockwise around as exp(-iωt), i.e. if the effort-amplitude is e, the actual effort at time t is given by

$$e_r(t) = \mathrm{Re}[e \cdot \exp(i\omega t)] \tag{7}$$

As the real effort shall have even parity by time reversal, and as the time-reversed signal-pointer turns the opposite way, the time-reversed effort-amplitude must be the *complex conjugate of the original:*

$$T[e_r] = e_r = \mathrm{Re}[e^* \cdot \exp(i\omega t)] \tag{8}$$

We therefore have the following rule for time-reversal of signal-amplitudes (scalar or

pseudoscalar):

$$T[e] = e^* \; ; \; T[f] = -f^* \qquad (9)$$

Thus, the imaginary part of the effort changes sign by time-reversal, while the imaginary part of the flow keeps its sign. From here it is seen that a complex scalar bond not can be reticulated to two real scalar bonds.

One possibility is to use a real pseudoscalar bond for the imaginary parts of flow and effort, but then we must indicate the imaginary part of the effort as a flow and the imaginary part of the flow as an effort. This then also agrees with the flow-orientation rule for the complex scalar bond, according to which both the real and the imaginary part of the flow shall change sign by orientation-shift, and when the imaginary part of the flow is an effort, it must belong to a pseudoscalar bond (figure 7b). Another possibility is to use two scalar bonds for the reticulation, but let the bond that carries the imaginary parts have imaginary variables, the effort ie_2 and the flow if_2, for, when e_2 changes sign by time-reversal, then ie_2, according to (9) will keep its sign.

In a similar way we find that a pseudoscalar complex bond can be reticulated to a real pseudoscalar and a real scalar bond or in a real and an imaginary pseudoscalar bond. In figure 8 below these rules are shown, now using dotted lines for pseudoscalar bonds and fully drawn lines for scalar bonds

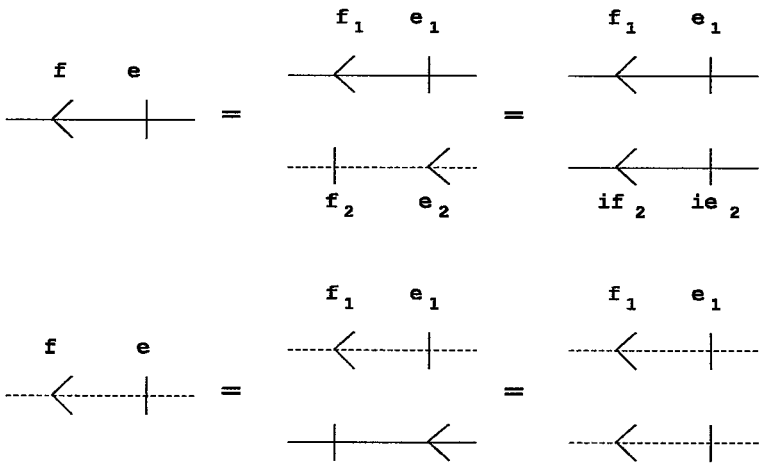


Figure 8. Translation of complex, one-dimensional bonds to real and imaginary bonds.

For a complex bond we have a free choice between regarding it as scalar or pseudoscalar. For, if we want to define a complex bond from simple (real) bonds it must be a scalar and a pseudoscalar bond that in relation to orientation are equal, such that one meaningfully may claim that they are oriented "the same way". When we then are going to form the complex bond by aggregation of the scalar and the pseudoscalar simple bond, we are free to choose, whether it shall be one or the other of these two that is to be "the upper" in relation to figure 8. If we call the effort and the flow in the scalar bond for E_1 and F_1 and the effort and the flow in the pseudoscalar bond for E_2 and F_2, then the effort and the flow in the complex bond may be defined as

$$e_s = E_1 + iF_2 \; ; \; f_s = F_1 + iE_2 \qquad \textbf{(10)}$$

which leads to a scalar, complex bond (hence the subscript), or the effort and the flow may be defined as

$$e_p = E_2 + iF_1 \; ; \; f_p = F_2 + iE_1 \qquad \textbf{(11)}$$

Which leads to a pseudoscalar bond. We thus arrive to a rule of the same character as the flow-orientation rule, as the two choices describe the same physical situation, and as

$$e_p = if_s^* \; ; \; f_p = ie_s^* \qquad \textbf{(12)}$$

we are free to re-define a scalar bond to a pseudoscalar if we exchange effort- and flow-indices without moving the symbolic expressions, complex conjugate these expressions and multiply them with i. This rule, that is called the rule of *shift of genus* is displayed on figure 9 below:

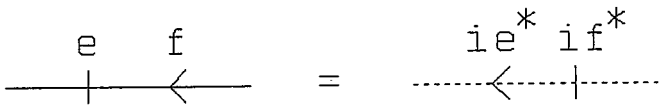


Figure 9. The rule of shift of genus

By the *genus* of a bond we shall understand a number γ, denoting the "type" of the bond with respect to time-reversal and orientation. For one-dimensional bonds (simple or complex) there are only the two possibilities $\gamma = 1$, scalar bond, and $\gamma = -1$, pseudoscalar bond. For vectorbonds we shall define genus as a complex number that may have a couple of other values. Bonds with genus 1 is the basic standard and the rule of shift of genus makes it possible to retrace possible pseudoscalar bonds to the scalar type. Transitions from $\gamma = -1$ to $\gamma = 1$ can be shown iconically by use of a *genus-gyrator*, as shown below:

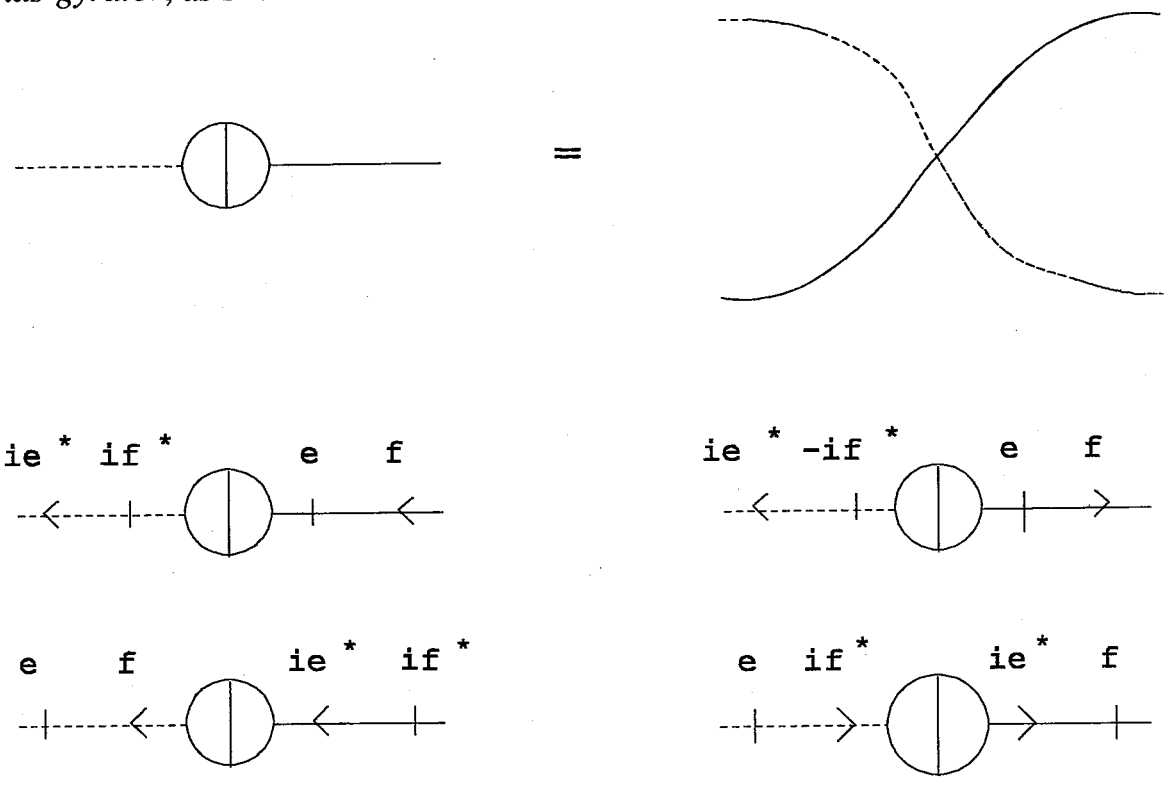


Figure 10. Genus-gyrator between a scalar and a pseudoscalar bond.

In the upper part of figure 10 is shown how the genus-gyrator is constructed by exchanging the up/down ordering of two simple bonds. It is evident from this that the icon for the element must be *symmetrical* with respect to exchange of the two ports. Below in the figure is seen different indications and symbol-assignments. The symmetry of the icon corresponds to a symmetry of the symbolic functions. When a variable in the left port is formed by complex conjugation of a variable in the right port and multiplying by i, the variable in the right port is formed by the same operation applied to the variable in the left port.

The genus-gyrator is an example of an *antilinear* two-port with gyrator-character. Anti-linearity comes from the complex conjugation of the input signals and the gyrator-character because an input effort in one port becomes an output flow in the other port, and vice versa. An *icon-symmetric* gyrator does not exist in the simple energy bond formalism; that it appears now is because the gyrator-parameter now may be an imaginary number, or, as we shall see later, an *antihermitean tensor.*

We shall now proceed to construction of the concept of the general vector bond, while we as earlier will let the generalization go on gradually, so that the connection to the simple formalism does not vanish out of sight. Let us therefore start by looking at the motion of a particle in 2 dimensions. We can use a normal cartesian coordinate system with base-vectors $\bar{e}_1$ and $\bar{e}_2$. The velocity vector of the particle can then be written out in coordinates as $\mathbf{v}=(v_1,v_2)$ and the force on the particle as $\mathbf{F}=(F_1,F_2)$. As the rate of energy-transfer to the particle is the scalar product of $\bar{F}$ and $\bar{v}$ can be written as

$$F \cdot v = F_1 v_1 + F_2 v_2 \qquad \textbf{(13)}$$

we may define a two-dimensional energy bond for the particle as shown in figure 11 below.

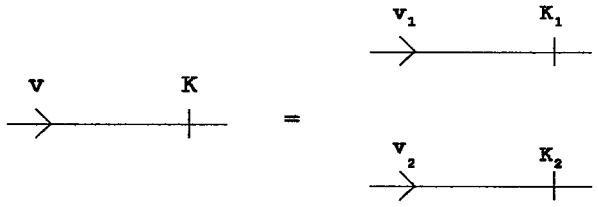


Figure 11. Two dimensional vector bond made of two simple bonds.

We shall now make a transition to another coordinate system with base-vectors e_1'and e_2', that are not necessarily perpendicular to each other, but still linearly independent. The transition from the old to the new base-vectors can then be defined with a 2*2 matrix A whose determinant is different from 0:

$$\begin{pmatrix} e'_1 \\ e'_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \qquad \textbf{(14)}$$

We can now choose to resolve the velocity-vector after the new base-vectors. The new velocity-coordinates then become

$$\begin{pmatrix} v'_1 \\ v'_2 \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \textbf{(15)}$$

where the matrix B is the *contragradient* matrix to A, defined as the transposed of the reciprocal of A:

$$B = A^{-1t} \text{ i.e. } B_{ij} = A^{-1}{}_{ji} \qquad \textbf{(16)}$$

If the coordinates of the force are transformed after the same formula, we cannot use the expression (13), for the energy flow in the "skew-angled" bond. Alternatively, we could try to find another transformation for the force-vector such that the form (13)

still holds, i.e. such that

$$F'_1 v'_1 + F'_2 v'_2 = F_1 v_1 + F_2 v_2 \qquad (17)$$

It is easily seen that the new force-coordinates then shall be defined as

$$\begin{pmatrix} F'_1 \\ F'_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \qquad (18)$$

Thus: in order to uphold the form (13) for the energy flow, making possible that the vector-bond can be reticulated out on scalar bonds, the effort (force) transform by the same matrix $\boldsymbol{A}$ as the base-vectors for the coordinate system, while the flow (velocity) transforms by the contragradient matrix $\boldsymbol{B}$. We therefore say that the flow transforms *contravariant*, while the effort transforms *covariant*. We might just as well have chosen to let the effort transform contravariant, but then the flow would have to transform covariant. The transition from the "rightangled" vectorbond (r) and the "skew-angled" (s) can now be reticulated in two ways by the simple energy bond technique depending on whether we use (18) or (15):

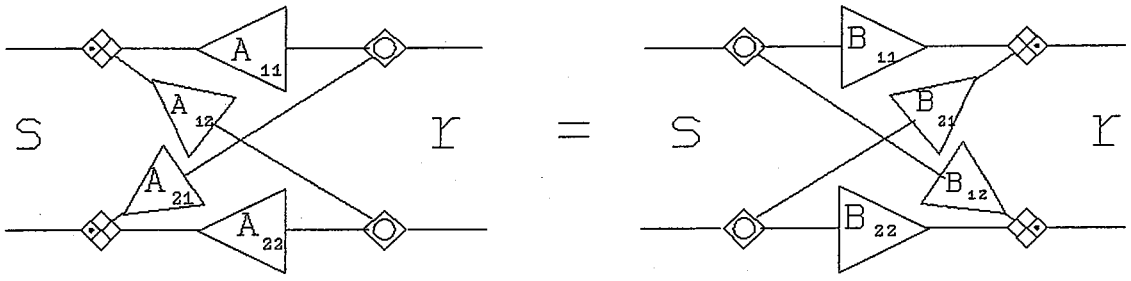


Figure 12. Covariant and contravariant scalar reticulation of coordinate transformation from "rightangled" (r) to "askew-angled" (s) coordinates in two dimensions.

We can now introduce a vector-bond-icon for the composite transformer in figure 12 (properly speaking, it is a *metrical transducer*, but so far there is no need to distinguish). We get at once the following rule for reversal of the icon:

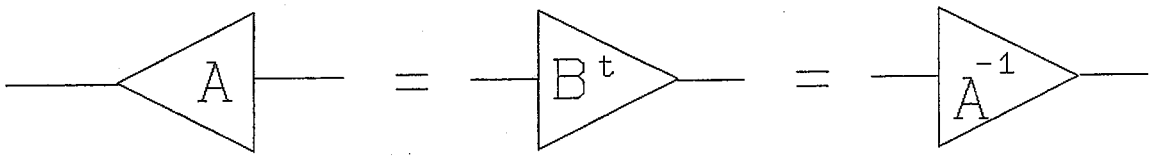


Figure 13. Iconic reversal of transformer

The reason why there appears a transposing of the B-matrix when tha A-icon is reversed is that the scalar transformer that connects the 1-bond on the primary side (the flat side of the icon) with the 2-bond on the secondary side (the pointed side) shall have the indication 21, and in figure 12 it is B_{12}.

Coordinate transformations that conserve the length of the axes and their orthogonality are characterized by the feature that the A- and B-matrices are identical. A matrix that is identical with its contragradient matrix is said to be *unitary*. If we only consider transformations corresponding to rotations of a usual (orthonormal) coordinate system, we cannot distinguish between a contravariant and a covariant vector, but for a skew-angled coordinate system it is necessary with this distinction. In the following we shall use a notation where the coordinates for the two vector-types are indicated differently; for contravariant vectors we place the coordinate-index above the symbol, for covariant below. If the flow is contravariant, we write its coordinates as f^i, and if the effort is covariant, we write its coordinates as e_i, where the index in both cases can have values from 1 to the bond's dimension d. The energy flow is written as

$$w = e_i f^i \qquad \textbf{(19)}$$

Presuming here that the vectors are real. We apply *Einstein's summation-convention*, i.e. when a double index occurs (here i), a summation is implied over this index from 1 to d. This rule shall be further sharpened by demanding that the two partners of the index-pair shall stand in different heights, one above, the other below and they must not belong to the same symbol.

If both flow and effort in the skew-angled system are expressed by their contravariant

coordinates the energy flow can be written

$$w = e^i g_{ij} f^j \tag{20}$$

Where g_{ij} are the matrix elements of *the metrical tensor*

$$g_{ij} = \sum_{k=1}^{d} A_{ik} A^t{}_{kj} = \sum_{k=1}^{d} A_{ik} A_{jk} \tag{21}$$

We have not here used the new summation-convention, because all indices of th As stand in the same height. In order to be able to use it we first introduce the metrical tensor for the orthonormal "right-angled" bond. This is simply the unit-matrix, Kronecker's δ, which in this case defines the *standard metric.*

$$G_{ij} = G^{ij} = \delta_{ij} = \begin{cases} 1 \text{ for } i = j \\ \\ 0 \text{ for } i \neq j \end{cases} \tag{22}$$

We then say that the standard metric is *euclidean.*

Instead of reckoning both flow and effort contravariant we could make them both covariant, and the energy flow in the skew-angled coordinate system can then be written

$$w = e_i g^{ij} f_j \tag{23}$$

and one sees easily that the g-matrix with both indices above can be calculated by in (21) replacing the A-matrices with B-matrices, and further that it is the reciprocal to the g-matrix with both indices below.

With help from the new summation-convention and the metric tensors g and G we

can now raise and lower indices on the matrices for the metric transducer A:

$$A_i{}^j = A_{ik} G^{kj} = A_{ij} \; ; \; A^i{}_j = g^{ik} A_{kj} = B_{ij} \qquad \textbf{(24)}$$

This rule is most easily remembered if one in the iconic representations of the metrical transducer takes care, as in figures 12 and 13, to have the bond with the standard metric G to the right and the bond with g-metric to the left.

In a bond with a given metric g we can similarly raise and lower indices for flow and effort, i.e. shift from covariant to contravariant representation of the vectors using the two g-matrices:

$$f_i = g_{ij} f^j \; ; \; e^i = g^{ij} e_j \qquad \textbf{(25)}$$

We can also by the raising-lowering rule define two matrix-representations of g where one index is above and the other below. Both of these turn out to be the unit matrix:

$$g_i{}^j = g_{ik} g^{kj} = \delta_{ij} \; ; \; g^i{}_j = g^{ik} g_{kj} = \delta_{ij} \qquad \textbf{(26)}$$

By using (22) and (24) we can now express the connection between the standard metric G and the given metric g:

$$g_{ij} = A_i{}^k A_j{}^l G_{kl} \; ; \; g^{ij} = A^i{}_k A^j{}_l G^{kl} \qquad \textbf{(27)}$$

The concept metrical tensor was introduced by Riemann for describing of the geometrical conditions, e.g. on curved surfaces, imbedded in a three-dimensional euclidean space. The equations (27) that express the transformation of the metrical tensor by the transformation-matrices for the coordinate-vectors are then applied as

definition of general (2. order) tensors in the Riemannian geometry.

Einstein could use Riemann's formalism unchanged for formulating the general relativity theory (GRT), and one therefore often say, that GRT is a *geometrical* description of the conditions in a *curved,* four-dimensional space-time continuum, while the special relativity theory (SRT) is about a *flat* space-time continuum. As we shall see later there are good reasons to be skeptical about that description. In a proper geometry of Riemannian type all distances between two different points are positive, which means that the metric tensor is *positive definite*. This is not valid in the relativity theories because one of the four dimensions is *time,* which one cannot regard as a spatial length. The non-definite tensor in SRT and GRT can in a simple way be related to the metric of the energy bond but not to any geometrical continuum. We can still apply Riemann's and Einstein's formalism within energy bond semiotics, but then it is no longer geometry it's all about.

4. General vector bonds.

The last chapter initiated a generalization of the concept of energy bonds, starting from the simple bond. The generalization proceeded in two directions: First, we introduced complex notation for one-dimensional bonds and the concept of *genus* in order to distinguish between scalar and pseudoscalar bonds. Second, we defined a vector-bond by aggregation of simple, scalar and real bonds, such that the efforts in the scalar bonds together formed an effort-vector and the flows a flow-vector. For such a vector-bond we then considered a coordinate-transformation by use of a *metrical transducer,* A, that changed the *metric* of the bond from the euclidean standard metric, corresponding to an orthonormal coordinate system (right-angled with equally long axes) to an arbitrary metric, corresponding to a skew-angled coordinate system. This arbitrary metric was described with a metric tensor g, and we saw that it was necessary for the vectors of the bond to distinguish between a *contravariant* and a *covariant* coordinate-representation. Correspondingly, 2. order tensors as g will have 4 different matrix-representations (and an nth order tensor 2^n different), that are distinguished from each other by placing the indices above or below.

We haven't yet discussed how the mirroring operations time-reversal and orientation-shift shall be connected with the indication-system of the vector bond. With the vector bond a new level of indication has emerged, namely the difference between covariant and contravariant indices, so the question is, whether the mirrorings shall have any influence on this level. For the vector bonds hitherto discussed there are several possibilities; we could take over the flow-orientation rule unchanged from the scalar bonds, i.e. just let the flow vector change sign and the effort vector stay unchanged by orientation-shift without raising or lowering indices, but then we would exclude some possible generalizations, that would be very fruitful. Therefore, we shall have to proceed cautiously and start by considering which types of generalizations, we are interested in, and then choose the mirroring rules, such that they are compatible with all these types and can be expressed in a common form.

The most obvious generalization is to allow vector bonds to be built of *complex scalar bonds*. In this case the energy flow must be written

$$w = \mathrm{Re}\,[e^{*}_{i} f^{i}] = \mathrm{Re}\,[e^{*i} f_{i}] \qquad \textbf{(28)}$$

If we now use the lowering rule

$$e_i = g_{ij} e^{j} \;;\; f_i = g_{ij} f^{j} \qquad \textbf{(29)}$$

where g now can contain complex elements, we get

$$\mathrm{Re}\,[e^{*i} g^{*}_{ji} f^{j}] = \mathrm{Re}\,[e^{*i} g_{ij} f^{j}] \qquad \textbf{(30)}$$

and this relation can only be fulfilled for arbitrary values of e^{i} and f^{j}, if g_{ij} is *hermitean*:

$$g^{+}_{ij} \doteq g^{*}_{ji} = g_{ij} \qquad \textbf{(31)}$$

The reciprocal matrix g^{ij} will then also be hermitean, so we simply say that *the tensor g is hermitean*, $g^{+} = g$. For real bonds it means that g is symmetrical, which one easily can convince oneself is the case for the earlier found expressions (21) and (27). As the matrix g_{ij} is hermitean, it has only real eigenvalues. With suitable coordinate transformations we may put it into a diagonal form, e.g. we may find a unitary transformation that makes it diagonal, where it is exactly the real eigenvalues that stand in the diagonal. However, we shall need much more than unitary transformations (which we are going to define more precisely later), but we shall be especially interested in such transformations that are continuously connected to the identity, so that we can think of such transformation formed gradually without sudden jumps. Such transformations form *Lie-groups*, described with a set of continuous parameters. For those changes of the g-matrix, that can be defined by such Lie-groups one can use *Sylvester's theorem of inertia*, stating that the number of positive eigenvalues and the number of negative eigenvalues are unchanged (and add to the dimension d). I.e. if we have a metric, where all eigenvalues are positive. then we can with a continuous transformation bring it to a diagonal form, where all elements are

positive. By an additional continuous *scale-transformation* the metric can be brought to *standardform* with only 1s in the diagonal. A metric with only positive eigenvalues therefore has an euclidean standard metric.

If there are p positive and n negative among the eigenvalues (p+n=d), the metric can be brought to standardform with p 1s and n -1s in the diagonal. By a suitable reordering of the coordinates we can make sure that it is the p first places in the diagonal that have 1s. The question is now: Can we reticulate such a non-euclidean standard-bond out on scalar bonds? (we would like to make the generalization such that this is possible).

Let us consider a two-dimensional standard-bond with the metric

$$(g_{ij})=(g^{ij})=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}=\sigma_z \qquad (32)$$

a so-called σ_z-metric (as g.. is equal to the Pauli-matrix σ_z).

If we in such a bond let both effort and flow be contravariant, the metric tensor will be reflected in the expression for the energy flow, that is

$$w=e^i g_{ij} f^j = e^1 f^1 - e^2 f^2 \qquad (33)$$

This means that bond 2 in the reticulation must have orientation against the vector bond's, when both flow and effort are reckoned contravariant. If we now change the effort to be covariant, the energy flow becomes

$$w=e_1 f^1 + e_2 f^2 \qquad (34)$$

The orientation is now the same in the scalar bonds 1 and 2, and as the effort-index has been lowered by using the σ_z-metric, e_2 has changed sign. "the effort" in the scalar bond 2 thus has changed sign by orientation-shift, i.e. according to the flow-

orientation rule e_2 is in reality a flow! Conversely, we realize that f_2 in the scalar bond must be indicated as an effort. When we in the transition from (33) to (34) has kept the variance of the flow-vector, but changed that of the effort-vector, it is still the same physical situation we describe, but if we change the flow-variance it will be another situation.

We are thus led to the following reticulation of a vector bond with a non-eudlidean standard metric (here σ_z-metric):

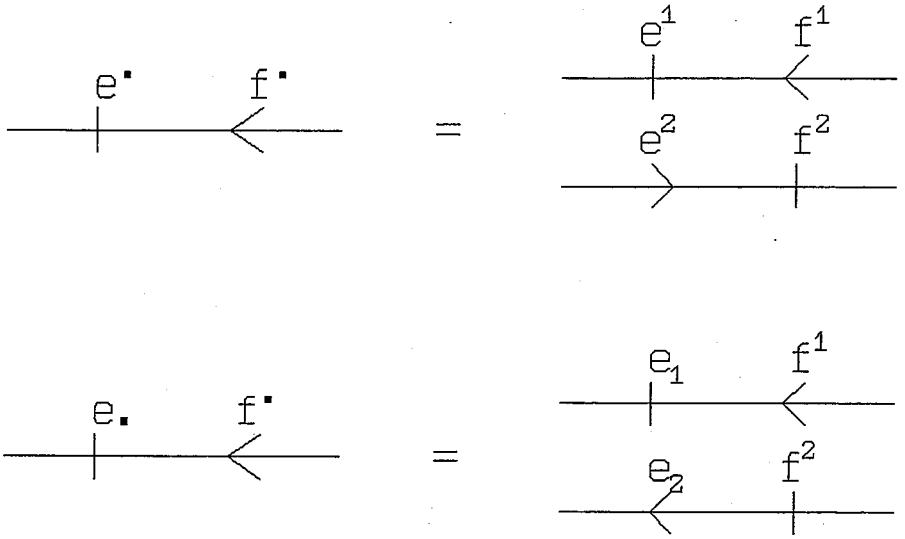


Figure 14. Scalar reticulations of vector bond in non-euclidean standard metric: Above: both vectors contravariant. Below: mixed variance.

For a non-euclidean bond we cannot just take over the flow-orientation rule from scalar bonds. If we have reticulated it out on scalar bonds and then shift orientation for all these and apply the flow-orientation rule, the components of the effort-vector will not all be unchanged, but all the deviant components, i.e. those that in the scalar reticulation appear as flows and on whose places there is a -1 in the diagonal matrix of the standard metric will change sign. So, if it before were the components of the covariant effort vector entering the scalar bonds, then it will after the orientation-shift be the contravariant effort-vector's coordinates that stand. Similar considerations apply for the flow-vector. Thus, it looks as if the flow-orientation rule needs the amendment that *the variance of both flow- and effort-vector shall be changed by orientation-shift.* This rule can also be applied to bonds with an euclidean standard metric, for in the euclidean metric there is no difference between the covariant and the contravariant vector-coordinates.

We shall now make an addition to the vector-tensor-notation. By a *tensor of nth order we shall mean a symbol with n index-places that each can be "up" or "down", contravariant or covariant. A genuine tensor transforms according to definite rules by coordinate-shift.* We have already seen the transformation rules applied to tensors of 1. and 2. order in the last chapter, namely for the energy bond vectors (1. order tensors) and the metric tensor (2. order tensor). The most important for now is to describe the index-system; we won't really need tensors of higher than 2. order. A tensor of nth order has 2^n matrix-representations, because each of the n indices can be above or below. When we shall denote a specific matrix-form we can therefore assign to the tensor-symbol a set of indexical sinsigns in the shape of dots, that solely with their up- or down-position denote whether the particular index-position shall be reckoned contravariant or covariant. These indexical sinsigns can then later be replaced with indexical legisigns as letters or numbers that denote specific coordinate-numbers between 1 and the dimension d.

Using the indexical sinsigns we can denote matrix-products where summation shall be performed over doubly appearing indexical legisigns without an explicit need to denote these legisigns. We can just denote the places with sinsigns (dots), and these can then *ad libitum* be filled out with letter-pairs after the summation rule, e.g.

$$e_{\bullet} = g_{\bullet\bullet} e^{\bullet} \sim e_i = g_{ij} e^j \qquad \textbf{(35)}$$

The rule for filling out the places with letters is thus, that if two dots stand in different heights and are separated with a single tensor-symbol, they shall be filled out with the same letter, and this letter-pair then cannot be used in other places of the same tensor-product, as they imply a summation from 1 to d over this index-pair. When a matrix-product is denoted with dots it is therefore important to place the factors in the right order, e.g. is

$$e^{\bullet} g_{\bullet\bullet} \sim e^j g_{ji} \qquad \textbf{(36)}$$

in general something different from (35).

The flow-orientation rule for vector bonds can be supplemented with another rule called *the rule of indifferent effort-variance*. If we with fixed vector-symbols and orientation of the vector-bond shift the effort-variance, then it is still the same physical situation we are describing. For when the effort-variance is shifted, the orientation shifts in all those scalar bonds, where the effort-vector's component is a flow, and simultaneously this component changes sign, which, according to the flow-orientation rule describes the same physical situation.

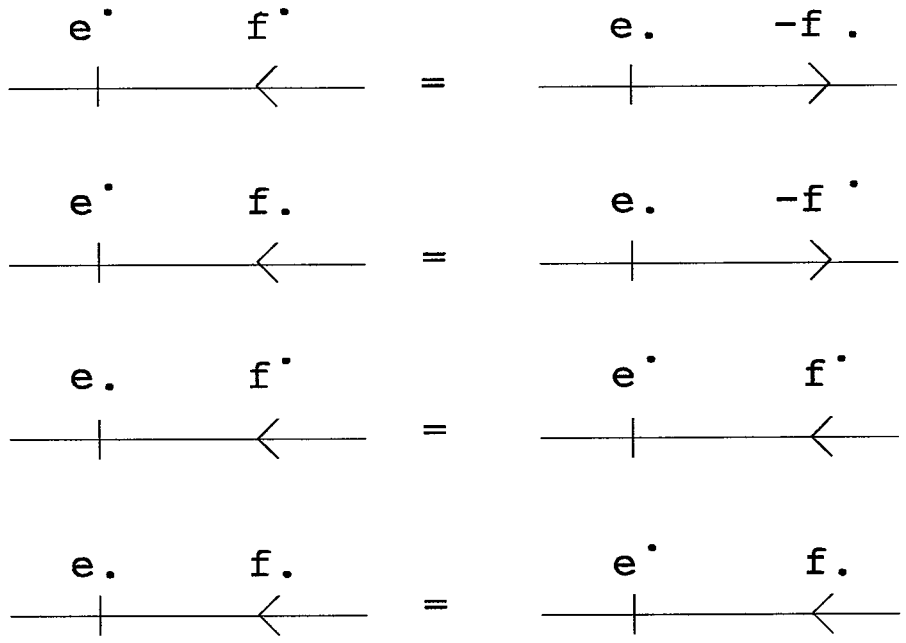


Figure 15. Flow-orientation rule for vector-bonds (the two utmost) and the rule of indifferent effort-variance (two lowest).

The validity of these rules can be checked for standard bonds that can be reticulated out on scalar bonds. For bonds with a general metric the rules become definitorical, and their strength lies in the fact that they can be formulated in the language of tensor-calculus, i.e. independent of definite coordinate systems. This is a relativity principle for the energy bond formalism and it corresponds closely to Einstein's general relativity-principle, as we shall see later.

To say that the validity of the rule of indifferent effort-variance can be shown by reticulating the bond on scalar bonds is a bit of circular reasoning, because it presumes that we know how a bond with a non-euclidean metric shall be reticulated out. This rule of reticulation, that was formulated in figure 14 presumes, on the other hand, that the two vector bonds shown in the figure are physically identical, i.e. that the rule of indifferent effort-variance is fulfilled. If we had chosen instead, that the flow-variance should be indifferent, we would not by reticulation of a non-euclidean bond be able to conclude that some of the effort-vector's components are efforts and others are flows, and with this rule we can employ the non-euclidean metrics for this purpose. If we were only interested in "unmixed" vectors, we could do with euclidean metrics, and then there would be no reason to talk about different variances at all.

A vector bond of standard metric might just as well be reticulated out on pseudoscalar bonds as on scalar, but generally there is nothing gained by it, and the rule of shift of genus (figure 9) makes it possible to translate pseudoscalar to scalar bonds, if the context justifies the use of complex bond-variables. When we reticulate a vector-bond out on one-dimensional bonds we shall in all cases demand that the one-dimensional bonds shall have *the same genus*, i.e. that the whole collection consists of either scalar or pseudoscalar bonds. We may then ascribe the whole vector bond a certain genus γ that is a scalar (a 0. order tensor, or a number), that (for now) only can be +1 or -1. If the dimension of the bond is 1, the metric tensor is also a number, g, that may be +1 or -1 (or ½ for complex amplitude bonds). In this way we can get the flow-orientation rule in the general form, figure 15, to be valid for one-dimensional bonds too (including pseudoscalar), and for such bonds are

$$\gamma = g \qquad \textbf{(37)}$$

The rules for time-reversal of effort and flow in one-dimensional bonds must then be reformulated to

$$T[e] = \gamma g\, e^{*} \;;\; T[f] = -\gamma g\, f^{*} \qquad \textbf{(38)}$$

This suggests that the time-reversal operation for vector-bonds must include a shift of variance, like the flow-orientation rule, which one easily can convince oneself about

by considering time-reversal of the bond on figure 14.

We shall attempt to make the discussion a bit more general by pretending that the genus γ for a vector-bond can be a *complex number*. For such a bond the metric g.. may contain complex elements. We shall, therefore, be a bit more cautious with operations that involve both lowering of an index and complex conjugation. We therefore require definitorical, that

$$\left.\begin{aligned} e_{\bullet}^{\ *} \doteq e^{*}{}_{\bullet} \doteq (g_{\bullet\bullet} e_{\bullet}\ )^{*} \\ e^{*\bullet} \doteq e^{\bullet *} \doteq (g^{\bullet\bullet} e_{\bullet}\ )^{*} \end{aligned}\right\} \qquad (39)$$

(and correspondingly for the effort-vector).

$$T[e_{\bullet}] = \gamma\, e^{*\bullet}\ ;\ T[e^{\bullet}] = \gamma e_{\bullet}^{\ *} \qquad (40)$$

(and correspondingly for the flow-vector). We may now define the time-reversed effort-vector by

When the time-reversal operator acts on a product we get the product of the time-reversed factors, i.e.

From this it is seen that the time-reversed metric tensor must be

$$T[g_{\bullet\bullet}] = g^{*\bullet\bullet}\ ;\ T[g^{\bullet\bullet}] = g^{*}{}_{\bullet\bullet} \qquad (41)$$

$$T[e_1{}^{\bullet}] = \gamma\, e_{1\bullet}\ ;\ T[e_2{}^{\bullet}] = \gamma\, e_{2\bullet} \qquad (42)$$

Because ***T*** is an antilinear operator, the time-reversed of a scalar must be the complex conjugate of the scalar. If we divide the effort vector in a real part and an imaginary part

$$e^{\bullet} = e_1^{\bullet} + i\, e_2^{\bullet} \qquad (43)$$

we get from (40) that

$$T[e_1^{\bullet}] = \gamma e_{1\bullet} \; ; \; T[e_2^{\bullet}] = \gamma e_{2\bullet} \qquad (44)$$

I.e. the rules for time-reversal of the complex conjugated effort-vectors must be

$$T[e^{*\bullet}] = \gamma e_{\bullet} \; ; \; T[e^{*}{}_{\bullet}] = \gamma e^{\bullet} \qquad (45)$$

Now, ***T*** must be a mirroring operation, so we shall have

$$T^2[e^{\bullet}] = T[\gamma g^{*}{}_{\bullet\bullet} e^{*\bullet}] = e^{\bullet} \qquad (46)$$

By time reversal of every factor and use of (42) and (45) i.e. the rules for time-reversal of the complex conjugated effort-vectors must be

$$T[f_{\bullet}] = -\gamma^{*} f^{*\bullet} \; ; \; T[f^{\bullet}] = -\gamma^{*} f^{*}{}_{\bullet} \qquad (45)$$

Now, ***T*** is a mirroring operation, so we shall have

$$T^2[e^{\bullet}] = T[\gamma g^{*}{}_{\bullet\bullet} e^{*\bullet}] = e^{\bullet} \qquad (46)$$

By time-reversal of each factor and use of (42) and (45) we get

$$\gamma^{*} \gamma g^{\bullet\bullet} e_{\bullet} = e^{\bullet} \qquad (47)$$

That is, in order for T to be a mirroring a complex genus of a vector bond must satisfy

$$\gamma^* \gamma = I \tag{48}$$

To find out how the flow-vector transforms by time-reversal we use that the energy flow must change its sign. Using (45) we get

$$T[e^{*\bullet} f_\bullet] = \gamma e_\bullet T[f_\bullet] = - e_\bullet f^{*\bullet} \tag{49}$$

And this gives together with (48)

$$T[f_\bullet] = -\gamma^* f^{*\bullet} \; ; \; T[f^\bullet] = -\gamma^* f^*_\bullet \tag{50}$$

The position of the dots in (40) and (50) seem to show that the time-reversed of a contravariant vector is covariant, and vice versa, but this would be an erroneous conclusion. The time-reversed symbols transform after other laws than the non-reversed, and the position of the dots in (4) and (50) refer to the non-reversed system, so from this one cannot conclude anything about variance-conditions in the time-reversed system. In fact we are facing yet another choice, and it proves convenient to conserve the variance by time-reversal, such that a time-reversed covariant vector is considered covariant in the time-reversed system. We therefore write:

$$\left.\begin{aligned} &T[e_\bullet] \doteq e^T_\bullet \gamma e^{*\bullet} \; ; \; T[e^\bullet] \doteq e^T_\bullet = \gamma \, e^*_\bullet \\ &T[f_\bullet] \doteq f^T_\bullet = -\gamma^* f^{*\bullet} \; ; \; T[f^\bullet] \doteq f^{T\bullet} = -\gamma^* f^*_\bullet \\ &T[g_{\bullet\bullet}] \doteq g^T_{\bullet\bullet} = g^{*\bullet} \; ; \; T[g^{\bullet\bullet}] \doteq g^{T\bullet\bullet} = g^*_{\bullet\bullet} \end{aligned}\right\} \tag{51}$$

Let us try to apply the time-reversal consideration to a situation where a vector-bond gives effort-input to a 1-port system, described by a response-tensor K. In this case there exists a symbolic relation between the two vectors of the energy bond, given by one of the four matrix-representations of the response-tensor K. By the time-reversal the flow- and effort vectors are replaced by their time-reversed edition, such that *the*

same symbolic relations hold for the time-reversed system as for the original. The time-reversal operation is not to be regarded as a reversal of the real physical time, but as a change of those measuring conventions that involve the use of a clock, corresponding to the insertion of an extra cogwheel in the clockwork, making the pointers turn in the opposite direction. It is *the same physical situation* that is described by the time-reversed symbols, and the symbolic relations, that are described with the basic iconic level of the model, plus the measuring prescriptions, shall therefore be unchanged.

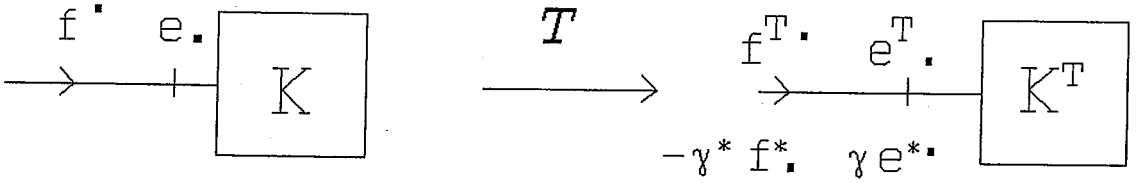


Figure 16. Time-reversal of 1-port system

For the two situations in figure 16 one gets:

$$f^{\bullet} = K^{\bullet\bullet} e_{\bullet} \; ; \; f^{T\bullet} = K^{T\bullet\bullet} e^{T}_{\bullet} \qquad \textbf{(52)}$$

From the expressions for the time-reversed vectors applied in the non-reversed system one gets

$$-\gamma^{*} f^{*}_{\bullet} = -\frac{\gamma^{*}}{\gamma} K^{*}_{\bullet\bullet} (\gamma e^{*\bullet}) \qquad \textbf{(53)}$$

By comparison of (52) and (53) and use of (48) we then find that variance-form of K^T, that has both indices above. From here it is easy to generalize to the three other variance-forms. Note, that a matrix-equation, where the variance is denoted with dots, but where the dots are in different heights on the two sides of the equals-sign is to be understood such that the first index (the left) is equal to the first and the second to the second:

$$\left.\begin{aligned} K^{T\bullet\bullet} &= -\gamma^{*2} K^{*}{}_{\bullet\bullet} \sim K^{T\,ij} = -\gamma^{*2} K^{*}{}_{ij} \\ K^{T}{}_{\bullet\bullet} &= -\gamma^{*2} K^{*\bullet\bullet} \sim K^{T}{}_{ij} = -\gamma^{*2} K^{*ij} \\ K^{T}{}_{\bullet}{}^{\bullet} &= -\gamma^{*2} K^{*\bullet}{}_{\bullet} \sim K^{T}{}_{i}{}^{j} = -\gamma^{*2} K^{*i}{}_{j} \\ K^{T\bullet}{}_{\bullet} &= -\gamma^{*2} K^{*}{}_{\bullet}{}^{\bullet} \sim K^{T\,i}{}_{j} = -\gamma^{*2} K^{*}{}_{i}{}^{j} \end{aligned}\right\} \qquad (54)$$

If we now time-reverse K^T again we shall get the original matrix K.. again, Remembering, that time-reversal of a scalar like γ^* amounts to a complex conjugation of the scalar, we get from (54) and (48), that $K^T = K$, as expected.

In addition to the original possibilities $\gamma=1$ (scalar $\gamma=-1$ (pseudoscalar bonds there are two other interesting viz. $\gamma=i$ and $\gamma=-i$. These two possibilities cannot be realized for one-dimensional bonds, but require at least two-dimensional complex bonds, that we shall call *spinor-bonds*. For spinor-bonds there is no difference in the time-reversal operations for efforts and flows, so they can't possibly be distinguished from one another. This means that they must have the same physical dimension, too. We shall not, though, at present treat such exotic types of bonds, but, by and large, keep to bonds with genus 1, that in the standard metric can be reticulated out on scalar bonds.

If we instead had looked at time-reversal of a system with flow-input, we would have found a relation corresponding to (54), but with γ entering instead of γ^*. However, (55) shows that we don't need to distinguish between γ and γ^* to an even power. We can therefore for an arbitrary 1-port, whether it has flow- or effort-input, write

$$K^T = -\gamma^2 \tilde{K} \qquad (55)$$

Where the tilde over the K comprises variance-shift for both indices, and complex conjugation.

The concept of a "1-port system" is now very general. Because the input-output bond now is a general vector-bond we have the possibility to define *an arbitrary multiport system* as a "1-port". Consequences of this possibility will be investigated in the next chapter.

By characterizing a vector-bond with a metric tensor g and a scalar genus γ we have obtained that the effect of the mirroring operations time-reversal and orientation-shift on the symbolic expressions for flow and effort can be expressed on tensor-form, i.e. independent of the coordinate system. This is perhaps not seen as entirely clear from the previous discussion, because those considerations that led to figures 14 and 15 were based on reticulation with scalar bonds, so they were only valid for bonds with genus 1. If the reticulation on figure 14 had used pseudoscalar bonds without further alterations in the setting of signs, the genus of the vector-bond would be -1, and the metric tensor would be $-\sigma_z$ instead of σ_z. The standard metric for bonds with genus -1 thus has 1 in the "deviant" places and -1 in the "normal" places in the diagonal. This difference then implies, that both the flow-orientation rule and the rule of indifferent effort-variance can be formulated as in figure 15, both for $\gamma=1$ and $\gamma=-1$. How these rules shall be formulated for the hypothetical spinor-bonds with $\gamma=i$ (or $\gamma=-i$) is a difficult question that we shall not consider now, because we in the most important applications only shall need the case $\gamma=1$.

We have defined *the standard metric* G.. as a diagonal matrix with 1 and -1 in the diagonal. Any metric can by suitable continuous transformations be brought to standard-form and therefore has a uniquely determined standard metric. By *the signature* of a metric we shall understand *the sum of the diagonal elements of the standard metric*. A d-dimensional metric will thus have the signature d if the standard metric is euclidean. In other cases it will be d-2n, where n is the number of deviant components in the effort-vector. Generally, the metric tensor is *hermitean*, but not necessarily dimension-less. The condition for g.. being dimension-less is that all components in each of the two energy-bond vectors have the same dimension. There can, however be good reasons to use a *dimensioned* (not dimension-less) metric, because we shall use coordinate systems like polar coordinates, where the different components have different dimensions. Among the dimension-less metrics we shall be specially interested in *neutral metrics*, characterized by

$$g_{\bullet\bullet} = (g_{\bullet\bullet})^{-1} = g^{\bullet\bullet} \tag{57}$$

To raise or lower an index is then a mirroring.

As an example of how to determine g.. for a given situation we look at the three-dimensional movement of a particle in *polar coordinates*. As the scalar product of two given vectors shall be a scalar, i.e. invariant by the shift from rectangular to polar coordinates, the so-called *norm-square*, i.e. the scalar product of a vector with itself also be invariant. The norm-square of the contravariant velocity-vector is determined by the metric g.. by

$$|v|^2 = v^i g_{ij} v^j \tag{58}$$

If the polar coordinates are called r, θ and φ we can determine

$$v^1 = \dot{r} \; ; \; v^2 = \dot{\theta} \; ; \; v^3 = \dot{\phi} \tag{59}$$

Where the dot over a symbol means that the quantity is derived with respect to time. The connection between the polar coordinates and the rectangular is given by

$$\left.\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \end{aligned}\right\} \tag{60}$$

Differentiating x, y, and z with respect to time we get

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix} \begin{pmatrix} \dot{r} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} \tag{61}$$

from where we find the norm-square of the velocity

$$|v|^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2 \qquad \textbf{(62)}$$

From (58), (59), and (62) we then get

$$g_{..} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix} \qquad \textbf{(63)}$$

This is an example of a dimensioned metric with euclidean standard metric. The signature is 3.

The set of neutral metrics with a given dimension d and signature s form an equivalence-class. The entire class is generated by a Lie-group of unitary transformations of that standard metric G.. that has the given signature. If the signature is equal to the dimension, the standard metric is euclidean, i.e. G.. is the unit matrix, and the equivalence-class contains no other metrics. If, on the contrary, the signature is less than the dimension there will be a manifold of equivalent neutral metrics. Let us, e.g. consider the two-dimensional metrics for complex bonds. The equivalence-class with s=0 here contains the three Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix};\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \textbf{(64)}$$

For which holds the relations

$$\left.\begin{array}{c} \sigma_x^2=\sigma_y^2=\sigma_z^2=1 \\ \\ \sigma_y\sigma_z=i\sigma_x \;;\; \sigma_z\sigma_x=i\sigma_y \;;\; \sigma_x\sigma_y=i\sigma_z \end{array}\right\} \qquad (65)$$

It further holds that the three Pauli matrices anti-commute with each other

The whole equivalence-class in this case forms a two-dimensional manifold

$$\sigma(\theta,\phi)=\sin\theta\cos\phi\,\sigma_x+\sin\theta\sin\phi\,\sigma_y+\cos\theta\,\sigma_z \qquad (66)$$

That is generated from the standard metric σ_z by the unitary matrix

$$U(\theta,\phi)=\cos\tfrac{\theta}{2}+i\sin\tfrac{\theta}{2}[\sin\phi\,\sigma_x-\cos\phi\,\sigma_y] \qquad (67)$$

Such that

$$\sigma(\theta,\phi)=U(\theta,\phi)\,\sigma_z\,U^{-1}(\theta,\phi) \qquad (68)$$

The angles θ and φ are polar angles for a point on the unit-sphere in three-dimensional space, so there is a one-to-one correspondence between the non-euclidean Pauli-metrics and the three-dimensional euclidean unit-vectors. The U-matrix in (66) is an example of a *1-unitary metric transducer*, an especially important type of ideal 2-port, that will be discussed in chapters 6 and 7. We note from (65) that the connection between U and the three-dimensional unit-vector is not one-valued, but two-valued: To a given unit-vector corresponds two different U-matrices.

5. Multiports as 1-ports

In this chapter we shall consider how the concept of the general vector-bond makes possible a generalization of the linear response theory, that in text 22 was developed for 1-port systems, described with complex scalar bonds.

By using complex signal-amplitudes, describing Fourier-components of real efforts and flows for a given Laplace-frequency $s=-i\omega$ one obtains that the reactive system-properties, describing storage of energy and involving integration over time, can be reduced to a purely algebraic relation of the same type as the passive response, that characterizes the sinks, but depends on the Laplace-frequency. An o-storage with the capacity C can be formally described as an o-sink with the conductance Cs and a x-storage with inductance or inertance L as a x-sink with impedance Ls. For a linear system with more scalar ports the relation between input- and output-variables can be expressed as a linear matrix-relation between an input- and an output-vector.

If we look at a system with d ports, of which p have flow-input and n=d-p have effort-input, we can put these bonds together to a d-dimensional vector-bond with flow-input, as shown in figure 17.

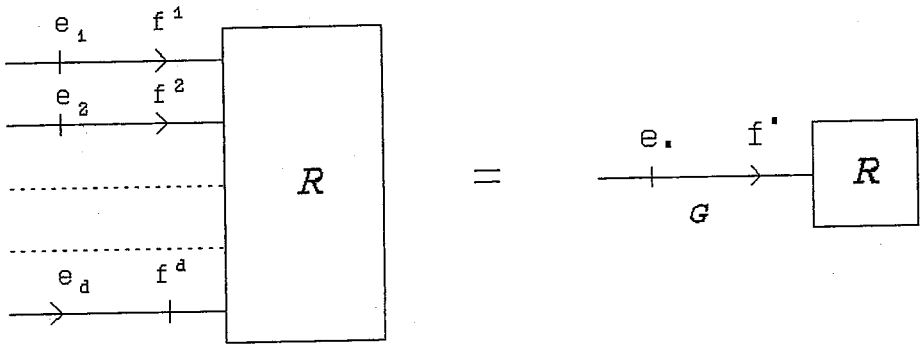


Figure 17. Definition of d-port with p flow-inputs and n effort-inputs as 1-port with d-dimensional flow-vector input

The metric G.. for the d-dimensional bond on figure 17 will then be of standard form with p 1s and n -1s in the diagonal:

$$G_{\bullet\bullet} = G^{\bullet\bullet} = \begin{pmatrix} 1 & 0 & \bullet & \bullet & 0 \\ 0 & 1 & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & \bullet & -1 \end{pmatrix} \tag{69}$$

When we, as in figure 17, have indicated contravariant flow-input and covariant effort-output, the linear relation between the flow- and the effort-vector will be given by the impedance-matrix R..

$$e_{\bullet} = R_{\bullet\bullet} f^{\bullet} \sim e_i = R_{ij} f^j \tag{70}$$

In the linear response theory the impedance-matrix is a function of the Laplace-frequency s. When s is real, the impedance matrix is also real and therefore easy to calculate. In the experimental situation where the measurement concerns ac-currents, s is imaginary, and when we determine the energy consumption of the system it is for a definite Fourier-frequency ω, and, hence, for an imaginary Laplace-frequency (s=-iω). The energy flow to the system is then given by (comp. (28)):

$$w = \tfrac{1}{2}[e^*_j f^j + e_i f^{*i}] = f^{*i} R^D_{ij} f^j \tag{71}$$

where R^D.. - the *dissipative part* of the impedance-matrix — is *the hermitean part*:

$$R^D.. = \tfrac{1}{2}(R.. + R^{\dagger}..) \quad R^D_{ij} = \tfrac{1}{2}(R_{ij} + R^*_{ji}) \tag{72}$$

Correspondingly, the *non-dissipative part* of R.. will be the same as the *non-hermitean part*:

$$R^{ND}..=\tfrac{1}{2}(R..-R^{+}..) \sim R^{ND}{}_{ij}=\tfrac{1}{2}(R_{ij}-R^{*}{}_{ji}) \qquad (73)$$

For the three other variance-forms of the impedance-tensor R we can correspondingly define a dissipative and a non-dissipative part by raising indices of $R^{D}..$ and $R^{ND}..$ with the metric tensor. For the mixed variances with one index above and one below, it will, however *not* be valid that R^{D} is the hermitean and R^{ND} the non-hermitean part, unless the metric is euclidean (G.. is the unit matrix, if all components of the flow-vector are flows in the scalar reticulation). The partition of R can then be expressed with a *tensor relation* without indication:

$$R = R^{D} + R^{ND} \qquad (74)$$

On diagrammatic form this division can be expressed by connecting the vector-bond to a junction, that must be of x-type for flow-input and of o-type for effort-input. For the system in figure 17 it must be an x-junction. Discussion of the junction-concept for general vector-bonds is difficult and will be postponed to a later chapter, but when the metric is of standard-form, as here, it is not difficult to see that the x-junction can be reticulated as scalar x-junctions for those bonds where the flow-coordinate is a flow and as scalar o-junctions for the deviant bonds, where the flow-coordinate is an effort.

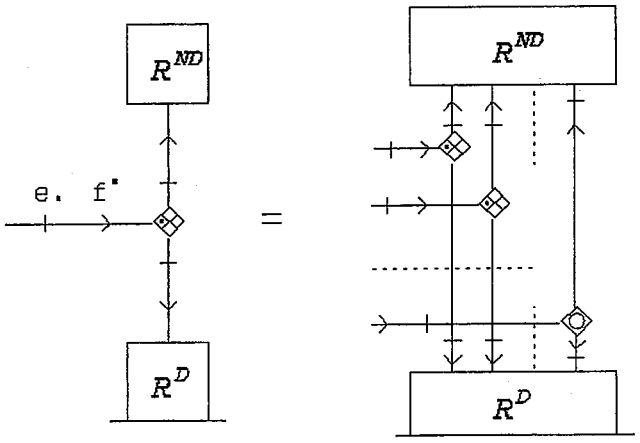


Figure 18. Division of tensor in a non-dissipative (ND) and a dissipative (D) part.

If a system develops according to its own internal laws, we may assume that these laws on the microscopic level are *reversible*, i.e. unchanged by time-reversal. This, however is not the same as saying that the macroscopic response of the system is reversible. For the response of the system to a signal from outside can only be characterized as a macroscopic signal if it raises above the noise-level in the system, and when it does, it will cause *dissipation*, because the macroscopic signal-energy will have a tendency to spread out to microscopic degrees of freedom, and from there it will not usually be able to return to the macro-level, as it is highly unlikely, that the micro-movements "swing to the beat and thereby make a coherent signal-like noise-contribution. If there is no external field, that can break the microscopic reversibility, it will only be the dissipative part of the system's response, that can break the time-reversal symmetry; the non-dissipative part of the response-tensor must be invariant to the time-reversal operation. In regard to (56) we must therefore have

$$R^{NDT} \doteq \tilde{R}^{ND} = R^{ND} \tag{75}$$

For the matrix-form with both indices below we get (comp. (54))

$$R^{NDT}{}_{\bullet\bullet} \doteq - R^{ND*}{}_{\bullet\bullet} = R^{ND}{}_{\bullet\bullet} \tag{76}$$

Now, $R^{ND}{}_{\bullet\bullet}$ is antihermitean, i.e. if we exchange the left-right sequence of the two indices and complex conjugate, we get the same matrix again with the opposite sign. Eq (76) therefore implies:

$$R^{ND}{}_{ij} = R^{ND\ ji} \tag{77}$$

For the dissipative part of R^D it must be valid, that it is *anti-reversible*, i.e. that it changes sign by time-reversal, for the dissipated energy flow, that is given by R^D,

changes sign by time-reversal, again under the assumption, that dissipation is the only thing that breaks reversibility, i.e. that no external fields break the microscopic reversibility, so we must have

$$R^{DT}{}_{\bullet\bullet} \doteq -R^{D*\bullet\bullet} = -R^{D}{}_{\bullet\bullet} \tag{78}$$

and, as R^D is hermitean, we get

$$R^{D}{}_{ij} = R^{D\,ji} \tag{79}$$

Thus, we have the same type of symmetry-relation for R^{ND} and $R^{D,}$ and, as R is the sum of these two, we have:

$$R_{ij} = R^{ji} \tag{80}$$

In the derivation of this symmetry-relation we have not assumed anything special about the metric, but only the general form of time-reversal (75), valid for genus 1 or -1. Furthermore, we have used the physical principle of microscopic reversibility. The rsult (80) is therefore more general than the point of departure of the analysis, where we did assume that the metric was on standard-form. If we look at one of the mixed variance-forms, we get for a general metric tensor

$$R_i{}^j = g_{ik} R^{kj} \tag{81}$$

If the metric is *neutral*, we are allowed to raise both indices on g, and simultaneously we can lower R's indices by using (80), so we get: (comp. (31)). If we further assume that the metric is *real*, we get:

$$R_i{}^j = g^{ik} R_{jk} = R_{jk} g^{*ki} \tag{82}$$

Here we have also used, that the metric tensor is hermitean (comp. (31)). If we further assume, that the metric is real, we get:

$$R_i^{\ j} = R_{jk}\, g^{ki} = R_j^{\ i} \tag{83}$$

This nice result, that *the R-matrices of mixed variance are symmetrical* (it is of course also valid, if the first index is above and the second below). thus presumes that the metric is neutral and real. In particular, it is valid when the metric is on standard-form.

The response-matrix of mixed variance gives the relation between input- and output-vectors in the energy bond, if these both are of the same variance, especially if they are both covariant we can use the form (80) with first index below and second above. In standard metric we can reticulate such a vector-bond out on scalar bonds by taking care that the bonds with the deviant components have the opposite orientation (comp. figure 14).

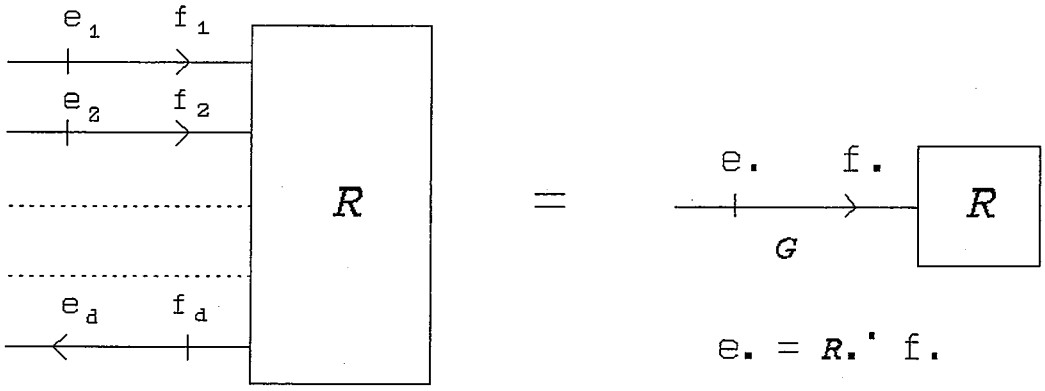
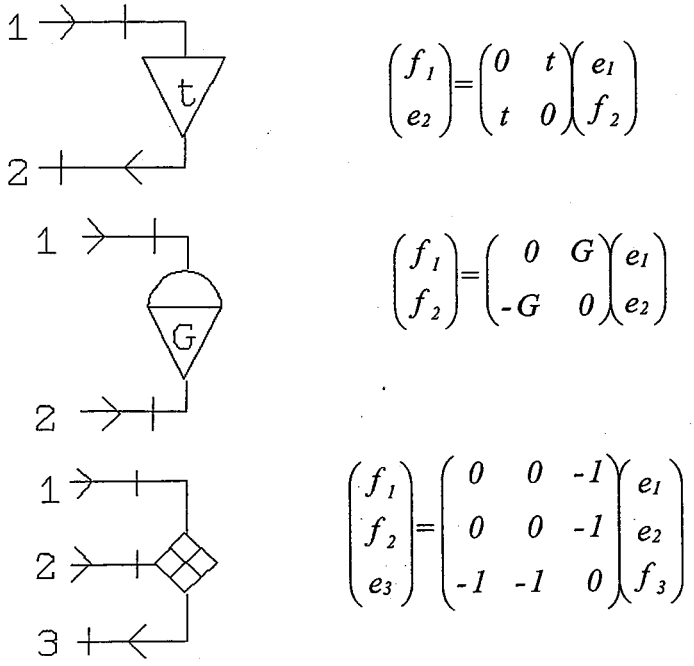


Figure 19. The relation between input- and output-vector of the same variance.

When we in a standard metric reticulate a vector-bond with the same variance of flow and effort out on scalar bonds, the signal signs in the scalar bonds all get "the same shape". Either the arrow points away from the stroke, which we call D-signs, or it points towards the stroke, which we call K-signs. We see then, that the

symmetry-rule (83) can be translated to a rule for scalarly reticulated multiports: *If we use the same type of signal-sign in all ports, either D-sign or K-sign, the response-matrix is symmetric.*

When one models with the simple energy bond formalism, one doesn't need that kind of rules. The equations follow unambiguously of the iconic model and the indications chosen, so one can try and see, whether the rule above is satisfied or not. For 1-port elements as sources and storages (in the complex response-theory) it is trivially satisfied, and it will also be valid for junctions and transformers, *but not for gyrators*. We shall use the name *reciprocal* about systems that satisfy the symmetry-rule. All simple elements except gyrators are reciprocal; gyrators are *antireciprocal*, because the K- or D-sign response-matrix for them are antisymmetrical.



$$\begin{pmatrix} f_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ f_2 \end{pmatrix}$$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & G \\ -G & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

$$\begin{pmatrix} f_1 \\ f_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ f_3 \end{pmatrix}$$

Figure 20. K-sign matrices for simple 2-ports and 3-ports.

Gyrators are *non-dissipative* and *antireversible* and thus break with the assumption, that the only source of irreversibility is the dissipation in the system. If the system is influenced by external fields that break the microscopic reversibility, then these fields will make themselves felt in a scalarly reticulated energy bond model via gyrators. On the other hand, if we in a system-reticulation only use reciprocal elements (no gyrators), then the only form of irreversibility lies in dissipation, i.e. in the sinks. The non-dissipative part of the system's

response is then taken care of solely by reversible elements and must therefore be invariant by time-reversal. For the dissipative part it must be valid, that the energy flow to all the sinks changes sign by time-reversal, and therefore the whole dissipative response-matrix must change sign, as presupposed in (78). It must therefore be so, that the total response-matrix for a system reticulated of reciprocal elements is reciprocal. This rule is known as *Brayton's theorem.*

If we allow complex transformer-parameters there seems to be problems with the reciprocity-properties. E.g. t=i in the response-matrix on figure 20 will give an anti-reversible response. It must therefore at present be stressed, that the rules in figure 20 are only valid for real parameters. Generalized rules for transformers and gyrators in the complex vector-bond-formalism will be treated in the next chapter.

If a system is influenced by external fields that break the microscopic reversibility, we can re-establish reversibility by allowing the time-reversal-operation include a reversal of these external fields. It may, e.g., be a magnetic field or Coriolis-forces that are extant. Let us for the sake of simplicity introduce a single symbol B for such fields, and let -B mean that all these fields are reversed. The reversibility and the anti-reversibility of, respectively the non-dissipative and the dissipative response, (76) and (78) can then be generalized to:

$$\left.\begin{aligned} R^{ND\,T}(-B)..\; &= R^{ND}(B).. \\ R^{D\,T}(-B)..\; &= -\,R^{D}(B).. \end{aligned}\right\} \qquad \textbf{(84)}$$

The symmetry relation (80) is then changed to

$$R_{ij}(B) = R^{ji}(-B) \qquad \textbf{(85)}$$

and, assuming that the metric is real and neutral we find the general reciprocity-relation

$$R_i^{\,j}(B) = R_j^{\,i}(-B) \qquad \textbf{(86)}$$

We have here an energy bond formulation of the famous reciprocity-theorem due to Lars Onsager (1931). This theorem is considered fundamental to non-equilibrium thermodynamics, but has a precursor in the theory of electrical networks (see text #8, p.85).
Let us as an example of the application of the reciprocity-theorem look at the *thermo-galvano-magnetic effects*. We consider a rectangular metal-plate with thickness Δz and edges Δx and Δy. A magnetic field B is applied in the z-direction, but first we assume that it is 0, so that only thermo-galvanic effects can occur. These effects comprise electrical resistance, heat conductance, and thermo-electricity (Seebeck- and Peltier-effect). It will be in accordance with experimental possibilities of control, if we assume that we control the electric current in the x- and y-direction and the temperature-differences in the same two directions. Thus, the system has 4 ports:
An electric x-port and an electric y-port, both with flow-input. The thermal effort in the x-direction can be defined as a Carnot-efficiency factor for the heat-current Q_x. If the temperature of the background-reservoir is T and the x-dependent local temperature is T(x) (x from 0 to Δx), The driving effort for the heat-flow given by the difference between the two Carnot-factors, that can then be approximated if the relative temperature differences are small:

$$\left(1-\frac{T}{T(0)}\right)-\left(1-\frac{T}{T(\Delta x)}\right)\approx\frac{T(0)-T(\Delta x)}{T}=\frac{\delta_x T}{T} \qquad (87)$$

And correspondingly for the effort in the thermal y-port, where the heat flow is Q_y. In the electric x-port the effort and flow are simply the electric voltage-drop from x=0 to x=Δx, V_x and the electric current in the x-direction J_x, and similar for the electric y-port. We can then describe the situation by defining a 4-dimensional vector-bond with flow-input to the system by defining the *covariant* flow-vector such that its first two components are the electric currents and its last two components are the thermal efforts. The *contravariant effort-vector* will
then have electric voltages as its first two components and the heat-currents as its two last. The energy flow to the system (the dissipation) is then

$$w = e^i f_i = V_x J_x + V_y J_y + Q_x \frac{\delta_x T}{T} + Q_y \frac{\delta_y T}{T} \qquad \textbf{(88)}$$

And the metric tensor for the vector-bond so defined will be

$$G.. = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \textbf{(89)}$$

According to the rule of indifferent effort-variance (figure 15) the same physical situation can be described with covariant effort, when the flow-variant is kept as covariant. This just requires, that in the two scalar bonds with deviant components (bond 3 and 4) the orientation is shifted and at the same time the sign is shifted for the symbolic expressions for the "efforts" (that are really flows). In this way we get the indication as shown below:

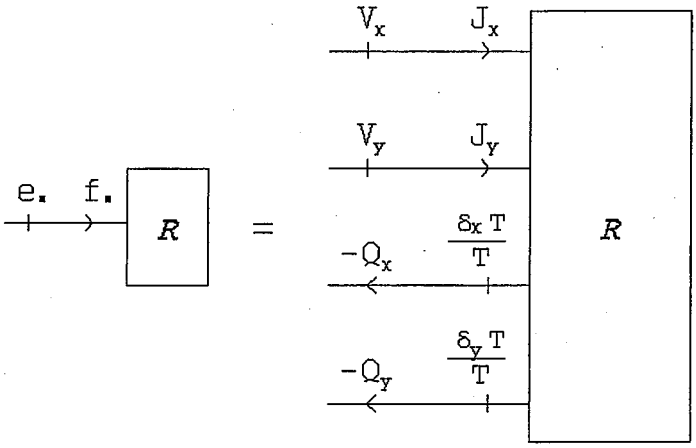


Figure 21. System with electric and thermal ports in the x- and y-direction.

If the system is kept isothermal, the efforts in the two lowest bonds in figure 21 are 0, and we may disregard the thermal bonds. The two electric bonds then form a two-dimensional euclidean bond, and the reciprocity-theorem (83) immediately gives that the electric resistance-tensor will be symmetric. We don't need to distinguish netween the different matrix-variants of the resistance-tensor, as the

metric is euclidean. If we disconnect the electric currents, we only have the two thermal ports, and reciprocity means that the heat-conductance tensor will be symmetric.

With all four bonds connected we shall simplify discussion by assuming that the material is isotropic. Then both the resistance- and the heat-conductance-tensor will be diagonal, and there will, all in all, be no couplings between the vector-components that belong to the x-direction and those that belong to the y-direction. The response-matrix is then:

$$R_{\bullet}^{\bullet} = \begin{pmatrix} R_x & 0 & P & 0 \\ 0 & R_y & 0 & P \\ P & 0 & -K_x & 0 \\ 0 & P & 0 & -K_y \end{pmatrix} \qquad \textbf{(90)}$$

Here the resistances R_x and R_y can be expressed by the specific resistance of the material and the heat conductances K_x and K_y by the specific heat conductance κ and the temperature T

$$\left.\begin{aligned} R_x &= \rho\frac{\Delta x}{\Delta y \Delta z} \;;\; R_y = \rho\frac{\Delta y}{\Delta z \Delta x} \\ K_x &= T\,\kappa\frac{\Delta y \Delta z}{\Delta x} \;;\; K_y = T\,\kappa\frac{\Delta z \Delta x}{\Delta y} \end{aligned}\right\} \qquad \textbf{(91)}$$

The combination of the scale-factors Δx, Δy and Δz in (91) express that the currents J and Q scale proportional to the cross-section area, while both the thermal and the electric efforts scale proportional to the length. The elements of the cross-section area are, so to speak parallel-connected (with an o-junction), while the length-elements are connected in series (with an x-junction). For the same reason the matrix elements in (91) that couple a thermal to an electric

current must be unscaled. For an isotropic sample there cannot be any coupling between the heat current in the x-direction and the electric current in the y-direction, and coupling-coefficients between currents in the x- and y-directions must be identical. A similar thing is valid for the couplings between electric and thermal efforts. There are therefore two pairs of thermoelectric coefficients, and members of a pair on the same side of the diagonal are identical by isotropy-reasons. The reciprocity theorem then implies, that coefficients standing symmetrically across the diagonal are identical, so there is only one thermoelectric coefficient, P, but it has two essentially different meanings. The P in the third row and first column denotes the ratio between the heat current and the electric current (in the x-direction) with an opposite sign, when there is no applied temperature-difference. With this meaning P is the *Peltier coefficient.* On the other hand, the P in the first row and third column is connected with the *Seebeck coefficient* or the *thermal voltage* S, that denotes the ratio between the electric voltage and the temperature difference (in the x-direction), when there is no electric current. The connection is given by

$$S \doteq \left(\frac{V}{\delta T} \right)_{J=0} = \frac{P}{T} \qquad \textbf{(92)}$$

This relation between two essentially different thermoelectric effects was first discovered empirically by Kelvin in 1854. The theoretical reason for it came somewhat later with the kinetic theories of Boltzmann and others, but it used a number of ad hoc assumptions, and first with Onsager's theorem the matter could be seen in its right perspective.

Thermoelectricity is a non-dissipative and reversible phenomenon. The division of the response-tensor in a dissipative and a non-dissipative part is done by finding its hermitean part (i.e. the symmetric part, as everything is real) and the anti-hermitean (antisymmetric) part of the matrix

$$R_{\bullet\bullet} = R_{\bullet}^{\bullet} G_{\bullet\bullet} = \begin{pmatrix} R_x & 0 & -P & 0 \\ 0 & R_y & 0 & -P \\ P & 0 & K_x & 0 \\ 0 & P & 0 & K_y \end{pmatrix} \qquad (93)$$

It follows from this, that the dissipative coefficients stand in the diagonal, and that the thermoelectric coefficients outside the diagonal describe non-dissipative effects. The division corresponding to figure 18 can now easily be reticulated:

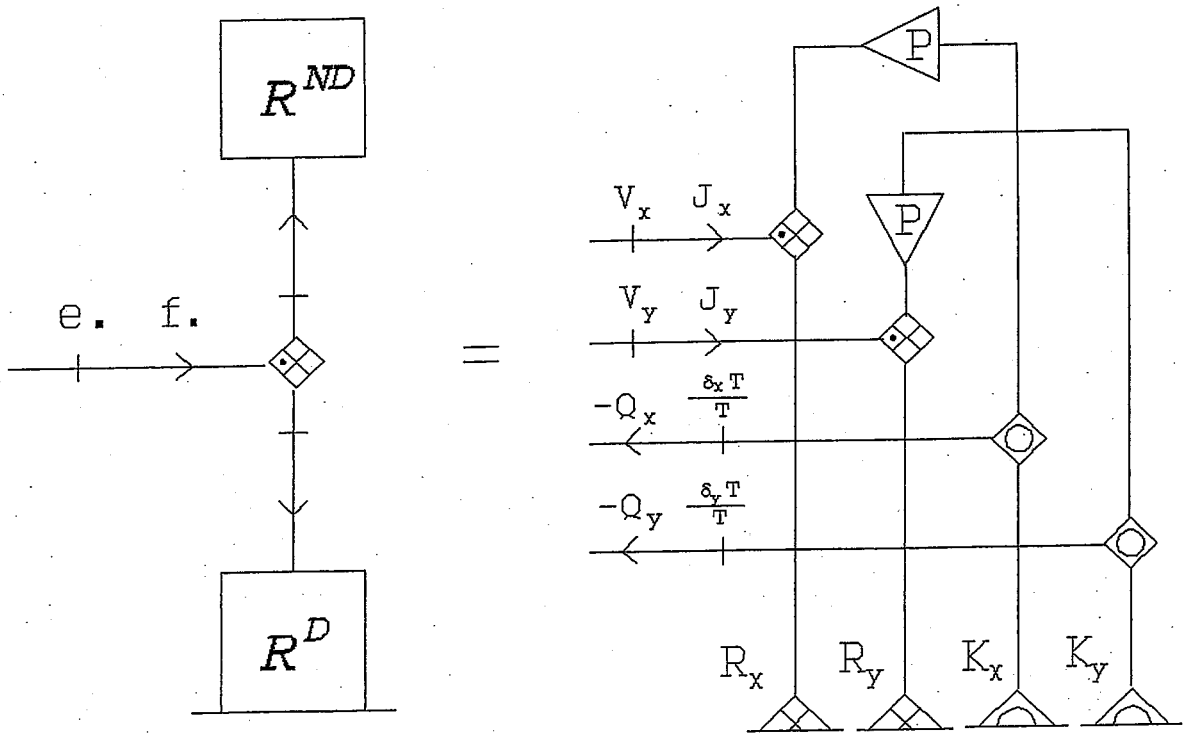


Figure 22. Reticulation of thermogalvanic effects.

According to the reciprocity theorem (86) it must be valid, that the symmetric part is an even function of the magnetic

The diagram in figure 22 consists of two separate parts, an x-part an a y-part that are uncoupled due to isotropy. When we now apply a magnetic field in the z-direction there will appear a coupling between these parts, corresponding to, that there now emerge magnetical field-dependent components where there was 0 in the response-matrix (90). We can now divide this matrix in a symmetric and an antisymmetric part

$$R(B)_{\bullet}^{\bullet} = R^{s}(B)_{\bullet}^{\bullet} + R^{a}(B)_{\bullet}^{\bullet} \tag{94}$$

According to the reciprocity theorem (86) it must be valid, that the symmetric part is an even function of the magnetic field and the antisymmetric part is an odd function of the magnetic field. For weak magnetic fields we may assume that the symmetric part is independent of B, while the antisymmetric part is linear in B. There is room for four new parameters in the antisymmetric part. When we take isotropy and the scaling of flows and efforts with the dimensions of the sample into account, we can do with three new thermo-galvano-magnetic specific parameters H, L, and E:

$$R^{a}(B)_{\bullet}^{\bullet} = B \begin{pmatrix} 0 & \frac{H}{\Delta z} & 0 & E\frac{\Delta x}{\Delta y} \\ -\frac{H}{\Delta z} & 0 & -E\frac{\Delta y}{\Delta x} & 0 \\ 0 & E\frac{\Delta y}{\Delta x} & 0 & -L\Delta z \\ -E\frac{\Delta x}{\Delta y} & 0 & L\Delta z & 0 \end{pmatrix} \tag{95}$$

To H corresponds *the isothermal Hall effect*, to L the *Leduc-Righi effect*, and to E corresponds two reciprocal effects called the *Ettingshausen effect* and the *Nernst effect*. For the Hall effect H is the traditionally defined Hall coefficient, but for the three other effects the traditionally defined coefficients deviate somewhat from

the quantities here introduced, as they enter in different combinations with the specific heat conductance. The precise definition of the different coefficients can be found in Callen's textbook on thermodynamics (Callen 1960). The energy bond parameters that are here introduced have a clearer physical meaning, when they enter an iconic reticulation, because it is clearly seen, whether they describe dissipative or non-dissipative effects, and if they are reversible or non-reversible (which is another distinction in the presence of a magnetic field).

With respect to the reticulation it is clear, that the effects that are linear in the magnetic field are attached to gyrators. When these gyrators are connected to the diagram in figure 22, it must be done with *weak causality*, because the magnetic field can be added *adiabatically*, i.e. as a slowly growing addition, that does not destroy the causality of the already existing thermo-galvanic effects. An x-gyrator having effort-output can be coupled to the two x-junctions at the electric ports, and similarly an o-gyrator with flow-output in both ports can be coupled to the two o-junctions at the thermal ports. Thus we get a description of, respectively, the Hall effect and the Leduc-Righi effect. The two Ettingshausen-Nernst gyrators shall connect an electric x-port to a thermal y-port, and vice versa, and this cannot happen with weak causality, unless we add an x-junction in the bonds to the thermal sinks. Thus, we arrive at the following reticulation:

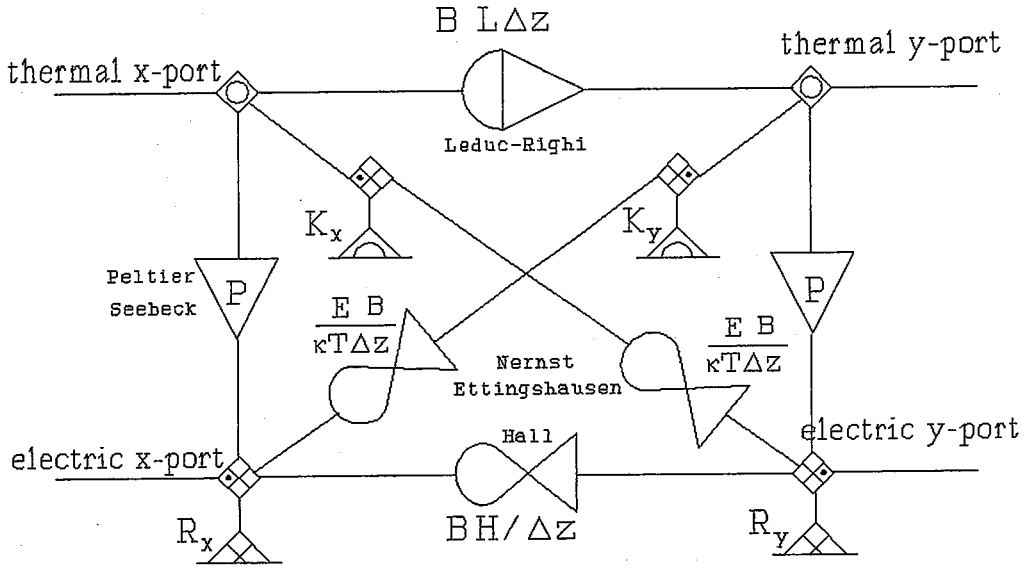


Figure 23. The thermo-galvano-magnetic effects.

When one writes the response-matrix for figure 23 with the on figure 22 given indications one will discover, that the resistances R_x and R_y enter in another way than in (90), because they are combined with a term of second order in the magnetic field. This is due to the fact that the electric current in the x-direction through the Ettingshausen-gyrator gives a thermal effort-input to the heat conduction-sink for the y-direction, and the resulting transverse heat current acts back through the same gyrator and thus contributes to the effort V_x. If one tries to measure the specific resistance of the material by sending a current in the x-direction with $J_y=0$ and $\delta_x T=0$ (so there is no Hall- and thermo-voltage, the result will depend on, whether the thermal y-port is controlled *isothermally,* $\delta_y T=0$ or *adiabatically* $Q_y=0$. The mentioned second order effect does only occur by isothermal control, so it is really the adiabatic specific resistance $\rho=\rho_a$ that determines R_x and R_y as in (91). The connection between the isothermal and the adiabatic specific resistance is found to be

$$\rho_i = \rho_a + \frac{(EB)^2}{\kappa T} \qquad (96)$$

The response-tensor for a multiport will in general by changed by causal shifts in

one or more ports. It will often be possible to find matrix-elements by a quick survey of *the signal routes* of the graph. A method for such a visual inspection, whereby one avoids to write equations builds on *Shannon's loop rule* (J. Gundermann, 1976). Signal routes can be perceived as railway tracks, where "the train", i.e. the signal can run both ways. Maybe it departs from some place as a flow-signal, runs to a "terminal station", where it is reflected and returns as an effort-signal. On the way it passes various "stations" in the form of energy bond icons, each of them multiplying the matrix element with a certain factor. Junctions act as switches, departing from the dominant bond. Thus, signals can only pass the junction from the dominating to a weak bond (strong causality) or from a weak to the dominating bond, but not from a weak to another weak bond. On figure 24 below we show some possible signal-route-structures for the diagram on figure 23. The left part of the figure shows the routes for the isothermal case, when there is effort-control in the thermal y-port, and the right part shows the routes for the adiabatic case with flow-control in the same port.

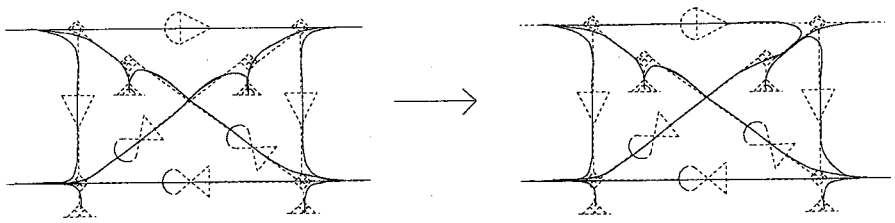


Figure 24. Change of signal-routes for figure 23.

The reciprocity theorem is a basic theorem for multiports, that doesn't have sense in the 1-port theory. Another basic theorem, that we are going to consider now, viz. the *Fluctuation-Dissipation theorem* (FD) on the contrary has such an obvious generalization from the 1-port theory, that it almost can be taken over directly. Therefore we shall not discuss it in great length. Let us consider a multiport which, like in figure 17 can be regarded as a 1-port with a d dimensional vector bond. We shall again let the input-vector be a contravariant flow-vector and the output-vector a covariant effort-vector. In general the response-matrix will have both a dissipative part R^{D}.. that is hermitean and a non-dissipative part R^{ND} that is non-hermitean. It is now clear, that the dissipative part alone is a source of noise in the effort-vector, for it is this part that contains all the sinks, that are the only sources of noise among the simple energy-bond-icons. We shall therefore pretend that R.. is hermitean, which just means that when R..

occurs in the following, it is really R^{ND}.. we mean.

A hermitean matrix can always be diagonalized by a unitary transformation, i.e. we can transform the flow-vector

$$f'^{\bullet} = U^{\bullet}{}_{\bullet} f^{\bullet} \quad ; \quad U^{*j}{}_{i} = U^{-1\,i}{}_{j} \tag{97}$$

And the effort-vector shall then be transformed such that the energy flow is conserved:

$$e'_{\bullet} f'^{*\bullet} = e_{\bullet} f^{*\bullet} \Rightarrow e_{\bullet} = e'_{\bullet} U^{*\bullet}{}_{\bullet} \tag{98}$$

It will then be possible to determine the unitary transformation U such that the transformed effort-vector is given by a diagonal matrix times the transformed flow:

$$e'^{\bullet} = R^{0}{}_{\bullet\bullet} f'^{\bullet} \tag{99}$$

The elements in the diagonal of R^0 will be the eigenvalues of the hermitean R.. and therefore real. Furthermore, they will be *positive*, because the dissipation is positive definite. (otherwise, we could make a perpetuum mobile). Figure 25 below shows in the upper part that U can be represented as a transformer (more about this in the next chapter, and in the lower part, that the diagonal R^0 can be reticulated as a collection of independent sinks.

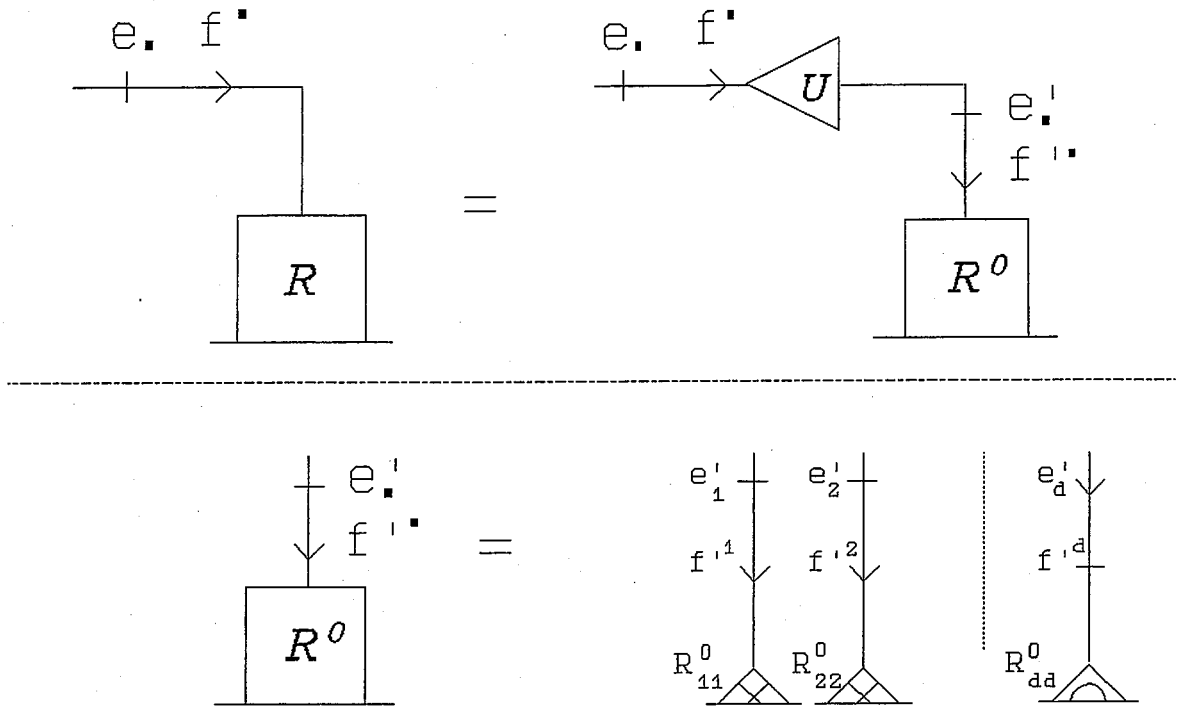


Figure 25. Diagonalization of dissipative system.

the system's response-matrix. However, it is only the real parts that contribute to the noise. From (97), (98), and (99) we get

$$e_i = U^{*k}{}_i e'_k = U^{*k}{}_i R^0_{kl} f''^l = U^{*k}{}_i R^0_{kl} U^l{}_j f^j \qquad \textbf{(100)}$$

In general both the diagonalizing transformer and the eigenvalues R^0_{ii} will be functions of the frequency ω, and each of the sinks on figure 25 therefore must be regarded as the real part of a complex impedance (or admittance), whose imaginary part belongs to the non-dissipative part of the system's response-matrix. However, it is only the real part that contributes to the noise. From (97),

(98), and (99) we get

$$e_i = U^{*k}{}_i e'_k = U^{*k}{}_i R^0_{kl} f'^l = U^{*k}{}_i R^0_{kl} U^l{}_j f^j \qquad \textbf{(100)}$$

So, the connection between R.. and R^0.. is given by

$$R_{ij} = U^{*k}{}_i R^0{}_{kl} U^l{}_j \qquad \textbf{(101)}$$

The two summations over the index-pairs k-k and l-l in (101) are really only one summation, as R^0.. is diagonal, but we disregard this in (101) in order to be able to use Einstein's summation-convention. Note also that (101) not can be expressed with the "dot-notation", because the left-right order of the two indices of the first factor is wrong. This deficiency will be repaired in the next chapter.
The noise from the scalar sinks in figure 25 is now given by the FD theorem as described in text# 22. The effort-noise from a x-sink R is described by the frequency-dependent *Poer spectrum*. If the noise is described as a Fourier-series over many densely packed frequencies ω_k

$$R_{ij} = U^{*k}{}_i R^0{}_{kl} U^l{}_j \qquad \textbf{(102)}$$

we may assume that the complex Fourier-coefficients c_k are statistically independent with randomly dispersed phases, except that the coefficients for ω_k and ω_{-k} shall be each other's complex conjugates, when the effort is real. The power spectrum is then defined by

$$\delta e'(t) = \sum_{k=-\infty}^{\infty} c_k \exp(-i\omega_k t) \qquad \textbf{(103)}$$

The factor ½ is due to the fact that the power spectrum is only defined for positive frequencies, but it includes an equal contribution from the negative. The last re-writing in (103) is based on the law of large numbers. The microscopic frequencies ω_k lie so closely that they are macroscopically undistinguishable, but

the sum of their mean-square-deviations will be equal to the mean-square deviation of their sum and proportional to the length of the small frequency-interval $\Delta\omega$, they belong to.

We can therefore just as well replace all these micro-noise-contributions by a "coarse-grained" noise-contribution, one for each of a series of frequency-intervals $\Delta\omega$ that are small in the macroscopic sense, but big enough in the microscopis sense to contain many micro-frequencies. The frequency-intervals so defined are disjunct and cover the whole frequency-axis, and in each of them we can imagine a noise-source, oscillating with the frequency ω, incoherent with and statistically independent of the noise-sources in the other frequency-intervals, and whose mean-square-deviation is proportional to the length of the frequency-interval it belongs to. The FD theorem for a sink $R(\omega)$ (i.e. the real part of the response-function) then says that

$$\frac{1}{2}\Phi_e(\omega)\Delta\omega = \sum_{\omega_k \in [\omega,\omega+\Delta\omega]} <|c_k|^2> \doteq <|\delta\tilde{e}'(\omega)|^2> \Delta\omega \qquad \textbf{(104)}$$

where E_{osc} is the energy of a harmonic oscillator with the resonance frequency ω in thermal equilibrium with the temperature T:

$$\Phi_e(\omega) = 2 <|\delta\tilde{e}'(\omega)|^2> = \frac{2}{\pi} R(\omega)\, E_{osc}(\omega,T) \qquad \textbf{(105)}$$

In the classical limit $\omega << K_B T$ is $E_{osc} = k_B T$, thus independent of the frequency.
It is now easy to generalize (104) to be valid for the entire dissipative field R^0.. being a collection of independent simple sinks. We define

$$E_{osc}(\omega,T) = \frac{1}{2}\hbar\omega \coth\frac{\hbar\omega}{2k_B T} = \frac{1}{2}\hbar|\omega| + \frac{\hbar|\omega|}{\exp\frac{\hbar|\omega|}{k_B T} - 1} \qquad \textbf{(105)}$$

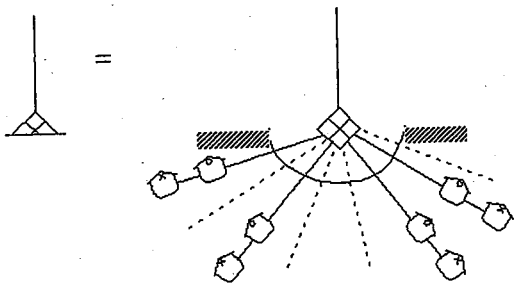
and have then, as R^0.. is diagonal and the noise-contributions from two different

sinks are statistically independent:

$$\Phi^0_{ij}(\omega)=2<\delta\tilde{e}'_i(\omega)\,\delta\tilde{e}'^{*}_j(\omega)> \qquad \textbf{(106)}$$

$$\Phi^0_{ij}(\omega)=\tfrac{2}{\pi}R^0_{ij}(\omega)\,E_{osc}(\omega,T) \qquad \textbf{(107)}$$

The FD theorem expresses that the response-function $Z(\omega)= R(\omega) + i\,X(\omega)$ can be "spectrally resolved" on harmonic oscillators with a spectral density, given by the real part $R(\omega)$. Especially, a pure frequency-independent sink and does not contain an imaginary part, have a spectral distribution that is uniform over all frequencies. It is this spectral resolution that is basic for the choice of the sink-icon



$$Z(\omega)=\frac{i}{\pi}\int_0^{\infty}R(\omega')\left[\frac{1}{\omega-\omega'}+\frac{1}{\omega+\omega'}\right]d\omega'$$

Figure 26. Spectral resolution of $Z\omega)=R(\omega)+iX(\omega)$.

The spectral resolution formula above presupposes that ω has a(small) positive imaginary part. The diagram shows the special case of a pure sink, where R is frequency-independent and the non-dissipative part of Z, $X(\omega)$ is 0. The quantity in square brackets behind the integral-sign is the response-function of a harmonic oscillator with resonance-frequency ω'.
We can now find the to (106) corresponding function for the original impedance R..

$$\Phi_{ij}(\omega) = 2 < \delta\, \tilde{e}_i(\omega)\, \delta\, \tilde{e}^*_j(\omega) > \qquad \textbf{(108)}$$

From (98) and (107) we then get (omitting the ωs):

$$\Phi_{ij} = 2 < U^{*k}{}_i\, \delta\, \tilde{e}'_k\, U^l{}_j\, \delta\, \tilde{e}'^*_l > = \tfrac{2}{\pi} E_{osc}\, U^{*k}{}_i\, R^0{}_{kl}\, U^l{}_j \qquad \textbf{(109)}$$

Finally, we get from (191) the FD theorem

$$\Phi_{ij} = 2 < U^{*k}{}_i\, \delta\, \tilde{e}'_k\, U^l{}_j\, \delta\, \tilde{e}'^*_l > = \tfrac{2}{\pi} E_{osc}\, U^{*k}{}_i\, R^0{}_{kl}\, U^l{}_j \qquad \textbf{(110)}$$

For a *passive* system, where the dissipative response-matrix is frequency-independent (as, e.g. the system on figure 23) we have in the classical limit a frequency-independent noise-spectrum, white noise. The immediate noise-correlation then diverges, being the integral of the spectrum over all frequencies. This is an example of *the ultraviolet catastrophe* of classical physics. In his discussion of the black body radiation Planck solved this problem by replacing the classical oscillator energy $k_B T$ with the quantum mechanical expression (105) *without the zero point term* $\frac{1}{2}\hbar\omega$. When this term is kept we apparently get an ultraviolet catastrophe that is even worse than the classical, so one may say it was lucky, that Planck was not farsighted enough to include the zero point energy. It has been discussed if this term ought to be omitted from the FD theorem, but this would lead to analytic difficulties. It must be correct that the zero point energy shall not figure in the spectrum of black body radiation, for this energy cannot be radiated away, as the oscillators cannot give it away. On the other hand, it will make itself felt by creating quantum fluctuations for a charged particle in vacuum. The zero point fluctuations are closely related to Heisenberg's uncertainty relations, and therefore it should not just be *renormalized away*. The problem of the ultraviolet catastrophe of quantum mechanics should rather be solved by realizing that the dissipative response matrices in the ultraviolet domain decrease with frequency faster than $1/\omega$ due to the inertia of elementary particles.

6. Two-ports and tensors

In this chapter we shall discuss two-ports that are *ideal* in the sense that they neither store nor dissipate energy. These two-ports can in general be characterized as *transmitters* that perform operations on the effort- and the flow-vector between a *primary* and a *secondary* port, but let the energy flow unchanged. The simplest types of transmitters are linear or anti-linear, and they were divided into *transformers, gyrators, and transducers.*

The proper energy bond is the basic transmitter — a *unit-transformer.* We can imagine two observers (1 and 2), that cannot see each other, but have mutual measuring prescriptions, definition of covariant and contravariant, can measure flow and effort, each in their own end of a long energy bond. These measurements then require, that they each choose an orientation, so let us imagine that they both choose the orientation from 1 to 2. We then know, that the same physical situation (as it must be, when it is two ends of the same energy bond) can be described equally well with covariant and contravariant effort (the rule of indifferent effort-variance, figure 15). That is, if both observers define the flow as contravariant, and if the orientation is the same, then they are free to choose, whether the effort shall be defined as covariant or contravariant. If now observer 1 chooses to measure the contravariant effort, and observer 2 to measure the covariant effort, the relation between the measured values will be

$$e^{(2)}{}_{\bullet} = g_{\bullet\bullet} e^{(1)\bullet} \tag{111}$$

where g.. is the metric tensor of the vector-bond. If the two observers agree about both orientation and variance of flow and effort, they will of course find the same effort, The relation between $e^{(1)}$ and $e^{(2)}$ will then be given by the unit-matrix, but this is identical with one of the mixed variants of the metric tensor, so in this case we can write

$$e^{(2)}{}_{\bullet} = g_{\bullet}{}^{\bullet} e^{(1)}{}_{\bullet} \; ; \; e^{(2)\bullet} = g^{\bullet}{}_{\bullet} e^{(1)\bullet} \tag{112}$$

It is in this sense we may assert, that the energy bond is a unit transformer and the metric tensor a unit tensor. If we provisionally think of a (2. order) tensor as a symbol with four matrix-variants, connected with the raising-lowering rule, it is clear that a tensor generally cannot have all its matrix-variants equal to the unit-matrix. We have earlier introduced the Kronecker-symbol δ, that is a unit-matrix with both indices down. We shall now introduce a related symbol Δ, that is a unit-matrix with both indices up. We then have

$$g_{\bullet}^{\ \bullet} = g^{\bullet}{}_{\bullet} = \delta_{\bullet\bullet} = \Delta^{\bullet\bullet} \tag{113}$$

and the raising-lowering rule gives

$$\begin{aligned} \delta_{\bullet}^{\ \bullet} = \delta^{\bullet}{}_{\bullet} = g^{\bullet\bullet} \ ; \ \delta^{\bullet\bullet} = (g^{\bullet\bullet})^2 \\ \Delta^{\bullet}{}_{\bullet} = \Delta_{\bullet}^{\ \bullet} = g_{\bullet\bullet} \ ; \ \Delta_{\bullet\bullet} = (g_{\bullet\bullet})^2 \end{aligned} \tag{114}$$

For a *neutral metric* (comp. (57)) is, thus, Δ=δ. By an expression like $(g..)^2$ we shall understand the usual row-column product of the matrix g.. with itself. This we cannot write as g..g.. with the new notation, because the dots are in the same height, but in such cases the two Kronecker symbols can be of help, as we may write

$$(g_{\bullet\bullet})^2 = g_{\bullet\bullet}\Delta^{\bullet\bullet}g_{\bullet\bullet} \ ; \ (g^{\bullet\bullet})^2 = g^{\bullet\bullet}\delta_{\bullet\bullet}g^{\bullet\bullet} \tag{115}$$

Although the unit transformer is nothing but the naked energy bond, it is still an advantage to have a special icon for it, that may be used, when we discuss observer 1 and 2's different indications of the same bond. As the bond has no inherent asymmetry between its two ends, it must be a symmetric icon, so we introduce the icon below in figure 27 for the unit transformer. Normally we introduce a tensor-symbol t to describe the *exchange-ratio* of a transformer, so for the unit-transformer t is equal to the metric tensor g.

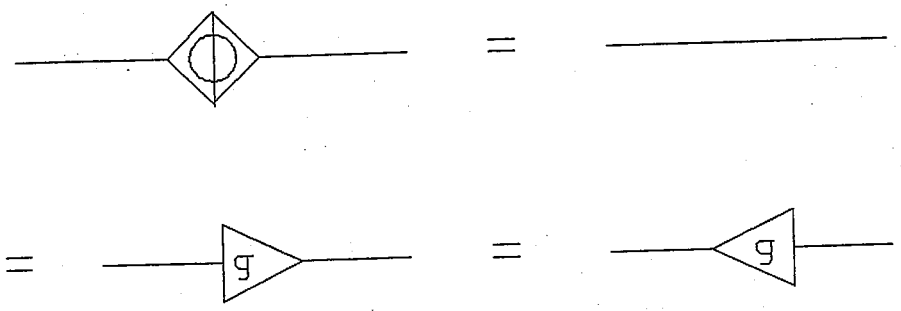


Figure 27. The unit-transformer.

We can now attempt to make a full survey over the different functions of a transformer. We shall place observer 1 to the right side, which we call the *primary* side, while observer 2 gets the *secondary* side to the left. For a symmetric transformer-icon of course it makes no difference what we call primary and secondary, but for the common triangular transformer-icon the primary side is the base-line of the triangle (the flat side) and the secondary side the opposite apex, and we shall normally place the primary side to the right, because it, as we shall see later, implies certain advantages, related to the common practice in mathematical formula place the quantity that is to be transformed, on the right side of the equal-sign and the result of the transformation on the left side (comp. the discussion of the "retrograde" reading of the sign-relation I-R-O in chapter 1).

The causality in the two ports is *relatively* fixed. We may freely choose the causality in the primary port, but by this choice the causality in the secondary port is fixed. For a transformer we must have the same causal sequence in the two bonds, That is, if the icon has effort-input in the primary port, it must have flow-input in the secondary port, and vice versa. The tensor-symbol of the transformer is defined by its operation on the effort when going from primary to secondary, and therefore it is most natural to choose effort-input on the primary side. However, it must be stressed, that the connection between primary- and secondary-effort is an *equation,* that is independent of which of the two efforts that is considered the input: if $e_2 = t\ e_1$, then $e_1 = t^{-1}e_2$. We shall therefore (temporarily) limit ourselves to consider effort-input on the primary side.

When the causality is fixed, there are 8 possibilities for indications of each of the two bonds, Because there are two possible orientations and two possible variances of each of the two vectors. So there are in principle 64 different functions of a transformer. Luckily, this number can be reduced to a few *prototypes* by use of the flow-orientation rule and the rule of indifferent effort-variance (figure 15). With the flow-orientation rule we can always take care, that the orientation of each bond goes from the primary to the secondary side. When we further apply the rule of indifferent effort-variance, there are only two types of bonds left, that cannot directly be translated to each other. We shall call them a- and b-bonds, that is, respectively, bonds with contravariant flow and bonds with covariant flow (implicit that orientation and causality are fixed, as described). There are then only four different prototypes of transformer-functions, that we shall take into account, viz. 2a-1a, 2a-1b, 2b-1a, and 2b-1b. For an icon-symmetric transformer like the unit-transformer the two mixed forms 2a-1b and 2b-1a will be identical, apart from orientation, but we retain them both for the sake of the general discussion. On figure 28 below we use the effort-symbol e and the flow-symbol f to denote the two input-vectors, i.e. primary effort and secondary flow, and the function of the element is denoted with symbolic expressions for the secondary effort and the primary flow. We have chosen to let the effort-variance follow the flow-variance, i.e. contravariant for a-bonds, covariant for b-bonds. Unfortunately we discover, that even for so simple an element as the unit transformer, we cannot at once denote the function in the two mixed cases 2a-1b and 2b-1a.

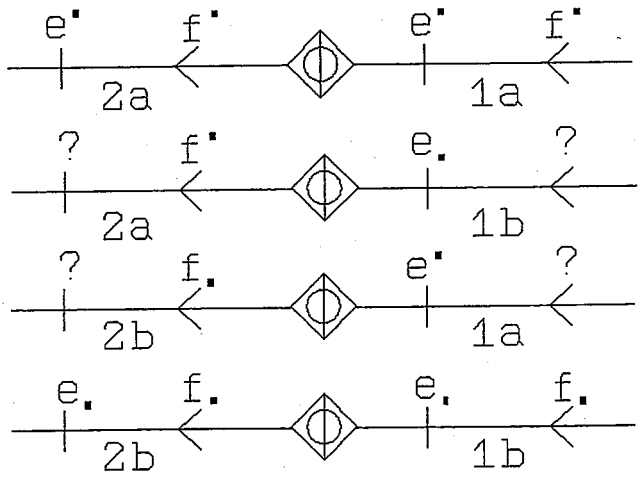


Figure 28. Known and unknown functions of the unit-transformer.

The problem of the missing translation from an a-bond to a b-bond of course has a solution; for semiotic reasons it must be possible to express the same physical situation both with covariant and contravariant flow, when the orientation is unchanged, but in the general case this will involve a symbol-transformation that is not trivial. We shall not in this chapter attempt to find the general expression for this transformation, but shall translate it to another problem, namely the function of the two-port o-junction, which for scalar and euclidean bonds is identical to the unit-transformer, but for general bonds something else.

The junctions of the energy bond formalism play the part of representing Kirchhoff's laws for networks. The node-law, that the sum of flows towards (or away from) a node is zero, is represented by the o-junction, the mesh law, that the sum of efforts around a mesh is zero, is represented by the x-junction. In order for these laws to have meaning for general vector-bonds we must have the same variance of flow and effort in all the attached bonds, when these are oriented in the same way relative to the junction. For the two-port o-junction we thus get the rule on top of figure 29, and this can then be translated to the rule below by means of the flow-orientation rule.

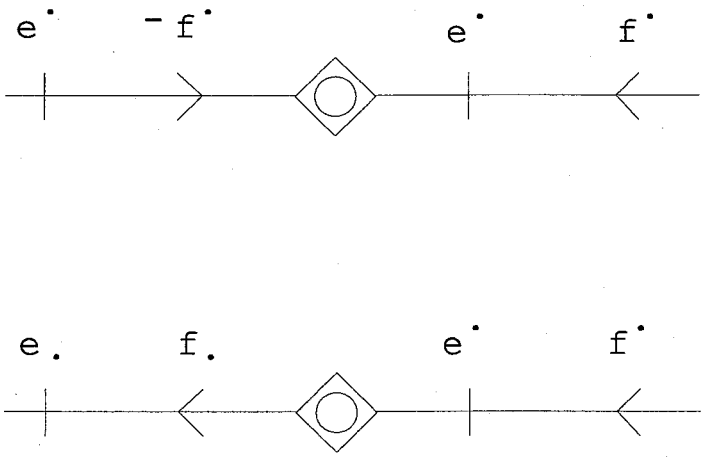


Figure 29. The two-port o-junction.

It can easily be seen, that the two-port o-junction is icon-symmetric, and that its function is a mirroring operation, i.e. if we cascade-couple two of them, we get a unit-transformer. The lower part on figure 29 shows that the two-port o-junction, in contradistinction to the unit-transformer, has a simple function, when it connects an a-bond with a b-bond. These two things together mean, that the function of the unit-transformer, when it connects two bonds of different a/b type and the same orientation will be the same as the function of the two-port o-

junction, when it connects two bonds of the same type and orientation. We shall use the tensor-symbol h for this function, although we shall later see that h is not a proper tensor, but something more basic, that we call a *protensor* (or proto-tensor). In figure 30 is shown, how the definition of the two-port as an icon-symmetric mirroring transformer with the exchange-ration h solves the problem of the missing functions in figure 28. The mirror-property of the h-transformer is expressed by the tensor-relation

$$h^2 = g \tag{116}$$

Whereof one can derive different matrix-relations, such as

$$\begin{aligned} &(h_{\bullet}^{\bullet})^2 \doteq h_{\bullet}^{\bullet} h_{\bullet}^{\bullet} = g_{\bullet}^{\bullet} = \delta_{\bullet\bullet} \\ &h_{\bullet\bullet} h^{\bullet}{}_{\bullet} = g_{\bullet\bullet} \; ; \; (h^{\bullet\bullet})^{-1} = h_{\bullet\bullet} \end{aligned} \tag{117}$$

However, one cannot conclude anything about, e.g. $(h..)^2$. We shall later show that for *dimensionless* metrics is

$$(h_{\bullet\bullet})^2 = \Delta_{\bullet\bullet} \; ; \; (h^{\bullet\bullet})^2 = \delta^{\bullet\bullet} \; ; \; h_{\bullet}^{\bullet} = h^{\bullet}{}_{\bullet} \tag{118}$$

And, further, that h=δ for *neutral metrics* (that are also dimensionless).

In figure 30 the upper part shows the protensor h as a transformer acting like the two-port o-junction, the middle part shows h as a mirroring, and the lowest part h as a relation-maker between a- and b-form of the same bond.

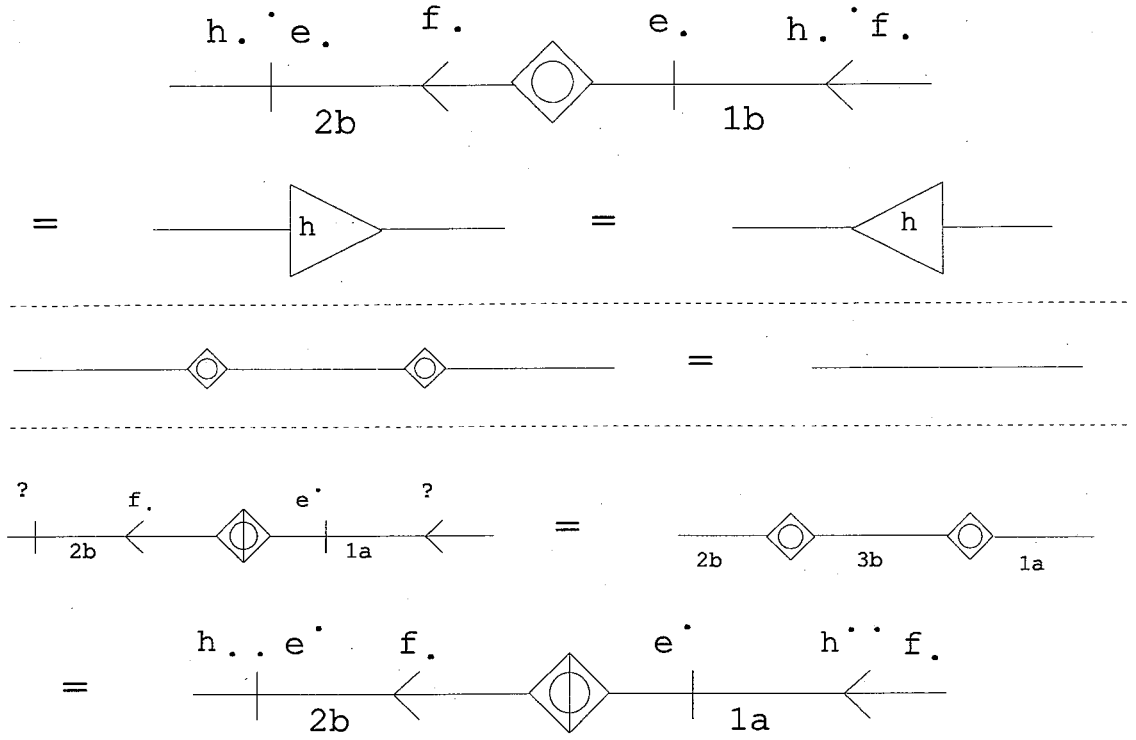


Figure 30. Roles of the protensor h.

Four tensor-like quantities involved, because the primary and the secondary bond each can be of a- or b-type. However, it is sufficient to know one tensor, then the others should be derivable of this and the protensor h (if one knows that).

We are now able to forward the general definition of a transformer. From the preceding it must be clear, that a single tensor is not enough, if we are to account for all the different functions of a transformer. There must be choose to let the tensor-symbol t describe the situation when both the attached bonds are of a-type (contravariant flows) and have orientations in the direction from the primary to the secondary side. Thus, depending on, whether the efforts are chosen contravariant or covariant, the relation between secondary and primary effort shall be

$$e^{(2)\bullet} = t^{\bullet}_{\bullet}\, e^{(1)\bullet} \quad ; \quad e^{(2)}_{\bullet} = t_{\bullet}^{\bullet}\, e^{(1)}_{\bullet} \tag{119}$$

The relation between primary- and secondary flow is then determined by the condition, that the energy flow shall be the same on the two sides. If we, firstly, put $f^{(1)\bullet} = t'^{\bullet}_{\bullet} f^{(2)\bullet}$ energy conservation gives

$$e^{(1)\,*}_{\bullet}\, f^{(1)\,\bullet} = e^{(2)\,*}_{\bullet}\, f^{(2)\,\bullet} \Rightarrow$$

$$e^{(1)\,*}{}_i\, t'^{i}{}_j\, f^{(2)j} = (\, t_j{}^{i}\, e^{(1)}{}_i\,)^{*}\, f^{(2)j} \qquad \textbf{(120)}$$

$$\Rightarrow \quad t'^{i}{}_{j} = t^{*}{}_{j}{}^{i}$$

This result leads us to the definition of *hermitean tensor-conjugation*. We say, that the tensor t' is the hermitean conjugate of the tensor t, $t' = t^{+}$, and this means for the tensor's four matrix-variants

$$\left.\begin{array}{l} t^{+}_{ij} = t^{*}_{ji} \;;\; t^{+}_{i}{}^{j} = t^{*j}{}_{i} \\ \\ t^{+i}{}_{j} = t^{*}{}_{j}{}^{i} \;;\; t^{+ij} = t^{*ji} \end{array}\right\} \qquad \textbf{(121)}$$

Comprisingly we may say that we hermitean conjugate a (2. order) tensor by for each of its four matrix-variants to complex conjugate and exchange the horizontal position of the two indices without changing their vertical position. In order to make it a bit easier to speak about the different matrix-variants of a tensor we introduce a numbering. The index positions in the horizontal dimension are regarded as a binary number, where the lower position corresponds to the digit 0 and the position above to the digit 1. For a 2. order tensor t we thus get variants with the numbers 0,1,2, and 3:

$$\left.\begin{array}{l} 0\text{ - variant } : t_{\bullet\bullet} \\ 1\text{ - variant } : t_{\bullet}^{\ \bullet} \\ 2\text{ - variant } : t^{\bullet}{}_{\bullet} \\ 3\text{ - variant } : t^{\bullet\bullet} \end{array}\right\} \qquad \textbf{(122)}$$

The hermitean tensor-conjugation, that in (121) is expressed for the four matrix-variants of a tensor, thus, acts like an "old fashioned" hermitean matrix conjugation for the variants 0 and 3, but is something different for variants 1 and 2. The reason for this is that we would like to use the raising/lowering rules in the usual way for the hermitean conjugated tensors, e.g. we would like to write $t^{+}{}_{\bullet}^{\ \bullet} = t^{+}{}_{\bullet\bullet}\, g^{\bullet\bullet}$ 124 and as

$$t^{+}{}_{ik}\, g^{kj} = t^{*}{}_{ki}\, g^{kj} = g^{*jk}\, t^{*}{}_{ki} = t^{*j}{}_{i} \qquad \textbf{(123)}$$

where we have used that the 3-variant of g is hermitean, we see that we are forced to define tensor-conjugation as in (121). For matrix conjugation it is valid, that the conjugated to a product is the product of the conjugated matrices in reversed order, and a similar rule is valid for tensor-conjugation:

$$t = uv \Leftrightarrow t^{+} = v^{+} u^{+} \qquad \textbf{(124)}$$

The proof of this rule can be made by looking at one of its possible matrix-variants. *If a tensor-relation is valid for one matrix-variant, it is valid for them all.* E.g. we get

$$t^{+}{}_{i}{}^{j} = t^{*j}{}_{i} = (u^{jk} v_{ki})^{*} = v^{+}{}_{ik} u^{+kj} \qquad \textbf{(125)}$$

When we use the word "tensor" it will often be tacitly implied that it is a 2. order tensor. Tensors of 1. order are called vectors, tensors of 0. order are called scalars. Tensors of 3. or 4. order shall not use in this context. they are inconvenient, because the "dot-notation" cannot be applied to tensors of higher order than 2. If a tensor (of 2. order) is equal to its own hermitean conjugate tensor we say that this tensor is *tensor-hermitean* or *metric hermitean*. If a tensor has the property that its ith matrix-variant is hermitean "in the old fashioned way", i.e. that this matrix is equal to its own matrix-hermitean conjugate (self-adjoined), we say that the tensor in question is i-hermitean. We may then formulate the following propositions (whose proof we leave to the reader):

If a tensor is 0-hermitean, it is also 3-hermitean and metric hermitean, but not necessarily 1-hermitean or 2-hermitean.

If a tensor is 1-hermitean, it is also 2-hermitean, but not necessarily metric hermitean.

By the metric reciprocal tensor to a given tensor t we shall understand a tensor t^{-1}, that obeys the relation

$$t^{-1}t = tt^{-1} = g \qquad (126)$$

This is not to be confused with reciprocity of the matrix-variants. It is *not* true that the i-matrix variant of t^{-1} is the reciprocal matrix to the i-matrix-variant of t. On the contrary, one finds that

The matrix-variant no. 0,1,2,3 of the reciprocal to a tensor t is equal to the reciprocal matrix to the matrix-variant 3,1,2,0 of t. E.g. we get from (126)

$$t^{-1}{}_{\bullet\bullet}\, t^{\bullet\bullet} = t^{-1}{}_{\bullet}^{\bullet}\, t_{\bullet}^{\bullet} = g_{\bullet}^{\bullet} \Rightarrow t^{-1}{}_{\bullet\bullet} = (t^{\bullet\bullet})^{-1} \wedge t^{-1}{}_{\bullet}^{\bullet} = (t_{\bullet}^{\bullet})^{-1} \qquad (127)$$

The condition for a tensor to have a reciprocal tensor is then, that all its matrix-variants have reciprocal matrices.

If a tensor has the property, that its reciprocal is equal to its hermitean conjugate tensor, we say that the tensor in question is *metric unitary* (or *tensor-unitary*). If, on the contrary, it has the property, that its ith matrix-variant is unitary in "the old fashioned way", i.e. that this matrix is the reciprocal to its hermitean conjugate matrix, we say that this tensor is *i-unitary*. A metric unitary tensor will generally

not be matrix-unitary in any of its variants; only if the metric is euclidean, these properties will coincide.

Let us now consider the transformer-definition, that was forwarded in (119) and (120). The definition was valid for the case, where both the primary and the secondary bond were of a-type, i.e. with contravariant flow, and where the orientation was in the direction from primary- to secondary port. By using the flow-orientation rule and the rule of indifferent effort-variance we can from here conclude about 16 different situations, of which only so many are shown in figure 31 that we have to use all the matrix-variants of t and t^+.

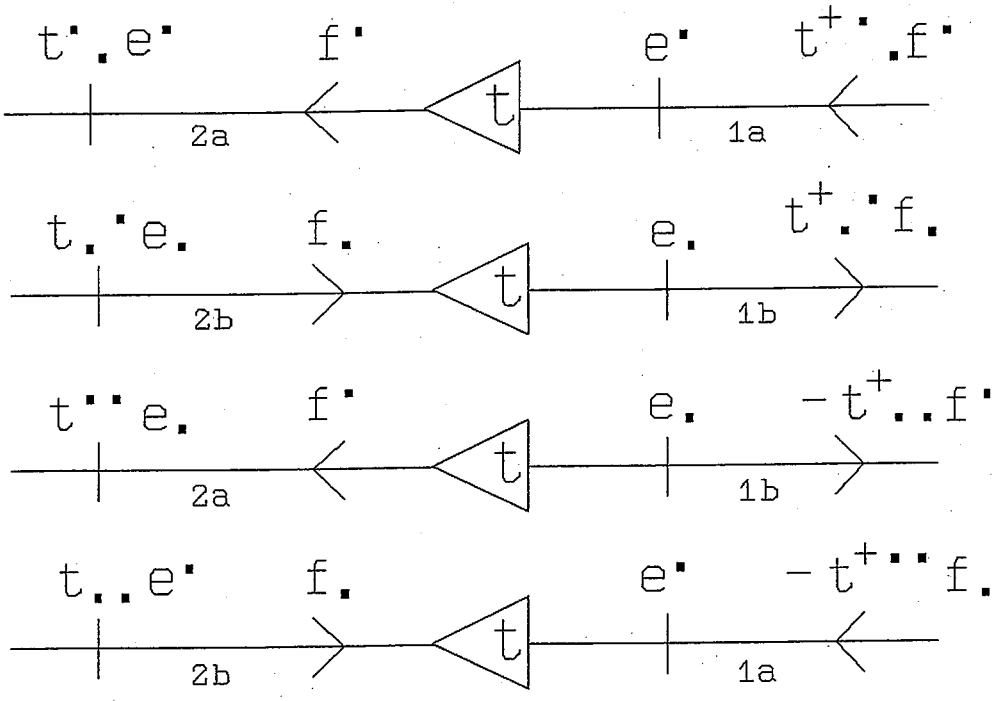


Figure 31. Transformer definitions with the tensor t.

There are, however 64 different indications of the two bonds of a transformer, and from the prototype on top of figure 31 we can only get 16 of them. The remaining 48 must be derived from three other prototypes, that with orientation from primary to secondary will be 2a-1b, 2b-1a, and 2b-1b. If the tensor-symbol t describes the transformer's function as in figure 31,(i.e. with the prototype 2a-1a), then it will be another tensor, $\check{t}$, that describes the function with the prototype 2b-

1b. We call $\check{t}$, the *dual* tensor to t (although this word is used in a number of other contexts, that have nothing to do with this). Which of the two prototypes one ascribes to t, is arbitrary (but it must, of course, be done consistently in a given context). Henceforth we shall use the convention from figure 31 and figure 32, but in (97), (98), and figure 32 we used, in fact, the opposite convention, which, however, we easily could change by renaming the applied tensor-symbol from U to $\check{U}$.

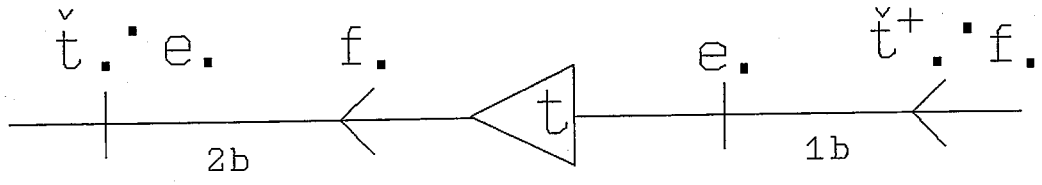


Figure 32. The dual tensor-function of the transformer in figure 31.

The difference between figure 31 and figure 32 does not lie in the transformer, but in the indication of the energy bonds. If we in figure 32 would change the symbol inside the transformer-icon to $\check{t}$, it would in general be a different transformer (the dual), and then we should at the same time change the symbol $\check{t}$ in the expressions for the output-variables to t. The duality-transformation is a mirroring, because the transition from a-indication to b-indication (with fixed orientation) is a mirroring.

We have earlier seen (figure 30), that the same physical situation can be expressed with both a-indication and b-indication (and the same orientation) by transforming both bond-vectors with the protensor h. This means, that we can always determine, which tensor-symbol, that expresses the function of a transformer t with given indications. First, we use the flow-orientation rule to orient both bonds in the direction of the transformer-icon, i.e. from primary to secondary. Then we compare the a/b-types of the re-oriented bonds with the prototype 2a-1a (figure 31 above). If there is a deviation (i.e. a b instead of an a), the tensor-symbol t in the expressions for the bond-variables shall be furnished with a factor h on that side of the t, where the deviation is found. In particular, we find for figure 32, where there is deviation on both sides:

$$\check{t} = hth \qquad \textbf{(128)}$$

The above rule rests heavily on the drawing-style, that we have the primary side to the right and the secondary side to the left. If the prototype had been named 1a-2a instead of 2a-1a, we should have placed the h on the opposite side of the t than that, where the deviation is found. The scheme below comprises the rule

Indication-form	Tensor-symbol
2a-1a	$t = h\overset{\vee}{t}h$
2a-1b	$th = h\overset{\vee}{t}$
2b-1a	$ht = \overset{\vee}{t}h$
2b-1bt	$\overset{\vee}{t} = hth$

A transformer or its tensor is said to be self-dual, if $\breve{t} = t$. The unit-tensor g, which commutes with all factors in a tensor-product, is self-dual, for $hgh=gh^2=g$. The protensor h is also self-dual, for $hhh=h^2h=h$.

By comparison of figure 30 and figure 32 it is seen that we have foreshadowed the self-duality of h. Further, we have presumed, that h is metric hermitean. The reason why, it necessarily must be so, is, that we by transition from an a-bond to a b-bond with the same orientation, describing the same physical situation, must apply the same transformation for flow and effort (the transformer h.. in figure 30 below). There is no difference in principle between flows and efforts, when time-reversal and orientation-shift are kept outside the picture, and we have just developed the concept of the general vector-bond from the idea of a standard metric, that can be reticulated out on scalar bonds, such that flows and efforts from the scalar bonds can be mixed in a flow-vector or an effort-vector. When h is metric hermitean and a mirroring, h is also metric unitary.

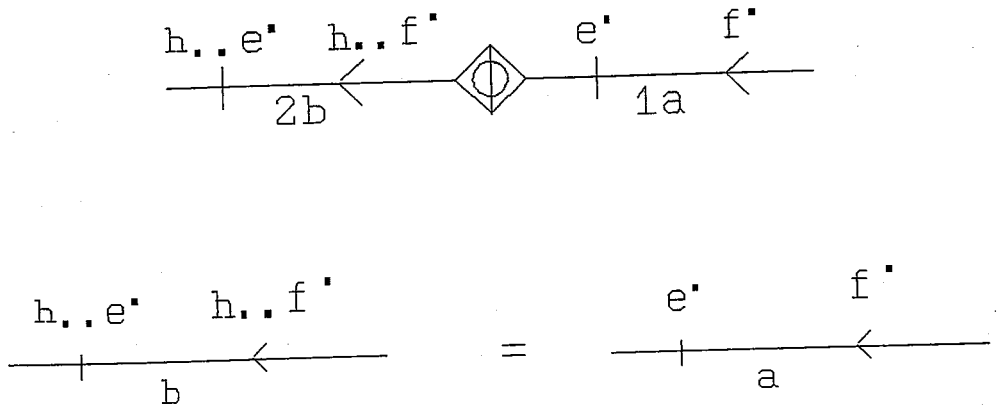


Figure 33. The same physical situation exposed with a- and b-bond (comp. figure 30 lowest).

For a dimensioned metric the 1- and 2-matrix-variants of h will also be dimensioned, and it has no meaning to diagonalize the or speak about their eigenvalues. On the other hand, if the metric is dimension-less, all matrix-variants of h also will be dimension-less. As the 1- and 2-variants are mirrorings (their square is the unit matrix, comp. (117)), they have the eigenvalues 1 and -1 and are hermitean. I.e. the protensor h is for a dimension-less metric 1-hermitean (and 2-hermitean), besides its being metric hermitean. We have therefore

$$\left.\begin{array}{l} h^{*}{}_{j}{}^{i} = h_{i}{}^{j} \;\wedge\; h^{*}{}_{j}{}^{i} = h^{i}{}_{j} \\ \\ \Rightarrow\; h_{\bullet}^{\bullet} = h^{\bullet}{}_{\bullet} \end{array}\right\} \qquad \textbf{(130)}$$

As $h_{\bullet}^{\bullet} = g_{\bullet\bullet} h^{\bullet\bullet}$ and $h^{\bullet}{}_{\bullet} = h^{\bullet\bullet} g_{\bullet\bullet}$, and as the 0-and 3-variants of h and g are reciprocal to each other, we find that *all matrix-variants of h commute with all matrix-variants of g.*

Because the Kronecker symbols δ and Δ can be expressed by g-matrices, we can further sharpen the proposition to: *All matrix-variants of g, h, δ, and Δ commute with each other for a dimension-less metric.* From this follows also that δ and Δ are self-dual. The rest of the results, that were foreshadowed in (118) can now be proved:

$$\left.\begin{aligned}(h_{\bullet\bullet})^2 &= h_{\bullet\bullet}\Delta^{\bullet\bullet}h_{\bullet\bullet} = h_{\bullet}^{\bullet}\Delta_{\bullet\bullet}h^{\bullet}{}_{\bullet} = (h_{\bullet}^{\bullet})^2\Delta_{\bullet\bullet} = \Delta_{\bullet\bullet}\\ (h^{\bullet\bullet})^2 &= h^{\bullet\bullet}\delta_{\bullet\bullet}h^{\bullet\bullet} = h^{\bullet}{}_{\bullet}\delta^{\bullet\bullet}h_{\bullet}^{\bullet} = (h^{\bullet}{}_{\bullet})^2\delta^{\bullet\bullet} = \delta^{\bullet\bullet}\end{aligned}\right\} \qquad \textbf{(130)}$$

Of special importance among the dimension-less metrics are the neutral metrics, where $(G..)^2 = \delta$ and $\Delta=\delta$. (We use a capital G for the metric tensor to emphasize that it is a neutral metric). For a neutral metric there is no difference between the 0- and the 3-variant of G, and this means, in fact, that we cannot distinguish between a bond, where both flow and effort are reckoned contravariant (an a-bond) and another, where the vector-symbols are the same, but where both are covariant, and the orientation is the same. From figure 33 therefore follows, that *for a neutral metric is $h=\delta$*. I.e. the 0- and 3- variants of h both are equal to the unit-matrix, and the 1- and 2-variants both are equal to G.. . If, especially, the metric is *euclidean*, $G=\delta$, all variants of G, h, δ, and Δ are equal to the unit-matrix. For a neutral metric, therefore, the i-variant of a tensor t will be equal to the 3-i-variant of the dual tensor:

$$\breve{t}_{\bullet}^{\bullet} = h_{\bullet\bullet}t^{\bullet}{}_{\bullet}h^{\bullet\bullet} = \delta_{\bullet\bullet}t^{\bullet}{}_{\bullet}\Delta^{\bullet\bullet} = t^{\bullet}{}_{\bullet} \qquad \textbf{(131)}$$

(and correspondingly for the other variants).

$$\breve{t}_{\bullet}^{\bullet} = h_{\bullet\bullet}t^{\bullet}{}_{\bullet}h^{\bullet\bullet} = \delta_{\bullet\bullet}t^{\bullet}{}_{\bullet}\Delta^{\bullet\bullet} = t^{\bullet}{}_{\bullet} \qquad \textbf{(132)}$$

In the previous chapter, when we discussed multi-ports as 1-ports in standard metric, nothing was said about the two dual representations, but the difference does also exist for 1-ports, and (131) shows that it is not trivial, even for a standard metric (unless it is euclidean). We cannot just change variance on both variables and then reckon, that the same input-output relation can be expressed by changing the vertical position on both indices of the response-matrix and keep the tensor-symbol. This symbol shall be replaced with the dual symbol, and (131) therefore expresses, that it is *the same response-matrix*, that shall be used with the changed variances, and therefore it cannot be the same tensor. This is immediately clear, when one thinks of a scalar reticulation as, e.g. figure 34, for the input-output conditions in such one depends only on the indications of the scalar bonds and not on, what we call covariant and contravariant in the vector-bond. One should not stare oneself blind on the vector-notation and believe that

everything in the world is tensors and Riemann geometry (as has been a strong tendency in physics after the relativity theories). A tensor is just a convenient quantity for calculations, that makes it possible to express relations that are independent of the coordinate-system, but if we loose the earth-connection to the underlying iconic level (that is our semiotic substitute for the physical reality) the tensors are worth nothing.

The icon for a transformer is asymmetrical in order to show, that we need to distinguish between the primary and the secondary side, when we are to account for the transformer's functions. This distinction, however, is a convention, that does not involve the underlying physical reality, so it must be possible to describe the same transformer with the icon reversed, if we at the same time exchanges the original tensor-symbol t with a new, t'. We then have to find the symbolic relation between t and t'.

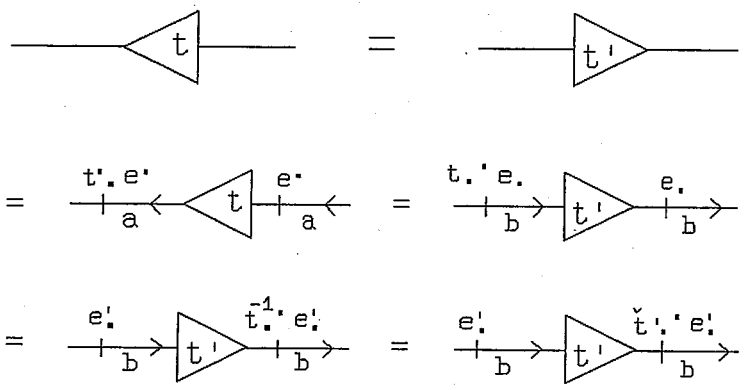


Figure 34. Reversal of the transformer-icon.

By use of the flow-orientation rule and the definition of the dual tensor-symbol, figure 32, we find in figure 34 by looking at the relation between the efforts in the two ports, that the connection between the original tensor's t and the reversed tensor's t' must be

$$t' = \breve{t}^{-1}$$

The condition for a transformer to be *icon-symmetric* is therefore

$$\breve{t}^{-1} = t \sim t\breve{t} = g \qquad \textbf{(133)}$$

If a transformer has the mirroring property $t^2=g$ and is self-dual, it must therefore be icon-symmetric, whereby it is confirmed, that g and h are icon-symmetric. Among the scalar transformers there is only one icon-symmetric transformer, apart from the unit-transformer, viz. the one with the exchange-ratio -1, and this was therefore in text #8 called *the symmetric transformer*. Now there are more possibilities for icon-symmetry, and we therefore rename the earlier "symmetric transformer" to *the negative unit-transformer*, but use the same icon for it (figure 35).

By use of this we can express the two-port x-junction. Transformers corresponding to δ and Δ are icon-symmetric if and only if the metric is neutral, but in that case they are identical to the two-port o-junction and don't need a new icon. For a non-neutral metric they are self-dual and reciprocal to each other, but not identical, and they therefore get an asymmetric icon, but only one, for by iconic reversal of the Δ-transformer we get the δ-transformer.

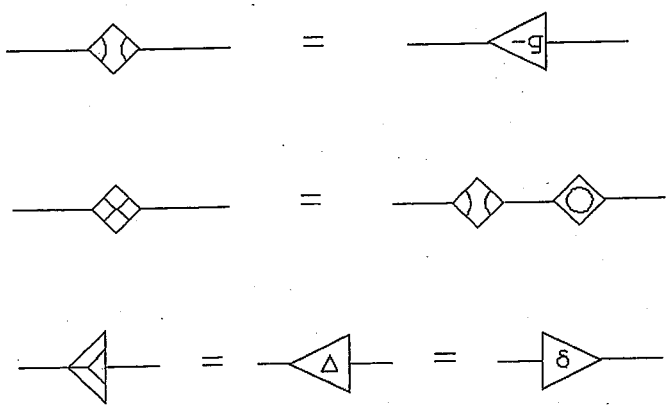


Figure 35. Icons for protensors.

Proper transformers within a given metric, i.e. such, that have a reciprocal, form in the mathematical sense a group with cascade-coupling, or concatenation as composition. In general cascade-coupling is not commutative. When we cascade-couple t_1 and t_2 and get a composite transformer $t=t_2t_1$ it means, that the 1- (or 2-) variant of t is the matrix product of the 1- (or 2-) variants of t_1 and t_2.

$$t_{\bullet}^{\bullet} e_{\bullet} = t_{2\bullet}^{\bullet} t_{1\bullet}^{\bullet} e_{\bullet} \qquad \textbf{(134)}$$

is meant that an effort-signal first meets the transformer t_1 and then t_2. This, in matrix-formula so much used way of writing is the main reason why we in definitions prefer to have the primary side to the right and the secondary side to the left, which is illustrated with the figure below:

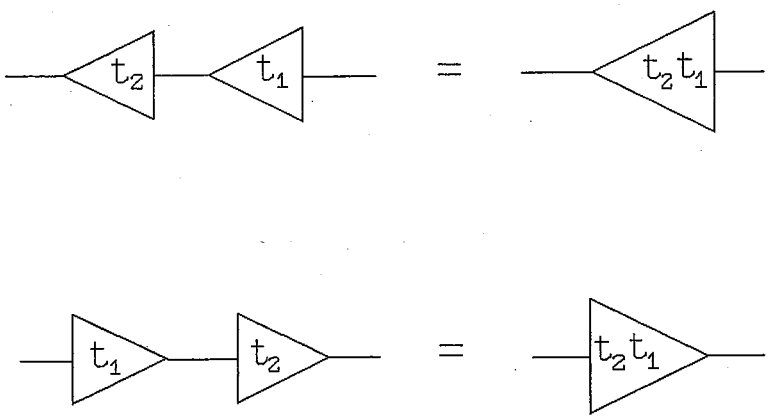


Figure 36. Concatenation of transformers.

A *gyrator* has, in contradistinction to a transformer, the same type of input in both ports, i.e. either effort-input in both ports or flow-input in both ports. If the causality is changed in the one port, it must also be changed in the other port, but for a definite gyrator both types of causality can be accepted. Still, it is an advantage to operate with two different icons, an x-gyrator, where the exchange-ratio is the ratio between the secondary effort and the primary flow, and an o-gyrator, where the exchange-ratio is the ratio between secondary flow and primary effort. In the definitions it is the most natural to let the x-gyrator have flow-input in both ports and let the o-gyrator have effort-input in both ports, but this is, as mentioned, not necessary; the reason why we use two different icons is not, that there are two different causal forms, but that there are two possible definitions of the exchange-ratio; *the shape of the icon serves to define the meaning of the symbol.*

As for the transformer we let the tensor-symbol describe the action of the gyrator, when both bonds are of a-type, and the orientation is in the direction from the primary side to the secondary side. For the indication-form 2b-1b we have to use the dual symbol, that we get from the original by multiplication on both sides with the protensor h, and if the indication-form only deviates on the one side, the symbol shall be multiplied with h on that side (again presuming, that we have primary on the right and secondary on the left). On figure 37 below is shown definitions of both gyrators and icon-reversal of an o-gyrator.

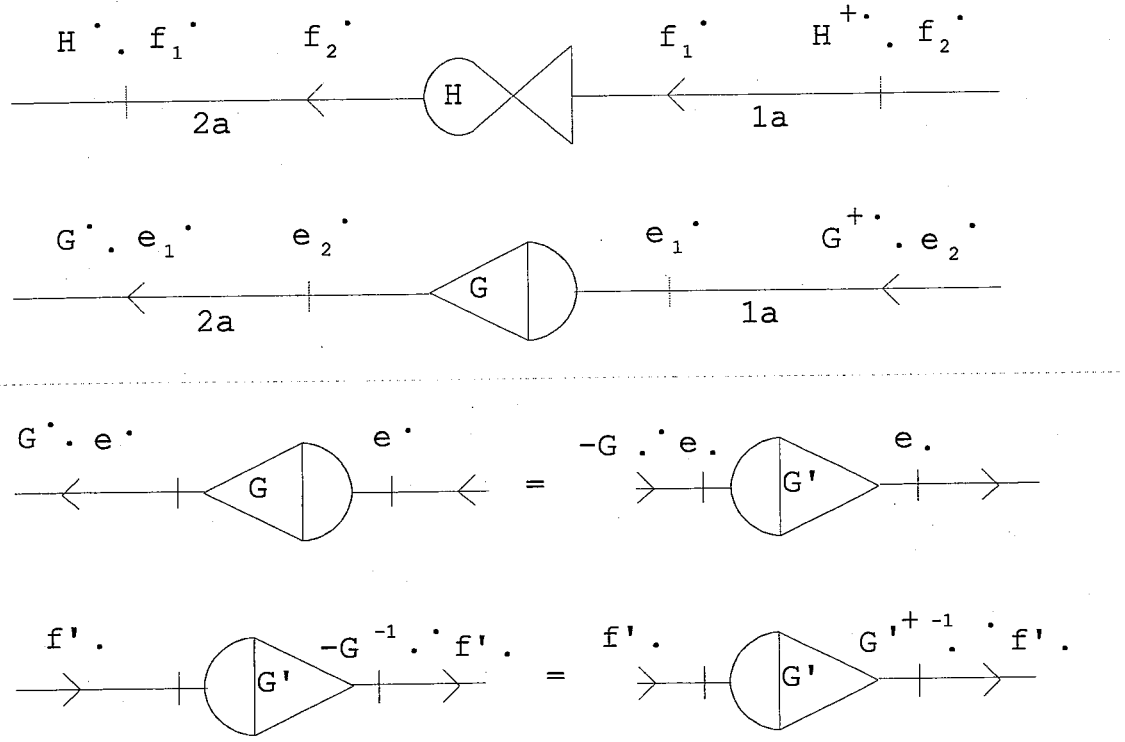


Figure 37. Above: Definition of x-gyrator H and o-gyrator G.
Below: Icon-reversal of gyrator.

From the derivation in the bottom of figure 37 we find, that the symbol-transformation of a gyrator-tensor G to G', when the icon is reversed, must be

$$G' = -\breve{G}^{+} \qquad \textbf{(135)}$$

We see from this, that the complex vector-bond-formalism opens for a possibility, that does not exist for simple bonds, viz. an Kicon-symmetric gyrator, for which G'=G. E.g. the o-gyrators with the exchange-ratio G = ±ig (where g is the metric tensor) will be icon-symmetric (and self-dual and metric anti-hermitean), and they can be pictured as shown in figure 38.

Another new possibility in the energy bond formalism is a 1-port gyrator. This was earlier called a *gyro-sink*, but as it is non-dissipative and plays an important part in describing mechanical forces, we now prefer to call it a *dynamo*. This is not a proper gyrator, but a non-dissipative 1-port Γ whose 0-variant Γ.. is *real and anti-symmetric*. Thus, a dynamo is metric antihermitean and not to be confused with an "imaginary sink", that may occur for scalar bonds in the form of

a storage for real frequencies (imaginary Laplace frequencies). If we aggregate the two scalar bonds to the gyrator on figure 20 to a two-dimensional vector-bond with euclidean metric, we get a dynamo. Icons for such (x-and o-dynamos) are also shown in figure 38.

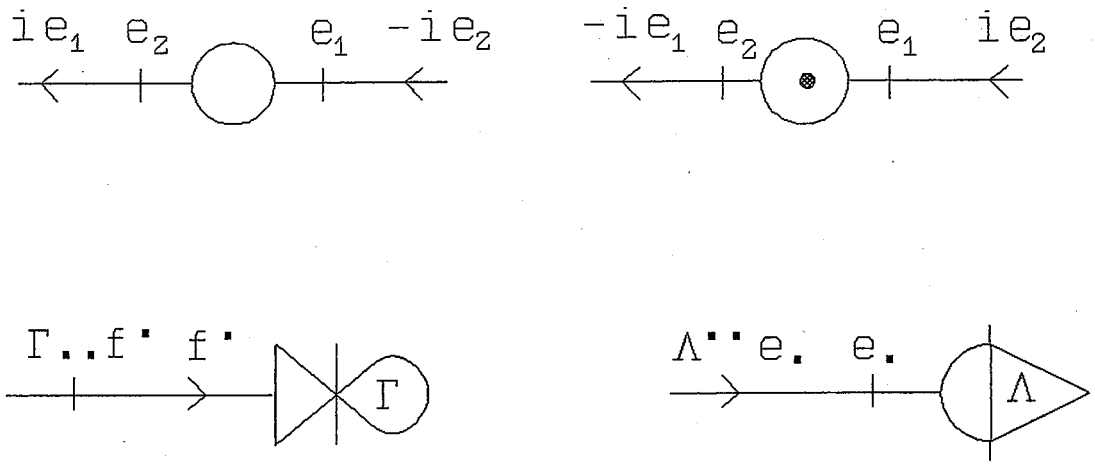


Figure 38. Icon-symmetric gyrators for complex scalar bonds and dynamos for real vector-bonds.

We have already before made acquaintance with a sort of icon-symmetric gyrator, viz. the genus-gyrator in figure 10. This, however, is anti-linear and connects two bonds with different genus, while the gyrators here considered all are linear and let genus and metric be unchanged.
Figure 39 below shows some rules for manipulation with icons and tensor-symbols.

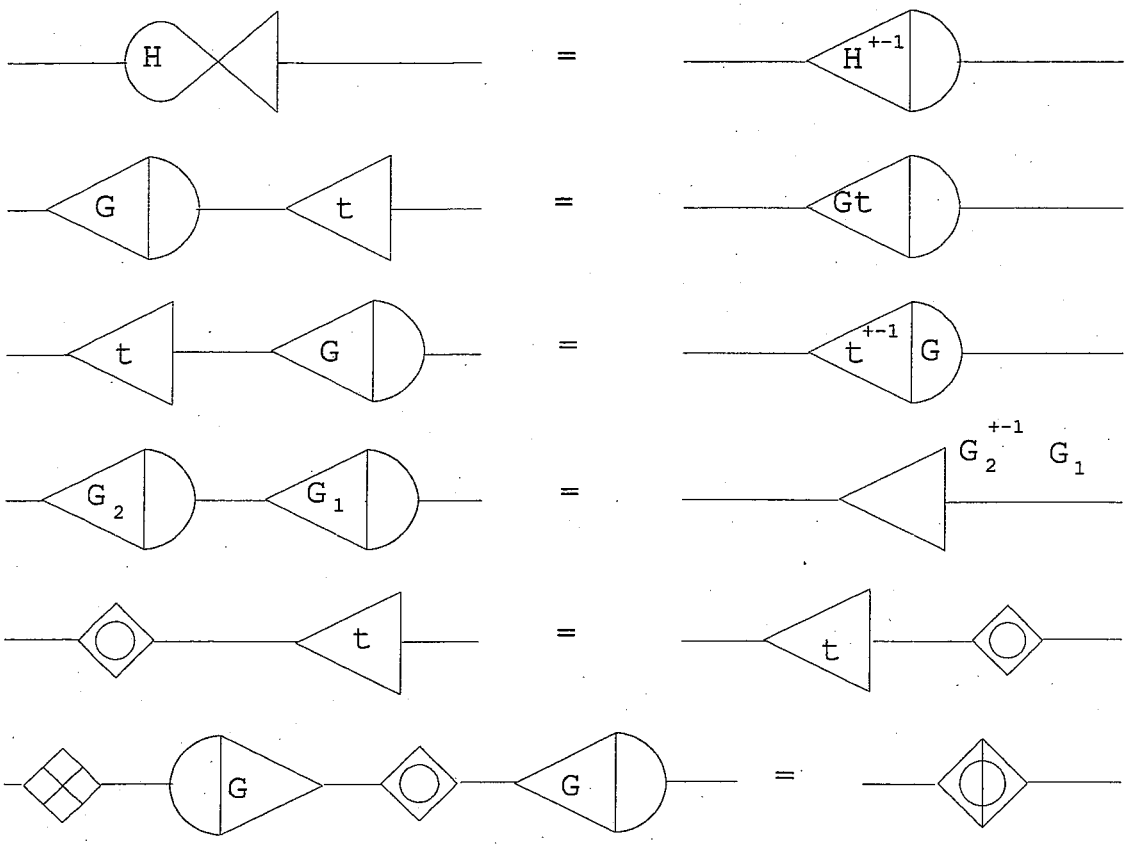


Figure 39. Icon-symbol-operations with two-ports.

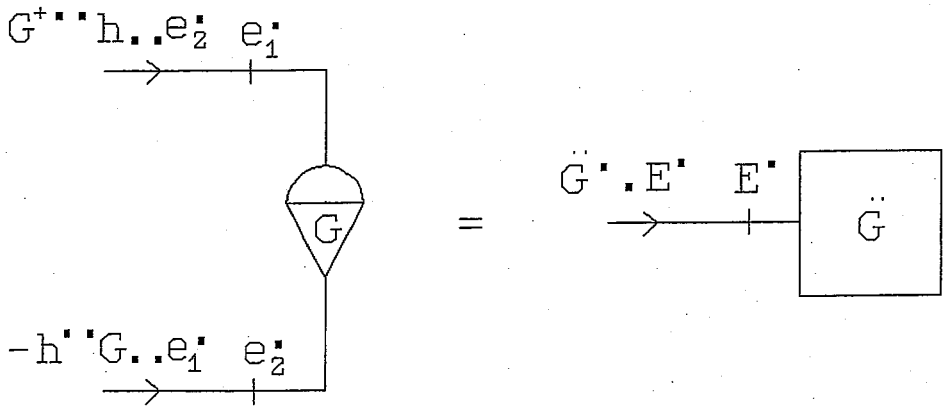


Figure 40. Gyrator defined as 1-port.

Both transformers and gyrators are non-dissipative, but not necessarily reversible elements. For simple energy bond models we have seen, that transformers are reversible and we can have anti-reciprocal transformers and reciprocal gyrators and some, that are neither the one nor the other. In order to survey the situation we shall use a technique, that was introduced in the previous chapter, namely to introduce new vector-bonds by aggregation of simpler bonds. The two vector-bonds that leads in to a transformer t can be put together to a vector-bond with the double dimension, that then leads to a 1-port $\ddot{t}$

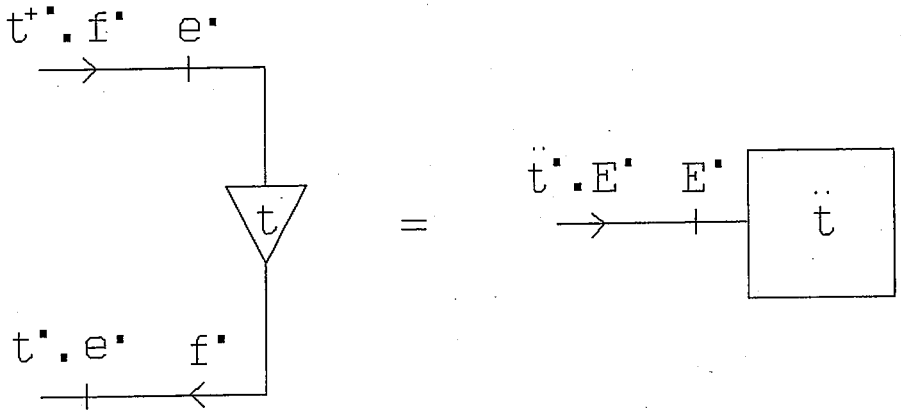


Figure 40. transformer defined as 1-port.

When the original d-dimensional bonds have the metric tensor g, the new, in figure 40 defined 2d-dimensional bond will have the metric

$$\ddot{g}_{..} = \begin{pmatrix} g_{..} & \mathbf{0} \\ \mathbf{0} & -g_{..} \end{pmatrix} \qquad \textbf{(136)}$$

where the bold o's stand for d*d null-matrices. The response-matrix, that is the 2-variant of the response-tensor for the 1-port so defined, is then

$$\ddot{t}^{\bullet}_{\bullet} = \begin{pmatrix} 0 & t^{+\bullet}_{\bullet} \\ t^{\bullet}_{\bullet} & 0 \end{pmatrix} \qquad \textbf{(137)}$$

$$\ddot{t}^{i}{}_{d+j} = \ddot{t}^{d+j}{}_{i} \Rightarrow t^{+i}{}_{j} \doteq t^{*}{}_{j}{}^{i} = t^{j}{}_{i} \qquad \textbf{(138)}$$

The condition, that the transformer is reciprocal, is that this matrix is symmetrical. Letting i and j be between 1 and d, we have:

$$A^{*i}{}_{k}\, g'_{ij}\, A^{j}{}_{l} = g_{kl} \qquad \textbf{(139)}$$

For an euclidean d-dimensional metric all matrix-variants of t are identical, and reciprocity is equivalent to, that *all matrix-elements of t are real*, but for non-euclidean standard metrics reciprocal transformers may have complex matrix-elements. E.g. we find for g.. $=\sigma_z$ that the transformer with the 1-variant σ_y is reciprocal (comp. (64)).
Similarly, we can now for a gyrator put the two d-dimensional bonds together to a 2d-dimensional bond, so that the gyrator becomes a 1-port:

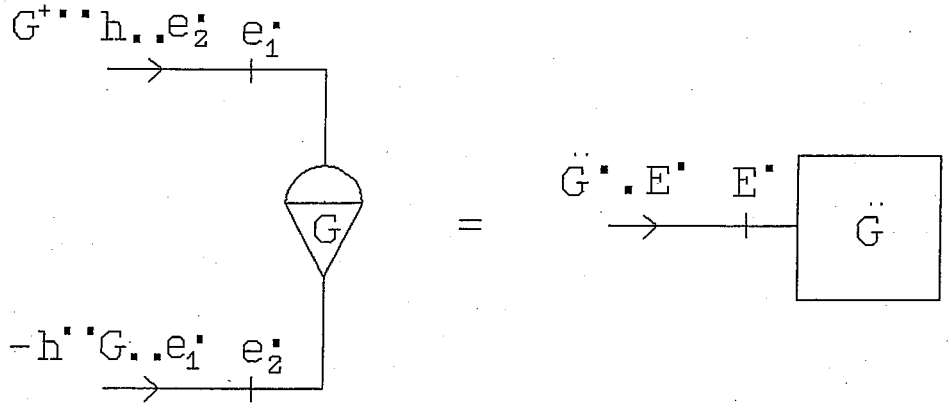


Figure 41. Gyrator defined as 1-port.

In this case the metric of the 2d-dimensional bond is:

$$\ddot{g}_{..} = \begin{pmatrix} g_{..} & \mathbf{0} \\ \mathbf{0} & g_{..} \end{pmatrix} \qquad \textbf{(140)}$$

and the response-matrix for the 1-port gyrator is

$$\ddot{G}^{\bullet}_{\bullet} = \begin{pmatrix} 0 & G^{+\bullet\bullet} h_{\bullet\bullet} \\ -h^{\bullet\bullet} G_{\bullet\bullet} & 0 \end{pmatrix} \qquad \textbf{(141)}$$

If the metric is euclidean, we may disregard the hs in (140), and the matrix-variants of G are identical. We then find, that the gyrator is anti-reciprocal, if and only if all its matrix-elements are real.

As the last form of ideal two-port we shall look at the *transducer*. In the simple energy-bond-formalism the transducer is defined as a transformer with dimension, and it cannot really be distinguished from a transformer in the diagrams. Here we shall establish a slightly different view. A *metric transducer* is a coordinate-transformation, that changes the metric. If flows and efforts have the same variance in the primary bond, we shall apply the same transformation to them to find flow and effort in the secondary bond. We have earlier, in chapter 3, looked on such a transformation from an orthonormal to a skew-angled coordinate-system, where the metric was changed from being euclidean to another metric (with euclidean standard metric). At that time we drew the metric transducer as a transformer, and we also saw, that the transformation could be reticulated out on scalar bonds, but only if flow and effort were of different variance, such that they transformed differently.
When we use mixed variances in a vector-bond, the metric becomes invisible, and a metrical transducer becomes undistinguishable from a transformer. Now, we shall make the metric visible, and that requires, that flow and effort have the same variance, so that they transform in the same way. In the figure below is shown such a transformation, attached to the icon for a metric transducer.

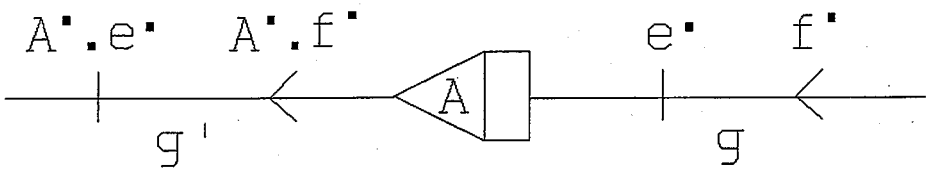


Figure 42. Metric transducer.

The metric g' is then determined by the demand that the energy-flow shall be conserved

$$w=(A^{i}_{k}e^{k})^{*}g'_{ij}A^{j}_{l}f^{l}=e^{*k}g_{kl}f^{l} \qquad \textbf{(142)}$$

Therefore, we get

$$A^{*i}_{k}g'_{ij}A^{j}_{l}=g_{kl} \qquad \textbf{(143)}$$

We have here rated the quantity A with index-places, as if it were a tensor, what it isn't in the usual sense. The tensors, we have met in connection with 1-ports function *within* a definite metric, with raising/lowering-rules for the index-places, connected with a definite metric tensor. Let us, for a short while use the name *endotensors* for such quantities, for then we can call a quantity like A for the metric transducer an *exotensor*, because it leads out from the metric, is an interface between two different metrics. We must also be prepared to accept, that if we want to apply the usual system with index-places for an exotensor, then the raising/lowering-rules will involve both metrics. If we define the raising/lowering-operation on A such, that it uses g' on the left side and g on the right side, then we may write

$$A^{\bullet}_{\bullet}=g'_{\bullet\bullet}A^{\bullet}\cdot g^{\bullet\bullet} \sim A^{l}_{i}=g'_{ij}A^{j}_{m}g^{ml} \qquad \textbf{(144)}$$

This is the same rule, as was used in (24). If we now further define hermitean conjugation of the exotensor in the same way as in (121), i.e.

$$A^{+}{}^{i}_{k}=A^{*i}_{k} \qquad \textbf{(145)}$$

Then (142) can be re-written to

$$A^{+}{}^{\bullet}_{\bullet}=A^{-1}{}^{\bullet}_{\bullet} \qquad \textbf{(146)}$$

And the connection between the two metrics (also (132)) becomes

$$g'_{\bullet\bullet} = A^{\bullet}_{\bullet} g_{\bullet\bullet} A^{+\bullet}_{\bullet} \qquad \textbf{(147)}$$

The raising/lowering rule for A, that one applies g' on the left side and g on the right side seems natural, when the drawing is like figure 42, but if the figure were reversed, so that the primary side and g were to the left, the rule would still be the same, and then it would not seem so natural. So we keep the habit to draw the primary side to the right. As a retribution we get a small problem when we raise and lower indices on A^{+}, for here we must use g' on the right side and g on the left side:

$$A^{+\bullet}_{\bullet} = A^{-1\bullet}_{\bullet} \qquad \textbf{(148)}$$

Equation (145) looks like the earlier definition of metric unitarity, but one should not confuse the two things, because the metric properties of A involve two different metrics and (145) is fulfilled per definition, i.e. it does not require special things of A, but serves to determine the relation between g' and g. However, we could imagine a situation, where A was metric unitary within the metric g, i.e. had the property (134), when the raising/lowering-operations and the hermitean tensor-conjugation are defined in the same way as we have done it for endotensors in the g-metric. In that case we would get from (146)

$$A^{+i}{}_{j} = A^{*\,i}_{j} = (g'_{jk} A^{k}_{l} g^{li})^{*} = g^{il} A^{+k}_{l} g'_{kj} \qquad \textbf{(149)}$$

That is, then the two metrics are identical, and A is not a metric transducer at all, but a metric unitary transformer. The metric unitary transformers within a given metric form a *group of metric-conserving coordinate-transformations*. They act in the same way as a metric transducer in the sense, that we must apply the same transformation on the contravariant flow and effort in the primary bond in order to find the contravariant flow and effort in the secondary bond, but this coordinate transformation does not result in a change of the metric.

For an euclidean metric the metric unitary transformations are the same as "old fashioned" unitary transformations, that conserve the coordinate system's right-angled-ness and equally long axes, i.e. they can be regarded as rotations of the coordinate system. For a non-euclidean standard metric as the two-dimensional σ_z metric, the metric unitary transformers will form a one-dimensional Lie-group

known as *the Lorentz group* from the special relativity theory. We shall return later to this group (chapter 9) and its physical meaning, but for now it is left to the reader to show that all members of this group can be written on the form

$$g'_{\bullet\bullet} = A_{\bullet}^{\bullet}\, A^{+}{}_{\bullet}^{\bullet}\, g_{\bullet\bullet} = g_{\bullet\bullet} \qquad \textbf{(148)}$$

I.e. then the two metrics are identical, and A is not a metrical transducer, but a metric unitary transformer. The metric unitary transformers within a given metric form a *group of metric conserving coordinate transformations*. They act in the same way as a metric transducer in the sense that we shall apply the same transformation to the contravariant flow and effort in the primary bond to find the contravariant flow and effort in the secondary bond, but this coordinate-transformation does not imply a change of the metric.

For a euclidean metric the metric unitary transformations are the same as "old fashioned" unitary transformations, that conserve the right angle and the equally long axes of the coordinate-system, so they can be described as rotations of the coordinate system. For a non-euclidean standard metric as the two-dimensional σ_z-metric, the metric unitary transformers will form a one-dimensional Lie-group, known as the *Lorentz group* from the special theory of relativity. We shall later return to this group and its physical significance, but for now it is left as an exercise to the reader to show that the members of this group can be expressed in the form

$$L(\chi)^{\bullet}_{\bullet} = \begin{pmatrix} \cosh\chi & \sinh\chi \\ \sinh\chi & \cosh\chi \end{pmatrix} \qquad \textbf{(149)}$$

So, a coordinate-transformation is described with a metric transducer or a metric unitary transformer. The transformation rules for a vector

$$e'_i = A_i^{\,j}\, e_j \;;\; e'^{i} = A^{i}{}_{j}\, e^{j} \qquad \textbf{(150)}$$

then lead to corresponding rules for transformation of tensors of higher order. We have already in (146) seen, how the metric tensor transforms, and the same rule is found for every other tensor. The concrete application of the rule depends

on the matrix-variance, and one can easily convince oneself, that the transformed tensor-symbol t' is subject to the same raising/lowering rules in the transformed metric g' as the original tensor t in the original metric g.

$$\left.\begin{array}{c} t' = A\,t\,A^{+} \\ t'_{\bullet\bullet} = A_{\bullet}{}^{\bullet} t_{\bullet\bullet}\, A^{+\bullet}{}_{\bullet} \sim t'_{ij} = A_i{}^{k}\, A^{*}{}_{j}{}^{l}\, t_{kl} \\ t'_{\bullet}{}^{\bullet} = A_{\bullet}{}^{\bullet} t_{\bullet}{}^{\bullet}\, A^{+}{}_{\bullet}{}^{\bullet} \sim t'_{i}{}^{j} = A_i{}^{k}\, A^{*j}{}_{l}\, t_{k}{}^{l} \\ etc. \\ t'^{\bullet\bullet}_{\bullet} = A_{\bullet}{}^{\bullet} t_{\bullet\bullet} g^{\bullet\bullet}\, A^{+}{}_{\bullet}{}^{\bullet} = \\ A_{\bullet}{}^{\bullet} t_{\bullet\bullet}\, A^{+\bullet}{}_{\bullet}\, A^{\bullet}{}_{\bullet} g^{\bullet\bullet}\, A^{+}{}_{\bullet}{}^{\bullet} = t'_{\bullet\bullet} g'^{\bullet\bullet} \end{array}\right\} \qquad \textbf{(151)}$$

The transformation rule applied to a transformer t corresponds to the transformer being pulled over to the other side of the metric transducer A:

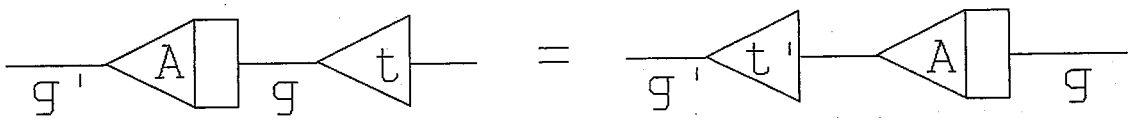


Figure 43. Transfer of transformer to another metric.

Tensors in general will be changed by a metric unitary transformation. Tensors that cannot change by such operations are not really tensors, because they do not conform to the transformation rule (151). This is the reason why we introduced the word *protensors*. Quantities like g, h, δ, and Δ depend only on the metric g and are therefore unchanged by metric unitary transformations, unlike real tensors, like the relativistic energy-momentum tensor and the electromagnetic field-tensor.

The transformation rules that build on the prototype figure 42 (a-bonds, orientation from primary to secondary) must be supplemented with other rules, building on other prototypes. If the orientation is the same, but the flow-variance in both bonds is changed from contra- to covariant, the symbol A must be replaced with the dual symbol

$$\breve{A} = h'Ah \tag{152}$$

where h' and h are the transformer-symbols for the two-port o-junction in the metrics g' and g. We shall assume (and show in the next chapter), that this symbol is uniquely determined by the metric. Eq. (152) corresponds to a rule saying that we can pull a junction over to the other side of a transducer, when we simultaneously change the transducer-symbol to the dual.

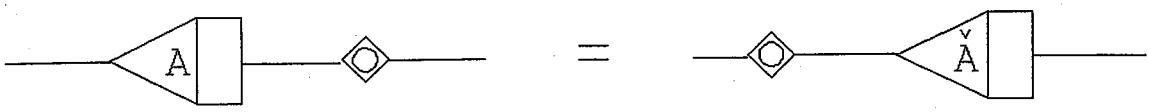


Figure 44. Transfer of junction to another metric.

The transformation rules (150)-(151) are usually regarded as the definition of, that a quantity is a tensor. As remarked above, this definition imply, that protensors like h, δ, and Δ are *not* tensors. While tensors are physical quantities, protensors are more like mathematical constants, as π and e. If h could not be regarded as a quantity, that is determined by the metric alone and is unchanged by the many coordinate-shifts, that may be performed within a given metric, we can't either define transformation rules for the proper tensors. Protensors are therefore more fundamental for the energy-bond-formalism than the proper tensors, apart from the metric tensor, that may be regarded both as a protensor and a proper tensor.

7. Junctions in a general metric.

The definition of junctions gives no problem: the o-junction is *effort-distributive and flow-conservative, while the x-junction is flow-distributive and effort-conservative.* A distributive relation determines the same value of a quantity in all bonds, while a conservative relation determines the sum of the quantities involved to be zero. These definitions then demand, that all the bonds attached to the junction are oriented in the same way in relation to the junction, that all the efforts have the same variance, and that all the flows have the same variance. Regarding causality, it follows from the definition, that a junction always must have one *dominating* bond, or a bond with *strong causality*, namely that bond, where the distributive quantity is input to the junction. Thus, for the o-junction there must be one bond with effort-input, and the other (weak) bonds have flow-input; for the x-junction there is one dominating bond with flow-input, and the weak bonds have effort-input.

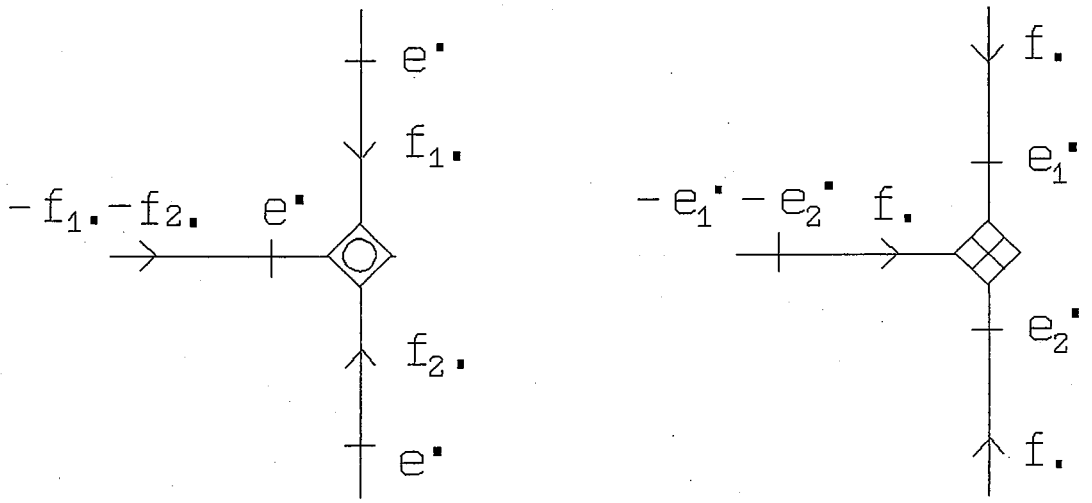


Figure 45. Definition of junctions.

If the orientation is shifted in one of the bonds with fixed flow-variance (or if the flow-variance is shifted with fixed orientation, the relations become more complicated, because they become involved with the protensor h. If we then insert a two-port junction in that bond where the orientation (or the flow-variance is shifted, we again get a simple relation. It is therefore convenient to operate with some alternative multiport-junctions ("multi" means 3 or more), where an extra two-port junction is built into one of the ports. From the simple energy-bond-formalism is known "the dotted x-junction", that is an x-junction with an extra two-port x-junction built into one or more ports.

However, there is not a correspondingly modified o-junction, and that is because the 2-port o-junction is an identity in the simple formalism. In the vector-bond formalism the 2-port o-junction is not trivial, so we'll need a "dotted o-junction", that we, for graphical reasons choose to draw in another way by prolonging one of the energy-bond-lines into the center of the junction

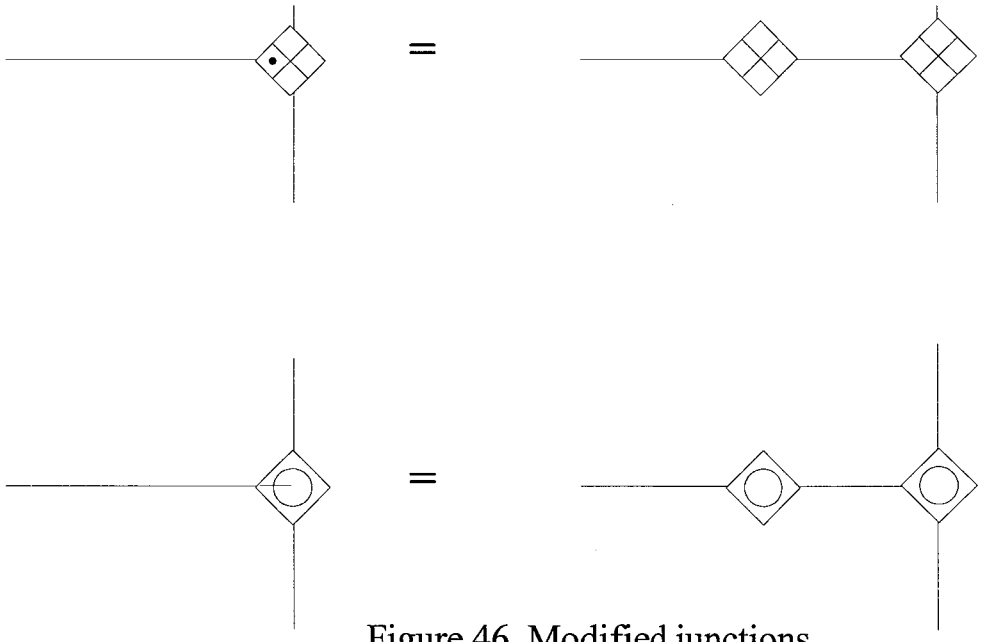


Figure 46. Modified junctions

By using junctions such modified, one will often obtain, that the protensor h does not appear explicitly in the equation-system for the model, and that is an advantage. All the same, we can't avoid discussing, how we determine h for a given metric g.

If the metric is *neutral*, we have seen that h = δ. We have further seen, that for *dimension-less* metrics the 1- and 2-variants of h are identical, and all matrix-variants of g, h, δ, and Δ commute with each other. Let us therefore as the first point try to determine h for a dimension-less metric g, that is not neutral.

A neutral metric G.. has the determinant 1 or -1, and we shall perform the restriction of the concept " a dimension-less metric" that we demand the same thing of it. If a metric is not dimensionless, we can de-dimension it with a diagonal transducer, a so-called *scale-transducer*, and when we have to do this anyway, we may just as well make sure that we change the determinant to 1 or -1. It is thus no real restriction in demanding this of the determinant of a dimension-less metric.

So, in the dimension-less metric is

$$h_{\bullet}^{\bullet} = h^{\bullet}_{\bullet} \wedge (h_{\bullet}^{\bullet})^2 = \delta_{\bullet\bullet} \qquad \textbf{(153)}$$

This means, that we to the metric g can associate a neutral metric G by the definition

$$G_{\bullet\bullet} = G^{\bullet\bullet} = h_{\bullet}^{\bullet} \qquad \textbf{(154)}$$

This neutral metric then has the same 1-variant of *its* h as the given dimension-less metric has of h. If g is neutral beforehand, G will be the same metric as g, and we may suppose, that G and g are of the same "family", i.e. have the same determinant and signature (same standard metric).
As all variants of h commute with all variants of g, all variants of G will also commute with all variants of g.

We'll call G *the proximal neutral metric* or *the projected neutral metric.* By a projection is understood an operator, that is *idempotent, i.e. obeys the condition*

$$P^2 = P \qquad \textbf{(155)}$$

and as the construction of a neutral metric from a dimension-les does not lead to anything new, when it is repeated, the operation may be regarded as a projection from the space of dimension-less metrics on to the space of neutral metrics. If we think of the projection of a point on a line, then the projected point is that point on the line, that is closest to the original point, hence the designation *proximal neutral metric*. In this metaphor, the line is that *equivalence class* of neutral metrics, that contains the metric G. This equivalence class will also contain the standard metric G_0.

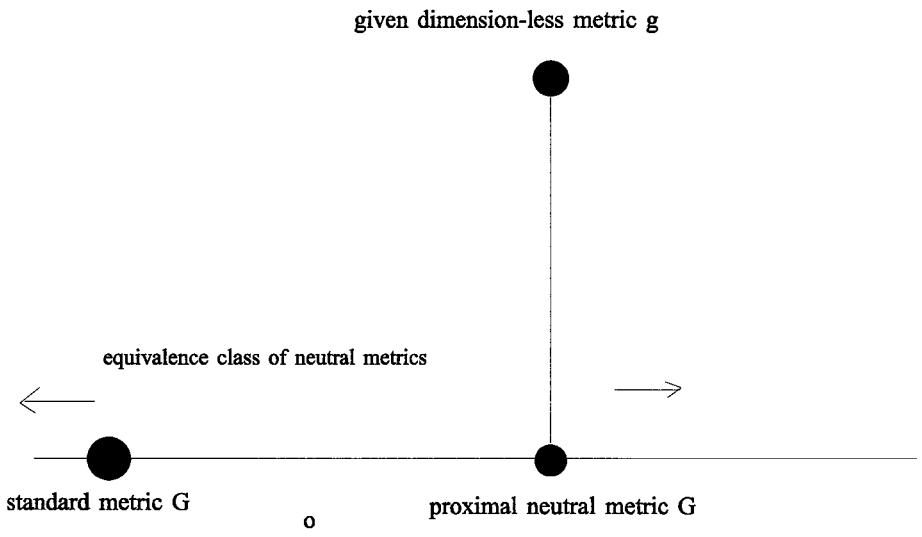


Figure 47. The Projection-metaphor for metrics.

That, which connects the neutral metrics in an equivalence-class, is *1-unitary transformations*. Let G_1 and G_2 be two neutral metrics in the same equivalence-class. Then they are connected through a 1-unitary transducer W

$$G_{2\bullet\bullet} = W_{\bullet}^{\bullet}\, G_{1\bullet\bullet}\, W^{+\bullet}_{\bullet} \qquad \textbf{(156)}$$

For 1-unitarity of W is a necessary and sufficient condition for, that multiplicative relations between matrices (including reciprocity) is passed on to the transformed matrices, and as the 1-unitary matrices form a group, (156) will constitute an equivalence class

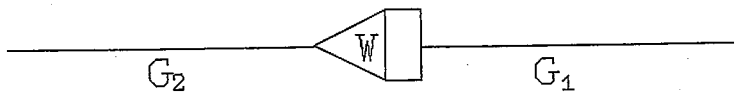


Figure 48. Two neutral metrics in the same equivalence-class connected with 1-unitary transducer.

As W is 1-unitary, we have

$$\left.\begin{aligned} W^{*\,i}{}_{j} = W^{-1\,j}{}_{i} = W^{+i}{}_{j} = W^{-1i}{}_{j} \\ \Rightarrow W^{\bullet}_{\bullet} = W^{\bullet}{}_{\bullet} \end{aligned}\right\} \qquad \textbf{(157)}$$

i.e. for 1-unitary transducers there is no difference of the 1- and 2-variant.

The connection between the to g proximal or projected neutral metric G and the standard metric G_0 is therefore also given by a 1-unitary transducer U:

$$G_{\bullet\bullet} = U^{\bullet}_{\bullet}\, G_{0\bullet\bullet}\, U^{+\bullet}{}_{\bullet} \qquad \textbf{(158)}$$

If the standard metric is euclidean, i.e. if $G_{0\bullet\bullet}$ is the unit matrix, U (1-variant) and U^+ (2-variant), that are each other's reciprocal matrices, will cancel each other, so $G=G_0$. The equivalence-class for an euclidean metric, thus, only contains the same euclidean metric. If the given metric has euclidean standard metric, then the proximal neutral metric also must be the euclidean metric. So in that case we have

$$G = \Delta \Rightarrow h^{\bullet}_{\bullet} = \Delta^{\bullet\bullet} \Rightarrow h_{\bullet\bullet} = g_{\bullet\bullet} \qquad \textbf{(159)}$$

This result can be shown to be valid generally, also for dimensioned metrics with positive diagonal elements, so we emphasize it as a general theorem:

If a metric g.. has an euclidean standard metric (is quasi-euclidean), the protensor h is identical with the metric tensor g.

This means, that when the concern is proper geometry in arbitrary, local coordinate systems, we don't need to worry about h, and the two-port o-junction will be a unit transformer. This result can, e.g., without further considerations be applied to description in polar coordinates, where g.. is given by (63).

For the general case, where the standard metric is not euclidean, we introduce the concept *standardization-transducer*, that is a metric transducer connecting the given metric with the standard metric. Such a transducer can be constructed in many ways, but we shall be particularly interested in one special way from G_0 to g, namely the one that passes the proximal neutral metric G. We shall write such a transducer on the form

$$S_{\bullet}^{\bullet} = H_{\bullet}^{\bullet} U_{\bullet}^{\bullet} \tag{160}$$

where U is 1-unitary and H 1-hermitean, and where the variants of H commute with the variants of the proximal neutral metric G. The connection between the two involved metrics is thus given by (158) and

$$g_{\bullet\bullet} = S_{\bullet}^{\bullet}\, G_{0\bullet\bullet}\, S^{+\bullet}{}_{\bullet} = H_{\bullet}^{\bullet} G_{\bullet\bullet}\, H^{+\bullet}{}_{\bullet} \tag{161}$$

and, as H is 1-hermitean and commutes with G.., we get

$$\left.\begin{aligned} H^{+\bullet}{}_{\bullet} &= H_{\bullet}^{\bullet} = H^{-1\bullet}{}_{\bullet} \\ g_{\bullet\bullet} &= (H_{\bullet}^{\bullet})^2 G_{\bullet\bullet} \end{aligned}\right\} \tag{162}$$

When this is combined with (153) and (154), we have

$$h_{\bullet\bullet} = g_{\bullet\bullet} h^{\bullet}{}_{\bullet} = g_{\bullet\bullet} G^{\bullet\bullet} = (H_{\bullet}^{\bullet})^2 \tag{163}$$

and we have hereby found the expression for the protensor h in a dimension-less metric. A proper mathematical proof, that the construction (160) is possible and unambiguous shall not be given now. Instead we shall proceed constructively and see, what has to be done to determine h from g.

In this chapter we shall restrict ourselves to two-dimensional, complex metrics with the signature 0. If the signature is 2, the standard metric is euclidean, and that case we have already considered. It will turn out, that the example is sufficiently general for the discussion of motion in accelerated frames of reference and shows the way for more complicated cases in general relativity.

So we look at the metric

$$g_{..} = \begin{pmatrix} a & b\,e^{-i\phi} \\ b\,e^{i\phi} & \frac{b^2-1}{a} \end{pmatrix} \qquad \textbf{(164)}$$

where a,b, and φ are real numbers. This metric is the most general expression for a two-dimensional dimension-less metric with the determinant -1. The standard metric is, as we shall se

$$G_{0..} = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \textbf{(165)}$$

so the signature is 0. The equivalence-class containing G_0 and the expression for the 1-unitary transducer U has been given earlier in (66) and (67). The proximal metric to (164) must be expressible on the form

$$G_{..} = \sigma(\theta,\phi) = \begin{pmatrix} \cos\theta & \sin\theta\,e^{-i\phi} \\ \sin\theta\,e^{i\phi} & -\cos\theta \end{pmatrix} \qquad \textbf{(166)}$$

As G.. shall commute with g.., it is easy to see, that it must be the same φ, that occurs in (164) and (166). We may start by writing g.. on the form

$$g.. = \tfrac{1}{2}\left(a + \frac{b^2 - 1}{a} \right)\delta.. + \rho\, G.. \tag{167}$$

where ρ is a real number, that can be found from the expression

$$\rho G.. = \tfrac{1}{2}\left(a - \frac{b^2 - 1}{a} \right)\sigma_z + b\cos\phi\,\sigma_x + b\sin\phi\,\sigma_y \tag{168}$$

For, as $(G..)^2 = \delta$, and as the Pauli matrices each have the square 1 and anti-commute with each other, ρ must be the square root of the sum of squares of the three coefficients to the Pauli matrices in (168). We therefore find:

$$\rho = \sqrt{\frac{1}{4}\left(a - \frac{b^2 - 1}{a} \right)^2 + b^2} = \sqrt{1 + \frac{1}{4}\left(a + \frac{b^2 - 1}{a} \right)^2} \tag{169}$$

the quantity θ in (166) is then given by

$$\theta = \arcsin \frac{b}{\rho} \tag{170}$$

and h.. can then be determined from (163) and (167):

$$h.. = \rho\,\delta.. + \tfrac{1}{2}\left(a + \frac{b^2 - 1}{a} \right) G.. \tag{171}$$

The hermitean transducer H, whose 1-variant according to (163) can be described as the square root of h.., will in general be calculated as an exponential function of a *generator,* that commutes with G... In this two-dimensional case where G.. is the spin-matrix $\sigma(\theta,\varphi)$ (comp. (166)), there are no other generators that commute with G.. than G.. itself, and we may therefore write

$$H_{\bullet}^{\bullet} = \exp(\tfrac{\chi}{2} G_{\bullet\bullet}) = \sum_{n=0}^{\infty} \frac{(\frac{\chi}{2} G_{\bullet\bullet})^n}{n!} = \cosh \tfrac{\chi}{2}\, \delta_{\bullet\bullet} + \sinh \tfrac{\chi}{2}\, G_{\bullet\bullet} \qquad \textbf{(172)}$$

so h.. is given by

$$h_{\bullet\bullet} = (H_{\bullet}^{\bullet})^2 = \exp(\chi G_{\bullet\bullet}) = \cosh \chi\, \delta_{\bullet\bullet} + \sinh \chi\, G_{\bullet\bullet} \qquad \textbf{(173)}$$

which accords with (171) and (169), when

$$\sinh \chi = \tfrac{1}{2}\left(a + \frac{b^2 - 1}{a} \right) \qquad \textbf{(174)}$$

The account of this example of the determination of h shows, that explicit expressions for the matrix-variants in general can be expected to be rather complicated, and that it will be a model-technical advantage, if one can avoid, that they occur in the equations. In the usual mechanical methods of reticulation, that we are going to consider later, it turns out, luckily, that junctions occur in "dotted" versions, so that we generally have no need to calculate h.

We still lack the discussion of, how we determine h, when the metric is dimensioned. This we shall also do with an example, that at the same time introduces the relativistic point of view, that we shall later follow up upon. We look at a particle, that can move in the two spatial dimensions of a flat disc, that rotates relative to the laboratory system with a constant angular velocity ω.

Let us first discuss the situation in the laboratory system, that we presume is an inertial system. Here we can by use of standard measuring rods and standard

clocks mark measurement numbers for the rectangular coordinates x and y, the polar coordinates r and θ, and the time t in every point. The coordinate systems are placed with origin in the center of the disc. We imagine, that the particle is equipped with its own "wrist-watch", that also is a standard clock, measuring the proper time of the particle, τ. By the velocity of the particle we shall mean a three-dimensional vector, where the two first coordinates denote the change of the spatial coordinates x and y, and the third the change of the coordinate time t, all measured in relation to the proper time τ:

$$U^1=\frac{dx}{d\tau}\ ;\ U^2=\frac{dy}{d\tau}\ ;\ U^3=\frac{dt}{d\tau} \qquad \textbf{(175)}$$

The first two coordinates of this contravariant flow-vector are flows, but the third is an effort, for t and τ change sign by time-reversal, while x and y do not. We therefore know, that the metric tensor for this coordinate system has a negative diagonal element on the third place, and that it, besides that, is diagonal, as we, temporarily, are in an inertial system. We may then assume, that this negative diagonal element, $-c^2$ is a constant of nature, if we believe, that the motion of a free particle in an inertial system can be expressed in a universal form. The constant c is, of course, the speed of light, but we really don't need to know that now:

$$g_{0\cdot\cdot}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -c^2 \end{pmatrix} \qquad \textbf{(176)}$$

Making now the transitiom to polar coordinates r and φ in the laboratory-system, but keeping the time t, the contravariant velocity vector is given by

$$V^1=\frac{dr}{d\tau}\ ;\ V^2=\frac{d\phi}{d\tau}\ ;\ V^3=\frac{dt}{d\tau} \qquad \textbf{(177)}$$

and the connection between the two velocity-vectors is

$$\begin{pmatrix} U^1 \\ U^2 \\ U^3 \end{pmatrix} = \begin{pmatrix} \cos\phi & -r\sin\phi & 0 \\ \sin\phi & r\cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V^1 \\ V^2 \\ V^3 \end{pmatrix} \tag{178}$$

The metric tensor g_1 for the polar laboratory- coordinate system can then be found from the condition

$$U^{\bullet} g_{0\bullet\bullet} U^{\bullet} = V^{\bullet} g_{1\bullet\bullet} V^{\bullet} \tag{179}$$

and one gets

$$g_{1\cdot\cdot} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & -c^2 \end{pmatrix} \tag{180}$$

i.e. a dimensioned diagonal metric, where the second and the third diagonal element depend differently of the measuring standards for length and time.

The metric g_1 can be brought to standard form at the same time as it is done dimension-less by a diagonal *scale-transducer* B, so there won't be need for further standardization of the dimension-less metric:

$$g_{1\bullet\bullet} = B_{\bullet}^{\bullet} G_{0\bullet\bullet} B^{+\bullet}{}_{\bullet} \tag{181}$$

where

$$G_{0\bullet\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} ; \; B_{\bullet}^{\bullet} = B^{+\bullet}{}_{\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & c \end{pmatrix} \tag{182}$$

In this case the scale-transducer B takes the part of the earlier used 1-hermitean transducer H, for B is 1-hermitean, and although we change the measuring scales of space and time, it will stay 1-hermitean, because it is diagonal. So, the 0-variant of the protensor for the metric g_1 becomes

$$h_{1\bullet\bullet} = (B_{\bullet}^{\bullet})^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix} \tag{183}$$

By use of the to (180) reciprocal matrix

$$g_1^{\bullet\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & -\frac{1}{c^2} \end{pmatrix} \tag{184}$$

we then find

$$h_{1\bullet}^{\bullet} = h_1^{\bullet}{}_{\bullet} = G_{0\bullet\bullet} \tag{185}$$

So in this case (153) is also valid, although the metric is dimensioned, but this is due to the metric being diagonal. When we now proceed to the conditions in the rotating coordinate system (the disc's), the relations are different.
In the disc's attached coordinate system we can use the coordinates

$$\left.\begin{aligned} r' &= r \\ \phi' &= \phi - \omega t \\ t' &= t \end{aligned}\right\} \tag{186}$$

and the velocity's contravariant coordinates are then

$$\left.\begin{aligned} V'^{1} &\doteq \frac{dr'}{d\tau} = V^{1} \\ V'^{2} &\doteq \frac{d\phi'}{d\tau} = V^{2} - \omega V^{3} \\ V'^{3} &\doteq \frac{dt'}{d\tau} = V^{3} \end{aligned}\right\} \qquad \textbf{(187)}$$

From the condition (comp. (179))

$$V^{\bullet} g_{1\bullet\bullet} V^{\bullet} = V'^{\bullet} g'_{\bullet\bullet} V'^{\bullet} \qquad \textbf{(188)}$$

we then find the metric tensor g' for the disc's coordinate system

$$g'_{\bullet\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & r^2\omega \\ 0 & r^2\omega & r^2\omega^2 - c^2 \end{pmatrix} \qquad \textbf{(189)}$$

We remark about this dimensioned metric, that the two first diagonal elements are positive, the last negative, but only inside the radius, where the rotation-velocity rω becomes equal to the speed of light c. Out there is a horizon "beyond which all is darkness", as Peirce said in the article "the Architecture of Theories" from 1891. Inside the horizon the signature is 1, but outside it is 3, so the metric outside the horizon cannot in any continuous way be brought in connection with the standard metric G_0. Said in another way: That part of the from the laboratory system measurable universe lying outside the horizon of the disc system, does not belong to the disc system's universe.

The metric g'.. (189) has the determinant $-r^2c^2$, and the reciprocal matrix is

$$g'^{\bullet\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} - \frac{\omega^2}{c^2} & \frac{\omega}{c^2} \\ 0 & \frac{\omega}{c^2} & -\frac{1}{c^2} \end{pmatrix} \qquad \textbf{(190)}$$

Now, when we shall bring g'.. on a dimension-less form with a diagonal transducer B, we shall at once take care, that the determinant of the de-dimensioned metric g.. becomes -1. It is only the second and the third diagonal element in B's 1-variant, that can have dimension, i.e. there are two scale-parameters, but the requirement that the determinant of g.. shall be -1 reduces it to one (temporarily) free parameter μ. WE therefore try with a scale-transducer of the form (comp. (182))

$$(h_{\bullet\bullet})^2 = \Delta_{\bullet\bullet} = (g_{\bullet\bullet})^2 \qquad \textbf{(191)}$$

The with B_μ de-dimensioned metric is then also μ-dependent, but, as we shall soon see, not all values of μ are equally good, if we shall determine the protensor h. Provisionally we use the form

$$g'^{\bullet\bullet}_{\bullet\bullet} = B_{\mu\bullet}^{\bullet}\, g_{\mu\bullet\bullet}\, B_{\mu}^{+\bullet\bullet} \qquad \textbf{(192)}$$

and thus, the μ-dependent dimensionless metric is:

$$g_{\mu\bullet\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu^2 & \beta \\ 0 & \beta & \frac{\beta^2 - 1}{\mu^2} \end{pmatrix} \qquad \textbf{(193)}$$

where

$$\beta = \frac{r\omega}{c} \tag{194}$$

We now have a situation that can be dealt with by the earlier developed method for two-dimensional metrics with the signature 0. for the metric (193) is effectively two-dimensional; we only need to take care of the lower 2*2 matrix, where all the elements are different from 0. This sub-matrix shall be compared to (164), and we see at once, that the φ occurring there must be zero, as the metric is real. From (166) we then get, that there must be a θ, such that the proximal neutral metric and the 1-variant of h is given by

$$G_{\bullet\bullet} = h_{\bullet}^{\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{pmatrix} \tag{195}$$

We must reckon, that θ is something definite, i.e. that θ determines μ, and not vice versa, for G is the proximal neutral metric, and should rather be independent on some arbitrary scale. Now it turns out so lucky in this case, that there is one definite choice of μ (that must be positive), that directly makes the metric (193) neutral and thus identical with (195), viz.

$$\mu = \sqrt[4]{1-\beta^2} \quad \theta = \arcsin\beta \tag{196}$$

The scale-transducer B_μ with this value of μ inserted directly takes over the part of the 1-hermitean transducer H in (163), i.e. we can fix the 0-variant of the protensor in the dimensioned metric g' to

$$h'_{\bullet\bullet} = (B_{\mu\bullet}{}^{\bullet})^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{r^2}{\sqrt{1-\beta^2}} & 0 \\ 0 & 0 & c^2\sqrt{1-\beta^2} \end{pmatrix} \tag{197}$$

By using (190) and the raising-rule we can then determine the 1- and 2-variant of h'. We introduce another dimension-less quantity, the relativistic time-dilation factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{198}$$

We then find

$$h'^{\bullet}_{\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\gamma} & \frac{r\beta\gamma}{c} \\ 0 & \frac{c\beta}{r\gamma} & -\frac{1}{\gamma} \end{pmatrix} ; \; h'^{\bullet}{}_{\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\gamma} & \frac{c\beta}{r\gamma} \\ 0 & \frac{r\beta\gamma}{c} & -\frac{1}{\gamma} \end{pmatrix} \tag{199}$$

Thus, the two h' variants are not identical for a dimensioned metric, and they are not hermitean, either (what they, naturally, can't be when the non-diagonal elements have different dimensions.

Besides, one can easily convince oneself, that h for the neutral metric (195) transforms like a tensor with the transducer B_μ and gives h' for the dimensioned metric. All in all, this is valid for the special 1-hermitean transducers H and B, we have considered in this chapter and used for the construction of h, that under them h transforms like a tensor. This transformation-property does, however, does not hold for arbitrary transducers, which is why we called h a protensor instead of a tensor.
The equations (163) and (183) show, that h.. is a *positive definite hermitean matrix*. For a dimension-less metric this means, that h.. has only positive eigenvalues, and for the dimensioned metric, we just considered, h' is diagonal and has only positive elements.

For a dimension-less metric we have earlier shown (130):

$$(h..)^2 = \Delta.. = (g..)^2 \qquad \textbf{(200)}$$

and this, together with h.. being positive definite, implies that h.. can be expressed as the positive square root of $(g..)^2$ or as "the absolute value of g..":

$$h.. = \sqrt{(g..)^2} = |g..| \qquad \textbf{(201)}$$

This formula makes possible a direct determination of h.. as a function of g.. for every dimension-less metric. One just has to find a 1-unitary metric transducer U that brings the metric to diagonal form $g_d..$

$$g_{d}{}_{\bullet\bullet} = U^{+}{}_{\bullet}^{\bullet}\, g_{\bullet\bullet} U^{\bullet}{}_{\bullet} \sim g_{\bullet\bullet} = U_{\bullet}^{\bullet}\, g_{d}{}_{\bullet\bullet} U^{+}{}_{\bullet}^{\bullet} \qquad \textbf{(202)}$$

It will then be the same transducer U, that appears in (160), i.e. the one, that diagonalizes the proximal neutral metric G.. to the standard form $G_0..$, for G.. can also be expressed as function of g.. by using (201)

$$G_{\bullet\bullet} = h_{\bullet}^{\bullet} = h_{\bullet\bullet} g^{\bullet\bullet} \qquad \textbf{(203)}$$

and all hermitean matrices, that can be expressed as functions of a certain hermitean matrix, can be diagonalized by the same 1-unitary transformation. A function of a hermitean matrix is determined by, first, diagonalizing the matrix and then taking the relevant function of the eigenvalues in the diagonal, and, finally, transforming back again. In order to determine h.. one thus has to diagonalize g.. with the transducer U as in (202), take the absolute value of the diagonal elements and then transform back again. This prescription, though, can only be used for dimension-less metrics; it makes no sense to diagonalize a matrix, if the diagonal elements have different dimensions.

Just like the 1-variant of h (or the 2-variant, that is the same for a dimension-less metric) defines a projected metric, viz. the proximal neutral metric G.., so the 0-variant h.. may define a projected metric $g_{qe..} = h..$, that is called *the projected quasi-euclidean metric*. By "quasi-euclidean" is meant, that this metric has a euclidean standard metric, because the eigenvalues of h.. are positive. This metric g_{qe} will have the same h.. as the original metric, so it is really a projection (comp. (155)).
The two transducers H and U that together can bring a dimension-less metric to standard form as in (160) and (161) may be regarded as internal properties of the given metric. It is therefore obvious to define them as protensors, i.e. as transformers with an equally fundamental status as the two-port o-junction h. We shall here limit ourselves to look at the definition of H as a protensor. We therefore introduce a transformer H', whose 1-variant shall be the same as the 1-variant of the transducer H. As H' is an endotensor and H an exotensor, the other variants of H' will be different from the other variants of H:

$$H'^{\bullet}_{\bullet} = H^{\bullet}_{\bullet} \; ; \; H'_{\bullet\bullet} \doteq H^{\bullet}_{\bullet} g_{\bullet\bullet} = H_{\bullet\bullet} G^{\bullet\bullet} g_{\bullet\bullet} = H_{\bullet\bullet} \Delta^{\bullet\bullet} h_{\bullet\bullet} \neq H_{\bullet\bullet} \qquad \textbf{(204)}$$

We introduce the following icon "a half o-junction" (comp. (163))

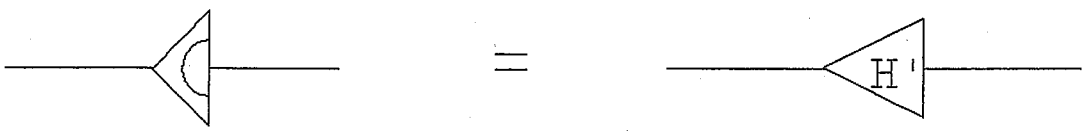


Figure 49. The protensor H', the half o-junction.

It is now clear, that all variants of H' commute with all variants of g and h. I.e. the 2-variant of H' is equal to the 1-variant, an H' is self-dual. (132) gives then, that icon-reversal of the half junction leads to the reciprocal of H', which leads to the formula below, that gives another justification for the choice of the icon (comp. figure 27).

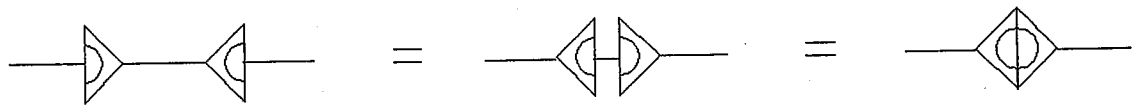


Figure 50. Icon-reversal of the half junction.

If the metric is neutral, g = G, is H'=h=δ, i.e. the half junction is identical with the whole two-port o-junction and needs no special icon.
If we concatenate two half junctions, we don't get a whole junction, for

$$(H'^{\bullet\bullet}_{\bullet})^2 = (H^{\bullet}_{\bullet})^2 = h_{\bullet\bullet}\Delta^{\bullet\bullet} = h^{\bullet}_{\bullet}\Delta^{\bullet}_{\bullet} \qquad \textbf{(205)}$$

which is brought forward by the following icon-formula (comp. figure 35)

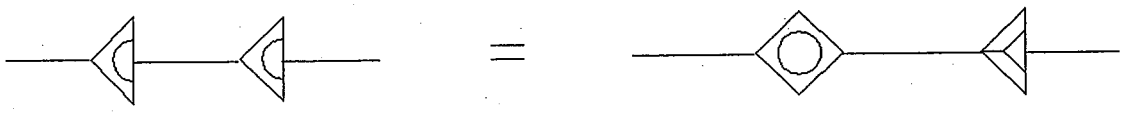


Figure 51. Concatenation of half junctions.

For quasi-euclidean metrics (that have euclidean standard metric) is h=g, so the whole two-port o-junction is not needed, but the protensors H' and Δ are non-trivial.

The question of, how these rules can be formulated for dimensioned metrics we shall leave for now.

8. Existential bond graphs.

The text has, until now, contained several examples of *iconic formulæ*, wherein appear two or more energy-bond-graphs with an equal sign between. Examples of such formulæ are found in the figures 27,30,35,36,39,53,44,50, and 51. We shall here be especially concerned with icon-formulæ, where two two-ports appear with an equal sign between.

Each of the two two-ports is then, with Peirce's terminology (chapter 1), a *rheme*, i.e. a solitary "word", that in the sign-classification has a 1 in the interpretant's place (3.place), i.e. the meaning of the rheme is potential, the lonely word says nothing, but when the words are put together after certain rules of language, *propositions* come into being (dicisigns), that are sentences with a meaning, that have a 2 in the interpretant's place. The equal-sign is in the iconic formulæ the connecting link between the two graphical rhemes, that creates the proposition (the *copula*).

Looking at a formula that equates two transformers t_1 and t_2, we can ascade-couple a new transformer t_3 on both sides of the equal-sign and in this way make the composite transformer on the right side to a unit-transformer. This allows us to bend the two ends of the left two-port together. The closed structure that emerges, when a two-port "bites itself in the tail", we call a ***sep***. The word means, presumably a *separator*, as it is a closed curve, that separates the area inside from the area outside, when it is drawn in a plane as on *the sheet of assertion* in Peirce's logical game "existential graphs".

When the sep is formed, we don't need the equal-sign any longer. The sep is a proposition, precisely like the original equation, so by letting the transmitter bite itself in the tail, it raises itself one sign-class, from 1 to 2 on the interpretant's place, from rheme to dicisign.

A sep *asserts something* about its components, and the existence of the sep is the proof of the proposition, it asserts. It forms its own closed energy-circle, that nothing can disturb. If there exists a two-port junction in a sep, one cannot add a third port to the junction, for that would break the integrity of the sep.

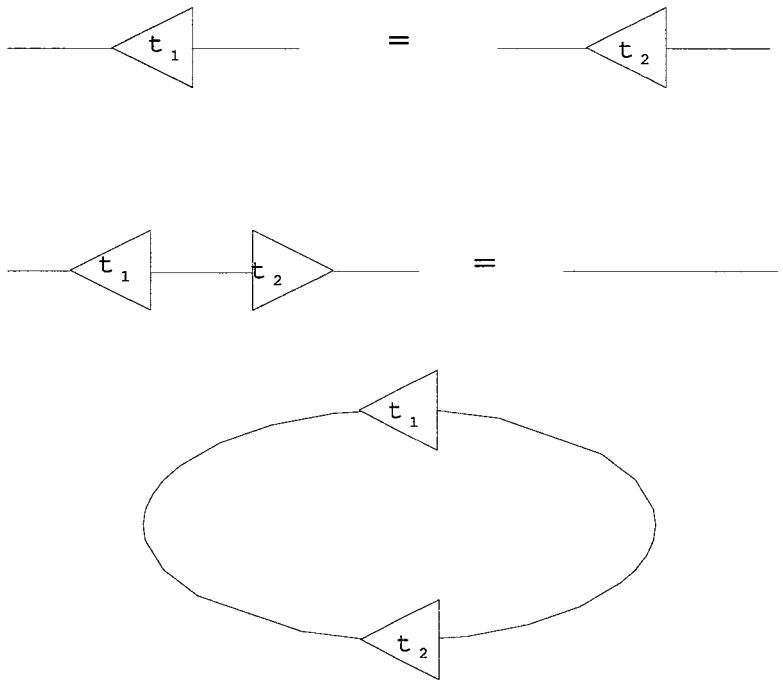


Figure 52. Formation of a sep (comp. figure 34 and eq. (132)).

If one writes the same tensor-symbol in the transformer-icons of a sep like figure 52, one hereby asserts, that the tensor in question is self-dual.

Peirce's existential graphs is a kind of game with the purpose of clarifying the meaning of logical propositions. We shall here only look at the simplest version, called the α-graphs, treating simple propositional logic. In the β-graphs bond graphs are introduced to a relation-logical analysis of sentences, and the γ-graphs introduce quantors, like existence- and all-quantor. The game proceeds by a participant put forward an *assertion* on the *sheet of assertion*, that is constructed for this purpose. The other participants must then try to refute or confirm the assertion. If an assertion can be proved false, it will imply certain sanctions against the one, who has put it forward. The false assertion must not be erased, but must be framed in a sep, whereby it is put apart from the rest of the sheet of assertion. A validated assertion, on the contrary, may well be erased, which just means that it is pushed away from the central (visible) part of the sheet, to where it may later be brought back.

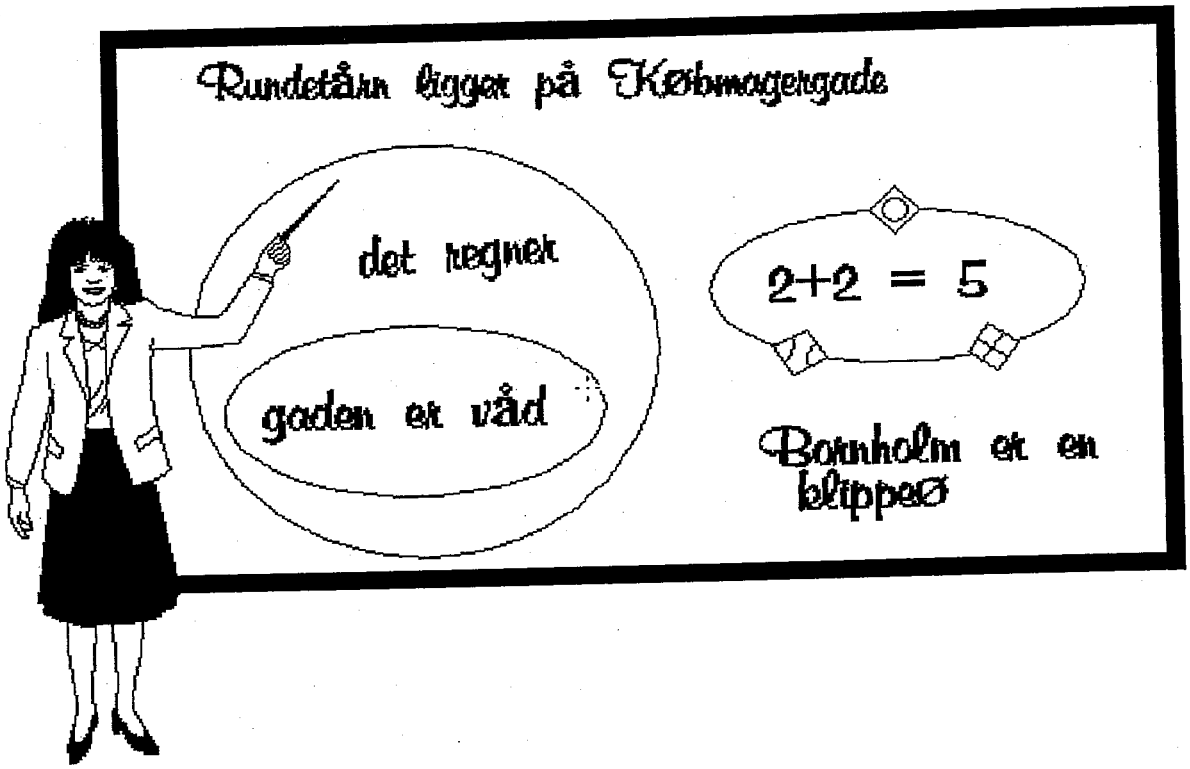


Figure 53. The sheet of assertion.

Peirce did never consider energy-bonds but only logical bonds as "lines of identity" in his β-graphs. The connection to the α-graphs in this context is, therefore, somewhat far-fetched. However, There are good reasons for studying closed energy-bond-loops as a means of logical expression, and this seems closely related to Peirce's concept of a sep in the α-graphs, and we shall pursue this relationship in the following.

The simplest type of sep is drawn as a closed curve, but it is not a curve in the usual mathematical sense, as was made clear by Peirce. We shall regard it as an energy bond that closes in itself, and thereby we also hit the essence of Peirce's conception. An energy bond does not change its character, if we allow it to cross itself, for such a cross-over is without physical significance, when there is no junction. An energy bond that looks like a curve with double-points, does not have real double-points and is therefore in a topological sense the same as a circular bond. If we equip the energy-bond-sep with structure in the form of built-in two-ports as in one of the examples in figure 53, it still has the same logical function as a simple sep: it delimits one part of the sheet from the rest, so it can be used for enclosing false assertions as "2+2=5".

A structure with a sep inside another sep is called a *scroll* by Peirce. We see one of these in figure 53, where in the outer sep stands the proposition "det regner" (it rains) and in the inner "gaden er våd" (the street is wet). This shall be conceived as the hypothetical, conditional proposition "if it rains, then the street is wet". The outer sep encloses a hypothetical universe, where one is allowed to formulate assertions, that cannot stand by themselves on the sheet of assertions, that only concerns the actual universe. The assertion in the inner sep shall then be regarded as the consequence of the hypothesis in the outer. How can this conception comply with the rule, that a sep is used to enclose false assertions? The sentence "it rains" is certainly not necessarily false.

In order to understand this we introduce a new construction, a ***blot***. A blot is just a sep, whose internal part is totally filled out, so that nothing can be written there. It shall be understood as a hypothetical universe where "anything goes", every assertion has already been put forward. If we inside a sep have the proposition P and a blot, as in figure 54, it can be read as "if P is true, then anything is true", and that is the same as saying "P is false". We can the let the blot float together with the internal rim and gradually let it shrink, until it finally is a cusp and disappears completely. We are then left with a single sep containing the assertion P, so the meaning of this must therefore be "P is false".

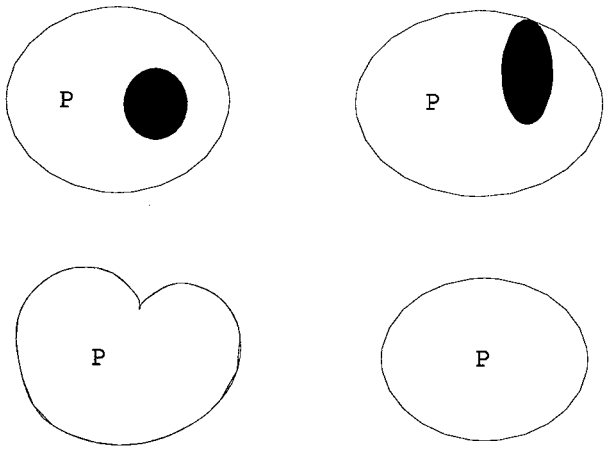


Figure 54. The sep as negation.

With this explanation the hypothetical conditional "if p then q" becomes the same as a *material implication*, that in the formal Boolean logic is construed as "either

p is false, or q is true, or both". In formal language:

$$p \Rightarrow q = \neg p \vee q \qquad \textbf{(206)}$$

When a sep is conceived as a negation of what is within, we can translate the conditional on figure 53 to "it is false, that it rains, and the street is not wet". The word "and" stands for a logical *conjunction*, that in formulæ is expressed with the sign $\wedge$, the counterpart to a *disjunction* denoted by $\vee$ (comp. (206)). The negation of a conjunction between two propositions is a disjunction of the negations of the two propositions, so for the material implication we have

$$\neg(p \wedge \neg q) = \neg p \vee q = p \Rightarrow q \qquad \textbf{(207)}$$

When two existential graphs occur independently on the sheet of assertion, e.g. the sentences "Rundetårn ligger på Købmagergade" (the Round Tower is in Købmager-street" and "Bornholm er en klippeø" (Bornholm is a rocky island) in figure 53, one of them can be removed without changing the meaning of the other. When they both appear the composite assertion is identical with the assertion of their logical conjunction. In this way we may translate traditionally formed combinations of propositions from the Boolean algebra by diverse seps as shown below in figure 55.

Algebraic manipulations with logical expressions the correspond to topological changes of the existential graphs according to certain rules, that may be derived from the preceding, such as:

If a sep contains one other sep, and the area between them is empty, both seps may be removed.

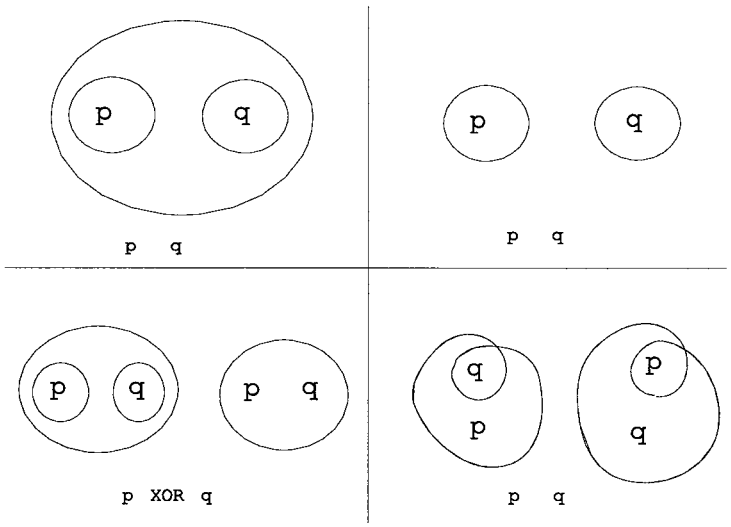


Figure 55. Logical combinations of propositions p and q: disjunction (inclusive or) neither nor, exclusive or, equivalence (bi-implication).

A blot within a sep may be erased, as it gets absorbed in the inner rim of the sep (comp. figure 54).

Peirce does not consider the sep as an existential graph, because it is without structure and therefore asserts nothing. When we expand the notion of a sep to encompass closed energy-bond-loops with two-ports built in, the sep becomes a graph, that asserts something about these two-ports, and the validity of the assertion will in some cases depend on, whether a symbolic relation is fulfilled. Let us consider some examples, that all can be derived from the preceding chapters.

All these examples are certain, or unconditioned, because they follow from the axioms of the energy-bond-formalism without further assumptions. Apart from the first, that is a definition of the unit-transformer, they are non-trivial, and they can be regarded as theorems. Like the simple sep, their existence is established, when one has realized the validity of the theorems.

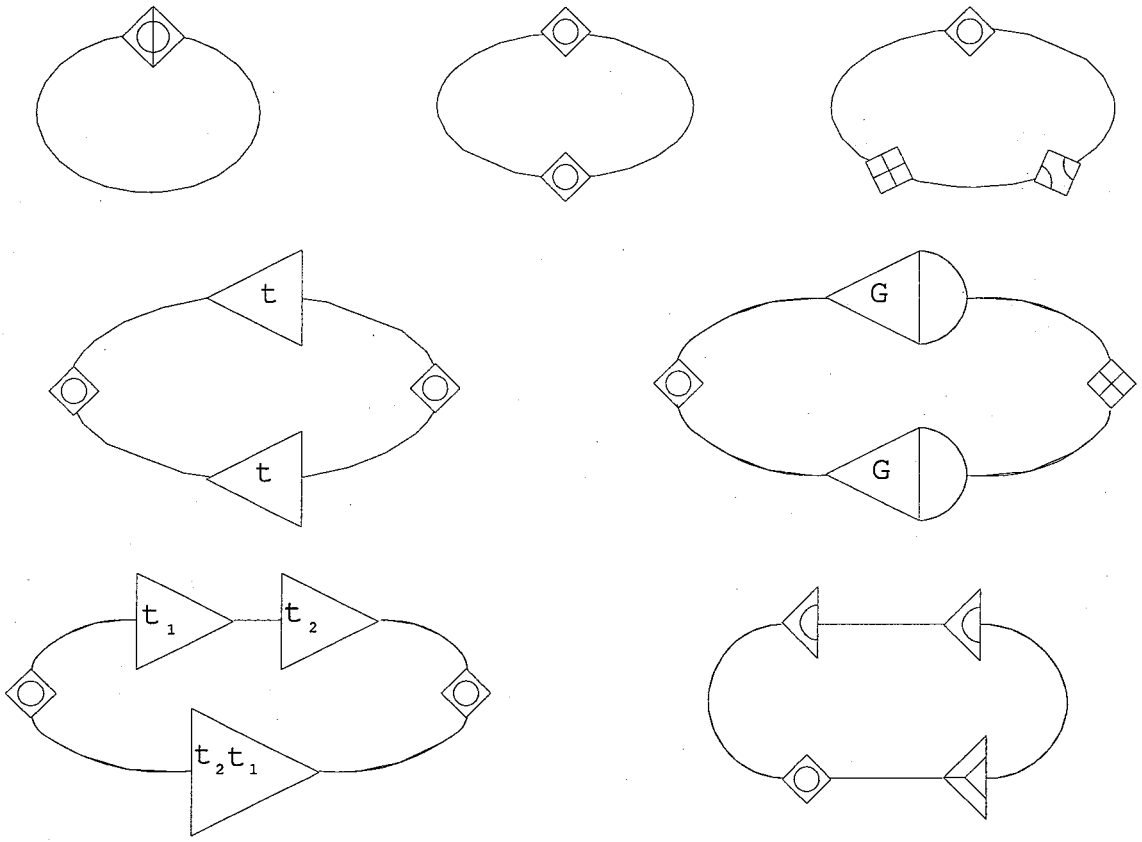


Figure 56. Examples of unconditioned seps.

If one encloses such a sep in another unconditioned sep, both seps will be annihilated.

A *conditioned* sep is only a true assertion, if some symbolic relation between the built-in two-ports is satisfied. As an example, the sep

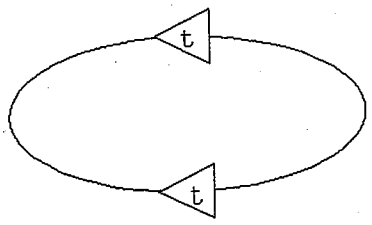


Figure 57. A conditioned sep.

is only true, if t is self-dual: $t = \breve{t}$. Such a sep is therefore uncertain and open to verification or falsification. The assertion, it puts forward, belongs to the external

side of the sep, if it is a sep. If the condition for the existence of the sep is written inside the loop, there appears a self-contradicting assertion, for if the condition is satisfied, the loop is a genuine sep, that negates, what stands inside. An extra sep outside it can make it a true assertion, but the two seps are then redundant, the assertion $t = \breve{t}$ can stand alone, or one may do with the one sep, figure 57.

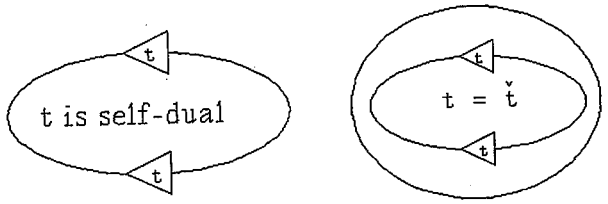


Figure 58. A self-contradicting and a redundant graph.

Peirce mentions in his second Lowell-lecture from 1903, that he had been forty years developing the existential graphs, which with its β- and γ-graphs is a much more intricate game, than the here given introductory considerations of the α-graphs may suggest, (though the energy-bond-seps gives it a new dimension, that Peirce didn't consider). The second Lowell-lecture, that the following quote is from, is one of more incomplete fragments on the graphs, collected in Carolyn Eisele's 4-volume edition "the new elements of mathematics by Charles S. Peirce". In the most frequently quoted collection of Peirce's manuscripts "Collected Papers" many mathematically inspired papers have been left out by the editor, the philosopher of religion Charles Haetshorne, which deficiency the mathematician Carolyn Eisely sought to repair with "New Elements", though it also appears incomplete with respect to the existential graphs, by Peirce himself regarded as his main work. The quote is:

> "Before beginning, let us distinctly recognize the purpose which this system of expression is designed to fulfill. It is intended to enable us to separate reasoning into its smallest steps so that each one may be examined by itself. Observe, then, that it is *not* the purpose of this system of expression to facilitate reasoning and to enable one to

> reach his conclusion in the speediest manner. Were that our object, we should seek a system of expression which should reduce many steps to one; while our object is to subdivide one step into as many as possible. Our system is intended to facilitate the *study* of reasoning but not to facilitate reasoning itself. Its character is quite contrary to that purpose."

In a similar vein one may remark, that the purpose of the energy-bond-technique *not* is to make modeling of physical systems easier, but rather to exhibit the semiotic logic, that is common to physical theories, whatever they belong to the classical or the modern disciplines.

9. SRT particle- and electro-dynamics.

The dynamics of the Special Relativity Theory (SRT) refers to *inertial systems*, that are supposed to have a universal metric. Spatial coordinates and times are measured by means of standard measuring rods and standard clocks, that are supposed to be unaffected by stress and temperature-changes. The spatial coordinate-system is of the usual three-right-angled type with equally long units of length marked on the three axes. Every point in this coordinate-system then has a unique set of three coordinates, measured by standard measuring rods, and we may imagine, that there in certain uniformly distributed reference-points are placed signs with the coordinate-values written upon. In every reference-point there shall be a standard clock, the coordinate clock, and the clocks shall be synchronized by means of light-signals, or signals of whatever other type, whose speed of propagation may be supposed to be independent of time, place, and direction.

A travelling particle we can imagine as a space-ship, that carries with it its own standard clock. The captain can then enter into his log-book:" At 12 o'clock, ship's time, we passed the reference-point (1,2,3), where the coordinate clock showed 11."

If the spatial coordinates are called x,y, and z, and the coordinate time t, we can define the contravariant velocity-vector as

$$V^1=\frac{dx}{d\tau}\ ;\ V^2=\frac{dy}{d\tau}\ ;\ V^3=\frac{dz}{d\tau}\ ;\ V^4=\frac{dt}{d\tau} \qquad \textbf{(208)}$$

As in chapter 7 we may then argue, that the metric tensor
will be diagonal with 1 in the three first places and $-c^2$ in the fourth place, where c is a constant of nature with the dimension of a velocity. We can de-dimension the metric by defining the contravariant coordinates as

$$X^1=x\ ;\ X^2=y\ ;\ X^3=z\ ;\ X^4=ct \qquad \textbf{(209)}$$

The contravariant velocity is then defined as

$$U^{\bullet} = \frac{dX^{\bullet}}{d\tau} \tag{210}$$

and the metric has the standard form

$$G_{\bullet\bullet} = G^{\bullet\bullet} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{211}$$

Eq. (210) looks like the relation between a o-level and a x-rate from the simple energy-bond-formalism. We shall therefore introduce a o-storage to contain the contravariant coordinate-vector. This storage must be *cyclic*, i.e. it must not cause any effort in the connected bond, where it as input has the contravariant velocity. We shall start by trying to describe the motion of a free particle in an inertial system, so there must not be any back-action from the coordinate-vector to the circumstances, that influence the particle's velocity.

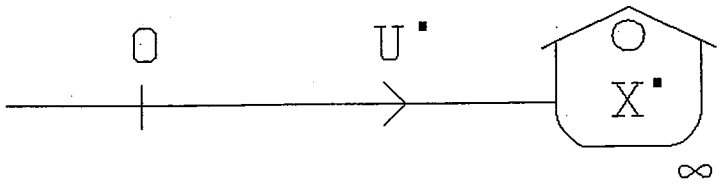


Figure 59. Coordinate-vector as cyclic level.

The contravariant 4-vector, that we now call the particle's velocity, *the 4-velocity*, of course must not be confused with

$$\vec{v} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = c\left(\frac{U^1}{U^4}, \frac{U^2}{U^4}, \frac{U^3}{U^4}\right) \tag{212}$$

the three-dimensional velocity. If the particle is at rest in the inertial system, we have

$$U^1=U^2=U^3=0\ ;\ U^4=c \tag{213}$$

The norm-square, i.e. the 4-velocity's scalar product with itself is thus (no need for complex conjugation, all is real)

$$U^{\bullet}G_{\bullet\bullet}U_{\bullet}=-c^2 \tag{214}$$

If the particle has the 3-velocity $\vec{v}$ 215, the norm-square must be the same. For then the particle will lie still in an inertial system that moves with the velocity $\vec{v}$ 216, relative to the original system S. The transition between the two inertial systems, that both have the metric (211), is a metric-conserving coordinate transformation, i.e. a metric unitary transformer M, and such one will conserve all scalar products, including the norm-square of the velocity. For two arbitrary 4-vectors v and w and their metric unitary transformed editions we have, namely

$$M^{-1}=M^{+} \Rightarrow v'^i w'_i \doteq M^i{}_j v^j M_i{}^k w_k = v^j M^{+}{}_j{}^i M_i{}^k w_k = v^i w_i \tag{215}$$

We therefore get from (212) and (214)

$$\left.\begin{aligned} &(U^1)^2+(U^2)^2+(U^3)^2-(U^4)^2=(U^4)^2\left[\frac{v^2}{c^2}-1\right]=-c^2 \\ &\Rightarrow\ U^4=\frac{c}{\sqrt{1-\frac{v^2}{c^2}}}=\gamma c\ ;\ U^i=\frac{v^i}{\sqrt{1-\frac{v^2}{c^2}}}=\gamma v^i \end{aligned}\right\} \tag{216}$$

We have here chosen the positive solution for U^4, that shall have the value c for v=0. Besides, we have introduced *Møller's convention*, that greek indices (here

iota, ι) run from 1 to 3, while latin indices run from 1 to 4. The three-dimensional vector (v^1, v^2, v^3) is thus the particle's normally defined velocity relative to the inertial system, (v_x, v_y, v_z). Finally, we have again introduced the abbreviation γ for the relativistic time-dilation-factor, as in (198). This factor can now be seen in the true light, for it denotes the ratio between the increase of the coordinate-time and the proper time (comp. (208) and (210)), and as it is greater than 1, it describes the relation, that the standard clock attached to the particle, seen from the inertial system, appear to run too slowly. On the other hand, one cannot say, that the coordinate clocks from the viewpoint of the particle run too fast, for it is not a definite coordinate clock, that enters the definition of the 4-velocity U, but that clock, that is right at the particle, that is all the time a new clock, when the particle moves. The single coordinate clock, seen from the particle, runs just as much too slow as the particle's clock, seen from the inertial system, for the situation is completely symmetric; We might just as well use an inertial system, that (momentarily) lies still relative to the particle, and consider the coordinate clock as a particle that moves with the opposite velocity relative to this inertial system, and this will give the same time dilation, as γ is unaffected by sign-shift of the velocity. The explanation, why the coordinate-time runs before the particle's proper time must then be, that the coordinate clocks appear not to be synchronized. The synchronization procedure, used in S, employs the fact, that light-signals have the same speed in all directions, does not lead to synchronization, seen from the particle's system S'. For if all inertial systems are equally good, an observer in S' must think, that the light-velocity in this system is the same in all directions, and then it can't be it in S. (Though, of course it is; the real reason for the discrepancy is that a fixed coordinate clock in S will move relative to S' during the time of flight of the synchronizing signal).

Let us now look a little closer at the metric unitary transformations that define the transition from one inertial system S to another S' that moves with the velocity relative to S. Let us first for convenience assume that the two coordinate systems have parallel axes, and that the velocity v of S' relative to S goes in the direction of the positive x-axis. It will then do to look at the one spatial coordinate x and the time-coordinate t, so that the transformation-tensor M can be described by 2*2 matrices. We shall further assume that coincidence of the two origos happens when both clocks at the origos show 0. We may then assume that the same linear transformation M that transforms the energy-bond-vectors, also transforms the coordinates:

$$\left.\begin{aligned} U'^{\bullet} = M^{\bullet}_{\bullet} U^{\bullet} \Rightarrow X'^{\bullet} = M^{\bullet}_{\bullet} X^{\bullet} \\ X^{\bullet} = \begin{pmatrix} x \\ ct \end{pmatrix};\ X'^{\bullet} = \begin{pmatrix} x' \\ c't \end{pmatrix};\ M^{\bullet}_{\bullet} = \begin{pmatrix} M^{1}_{1} & M^{1}_{4} \\ M^{4}_{1} & M^{4}_{4} \end{pmatrix} \end{aligned}\right\} \quad \textbf{(217)}$$

When we thus have restricted ourselves to the 1-4 subspace, the metric G.. is given by σ_z. The relevant metric unitary transformers must have the determinant 1 of their 1- and 2-variants, for if the velocity v is 0, these matrices shall be the unit matrix. The determinant of the 1- or 2-variant of a metric unitary and real transformer can only be 1 or -1, because

$$\det(M^{\bullet}_{\bullet}) = \det(M^{+\bullet}_{\bullet}) = \det(M^{-1\bullet}_{\bullet}) \quad \textbf{(218)}$$

and, as the matrix can be generated by a continuous change of the unit matrix, the determinant can only be 1. We then have:

$$\left.\begin{aligned} M^{+\bullet}_{\bullet} = \begin{pmatrix} M_{1}^{1} & M_{4}^{1} \\ M_{1}^{4} & M_{4}^{4} \end{pmatrix} = \begin{pmatrix} M^{1}_{1} & -M^{4}_{1} \\ -M^{1}_{4} & M^{4}_{4} \end{pmatrix} \\ = M^{-1\bullet}_{\bullet} = \begin{pmatrix} M^{4}_{4} & -M^{1}_{4} \\ -M^{4}_{1} & M^{1}_{1} \end{pmatrix} \end{aligned}\right\} \quad \textbf{(219)}$$

that is

$$M^{1}_{1} = M^{4}_{4} \wedge M^{1}_{4} = M^{4}_{1} \wedge M^{1}_{1} M^{4}_{4} - M^{1}_{4} M^{4}_{1} = 1 \quad \textbf{(220)}$$

And therefore M can be expressed on the form (comp. (149))

$$M^{\bullet}_{\bullet} = L(\chi)^{\bullet}_{\bullet} = \begin{pmatrix} \cosh\chi & \sinh\chi \\ \sinh\chi & \cosh\chi \end{pmatrix} \qquad \textbf{(221)}$$

The quantity χ (sometimes called *the rapidity*) is the parameter for that Lie-group that is called the special Lorentz group. If v=0 then $\chi = 0$. Further, it is seen, that the to (221) reciprocal matrix is found by inserting the parameter-value $-\chi$. If we follow the origo in S', is x'=0 and x=vt, and we then have from (217) and (221):

$$0 = \cosh\chi\, vt + \sinh\chi\, ct \;\Rightarrow\; \tanh\chi = -\frac{v}{c} \qquad \textbf{(222)}$$

With the sign γ from (221) ($\gamma = \cosh\chi$) we then find the expression for the special Lorentz transformation

$$x' = \gamma(x - vt)\;;\; t' = \gamma\left(t - \frac{xv}{c^2}\right) \qquad \textbf{(223)}$$

The special Lorentz-group (221) is commutative (Abelian) and has further the nice property, that

$$L(\chi_1)^{\bullet}_{\bullet} L(\chi_2)^{\bullet}_{\bullet} = L(\chi_1 + \chi_2)^{\bullet}_{\bullet} \qquad \textbf{(224)}$$

As the protensor h =δ for the standard metric, we have

$$\breve{L}(\chi)^{\bullet}_{\bullet} = \delta^{\bullet\bullet} L(\chi)^{\bullet}_{\bullet} \delta_{\bullet\bullet} = L(\chi)^{\bullet}_{\bullet} = L(-\chi)^{\bullet}_{\bullet} = L^{-1}(\chi)^{\bullet}_{\bullet} \qquad \textbf{(225)}$$

From where we get the perhaps somewhat surprising result, that the metric unitary transformer $L(\chi)$ is icon-symmetric (comp. (133)).

The property (224) suggests, that the Lorentz-matrix (221) must be expressible by means of a generator L and the parameter χ on the form

$$L(\chi)^{\bullet}_{\bullet} = \exp(\chi L^{\bullet}_{\bullet}) = \sum_{n=0}^{\infty} \frac{(\chi L^{\bullet}_{\bullet})^n}{n!} \tag{226}$$

This formula will work, if the generator is given by

$$L^{\bullet}_{\bullet} = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{227}$$

A generator for a 1-parameter Lie-group is generally defined as in (226). The connection between the hermiticity-properties of the generator and the generated Lie-group can be summed up as follows (proof left to the reader):

If the generator is 1-/metric hermitean, the group is 1-/metric hermitean

If the generator is 1-/metric antihermitean, the group is 1-/metric unitary.

For the special Lorentz-group the generator is 1-hermitean and metric anti-hermitean, so the group is 1-hermitean an metric unitary.

As all members of the Lie-group have the determinant 1, it must be valid that *the trace* of the generator, i.e. the sum of its diagonal elements, is 0. We can, namely, re-write (226) to

$$L(\chi)^{\bullet}_{\bullet} = \lim_{n\to\infty} \left(1 + \frac{\chi}{n} L^{\bullet}_{\bullet}\right)^n \tag{228}$$

(somewhat sloppily we allow ourselves to write 1 in the parenthesis, where it, properly, should have been the unit-matrix). Thus, we get

$$\left.\begin{aligned} \det[L(\chi)^{\bullet}{}_{\bullet}] &= \lim_{n\to\infty}\left[\det\left(1+\frac{\chi}{n}L^{\bullet}{}_{\bullet}\right)\right]^n = \\ \lim_{n\to\infty}&\left(1+\frac{\chi}{n}\mathrm{sp}(L^{\bullet}{}_{\bullet})+O(\tfrac{1}{n^2})\right)^n = \exp[\chi\ \mathrm{sp}(L^{\bullet}{}_{\bullet})] \end{aligned}\right\} \qquad \textbf{(229)}$$

We can express (228) with the diagram

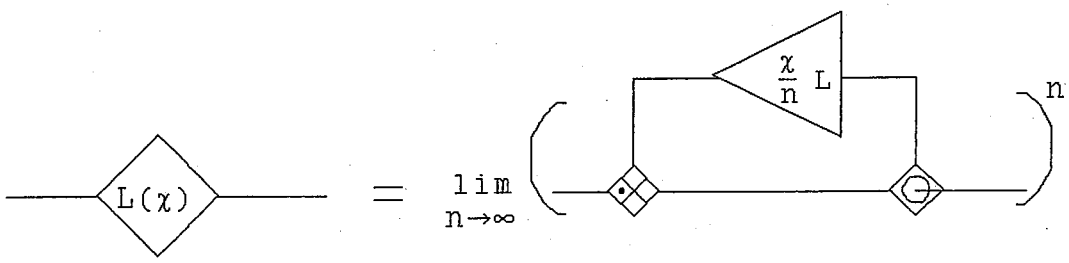


Figure 60. Lie-group element and generator.

In general we cannot presume, that a metric unitary transformer, so generated, is icon-symmetric. The generator L for the Lorentz-group is not icon-symmetric.

From (222) and (224) we may derive the relativistic law for addition of velocities. If we have one more inertial system S", moving with velocity w relative to S' (still in the direction of the positive x-axis), then the transition from S' to S" can be described with a metric unitary transformer, corresponding to (221), whose rapidity-parameter χ' is given by w like χ is given by v in (222). The transition from S to S" is the, according to (224) given by the parameter $\chi+\chi'$, so the velocity of S" relative to S must be

$$-c\tanh(\chi+\chi') = \frac{v+w}{1+\frac{vw}{c^2}} \qquad \textbf{(230)}$$

A consequence of this formula is, that one never by successive addition of small velocities can reach velocities greater than or equal to c.

Until now we have only looked at one special type of Lorentz-transformation. Let us now try to describe the total group. The Lorentz-group describe all possible metric unitary transformations from one inertial system to another, so it includes also usual rotations of the coordinate system, without relative motion of the systems. Such transformations have no matrix elements that mix the spatial and the temporal components of a 4-vector, and they will therefore be 1-unitary (or 2-unitary, which is the same) at the same time as being metric unitary. E.g. a rotation of the angle φ around the z-axis will correspond to the transformation matrix

$$D_z(\phi)^{\bullet}{}_{\bullet} = \begin{pmatrix} \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{231}$$

This is again a 1-parameter Lie-group, where all the members have the determinant 1, and as before we can generate it with a generator having the trace 0. Such a generator must then be 1-antihermitean in order for it to be metric antihermitean, as it only involves the three spatial coordinates. The generated group is both 1-unitary and metric unitary. It is easy to find the generator, because we have:

$$\left.\begin{aligned} D_z{}^{\bullet}{}_{\bullet} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \\ (D_z{}^{\bullet}{}_{\bullet})^2 &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \; (D_z{}^{\bullet}{}_{\bullet})^3 = -D_z{}^{\bullet}{}_{\bullet} \end{aligned}\right\} \tag{232}$$

and thereby

$$\left.\begin{array}{c} \exp(\phi\, D_z^{\bullet}{}_{\bullet}) \doteq \sum_{n=0}^{\infty} \frac{(\phi\, D_z^{\bullet}{}_{\bullet})^n}{n!} = \\ 1 + \left(\sum_{n=0}^{\infty} \frac{(-1)^n \phi^{2n+1}}{(2n+1)!}\right) D_z^{\bullet}{}_{\bullet} - \left(\sum_{n=1}^{\infty} \frac{(-\phi^2)^n}{(2n)!}\right) (D_z^{\bullet}{}_{\bullet})^2 \\ = 1 + \sin\phi\, D_z^{\bullet}{}_{\bullet} - (\cos\phi - 1)(D_z^{\bullet}{}_{\bullet})^2 = D_z(\phi)^{\bullet}{}_{\bullet} \end{array}\right\} \quad \textbf{(233)}$$

$$D_{\bar{n}} = n_x D_x + n_y D_y + n_z D_z \quad \textbf{(234)}$$

and we have the commutator-relations (index-dots omitted)

(we are now again writing 1 where there ought to be the unit matrix $G^{\bullet}{}_{\bullet}$ 235)
Correspondingly we can get generators for rotations about the x-axis and the y-axis. The two other rotation generators are

$$D_x^{\bullet}{}_{\bullet} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \; D_y^{\bullet}{}_{\bullet} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \textbf{(234)}$$

and we have the commutator-relations (index-dots omitted)

$$D_x = [D_y, D_z] \; ; \; D_y = [D_z, D_x] \; ; \; D_z = [D_x, D_y] \quad \textbf{(235)}$$

By means of these three rotation-generators for the three coordinate axes we can make a generator for rotation about every unit vector

$$D_{\bar{n}} = n_x D_x + n_y D_y + n_z D_z \qquad \textbf{(236)}$$

The direction of the unit vector is described by two parameters, e.g. longitude and latitude on the unit sphere. A rotation through an arbitrary angle φ about the unit vector is then generated by

$$D_{\bar{n}}(\phi) = \exp(\phi D_{\bar{n}}) \qquad \textbf{(237)}$$

In this way all possible rotations of the rigid coordinate system are generated as a three-parameter Lie-group.

Looking now at the proper Lorentz-transformations, where the one inertial system moves relative to the other with parallel axes, then we can in a similar way find three generators, describing motions in the x-, y-, and z-directions. We have earlier looked at the first of these generators, but now we write them all out as 4*4 matrices (understanding it to be the 2-variants):

$$L_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} ; \; L_y = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} ; \; L_z = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \textbf{(238)}$$

These have the commutator-relations

$$[L_x, L_y] = D_z \; ; \; [L_y, L_z] = D_x \; ; \; [L_z, L_x] = -D_y \qquad \textbf{(239)}$$

The L-generators have a vector-character, like the Ds. We can make a scalar product of (L_x, L_y, L_z) with a unit vector (n_x, n_y, n_z) like in (236) and thereby

construct a generator for motion in the direction of this unit vector, so that an arbitrary Lorentz transformation without rotation (axis-parallel motion) can be made after the recipe

$$L_{\bar{n}}(\chi) = \exp(\chi L_{\bar{n}}) \tag{240}$$

If we compose two Lorentz transformations without rotation, going in two different directions, it will turn out, that the resulting transformation is *not* a Lorentz transformation without rotation. In other words: Even though the second coordinate system has its axes parallel to the first's, and the third with the second's, then the third coordinate system's axes will not be parallel to the first's. This phenomenon, known as the *Thomas precession* (see e.g. Møller) is connected with the fact, that the commutator relations of the Ls (239) involve the Ds.

The 6 generators that are now described, span together the space of metric anti-hermitean 4*4 matrices. The total group of *proper* Lorentz transformations (having determinant 1) can therefore be defined as a 6-parameter Lie-group by the construction

$$\left.\begin{aligned} L(\phi_x,\phi_y,\phi_z,\chi_x,\chi_y,\chi_z) &= \exp(K), \text{ where} \\ K &= \phi_x D_x + \phi_y D_y + \phi_z D_z + \chi_x L_x + \chi_y L_y + \chi_z L_z \end{aligned}\right\} \tag{241}$$

Thus, we have sketched the basic *kinematic* of SRT.

In order to treat the *dynamics* in the motion of a particle, we have to discuss, what a force is, and how it changes the velocity of the particle. In the energy-bond-description of classical (non-relativistic) dynamics Newton's second law is reticulated by means of a linear x-storage with an inertance, that is the mass of the particle, an x-level, that is the momentum of the particle, and an effort-input, that is the force on the particle. The flow-output from the x-storage is the particle's velocity. The simplest possible generalization of this picture to SRT proceeds to use the same reticulation in the four-dimensional energy-bond-formulation. We

must therefore start by defining a linear x-storage for general vector-bonds.

A linear storage has an energy-function, that is quadratic in the level variable, and the output from the storage is determined by differentiation of the energy-function with respect to the level-variable. The input-rate to an x-storage is an effort-vector, that is the time-derivative of the x-level. If we let the effort (the force) be covariant, then the x-level (the momentum) must also be a covariant vector. The derivative of the scalar energy-function with respect to the momentum, i.e. the output-flow (the velocity) will then be contravariant. If we call the momentum P., the energy-function ε(P.), and the force F. we have namely

$$\sum_{i=1}^{4}\frac{\partial \varepsilon}{\partial P_i}F_i=\sum_{i=1}^{4}\frac{\partial \varepsilon}{\partial P_i}\frac{\mathrm{d}P_i}{\mathrm{d}\tau}=\mathrm{w}=\mathrm{U}^{\mathrm{i}}\mathrm{F}_{\mathrm{i}} \qquad \textbf{(242)}$$

where w is the energy flow to the storage, being a scalar, that is expressed as a sum of products of the covariant effort-coordinates with the contravariant flow-coordinates.
The connection between the energy-function and the output-flow for the linear storage is then

$$\left.\begin{aligned}\varepsilon(P_\bullet)&=\frac{1}{2m}P_\bullet G^{\bullet\bullet}P_\bullet\\ U^\bullet&=\frac{\partial \varepsilon}{\partial P_\bullet}=\frac{1}{m}G^{\bullet\bullet}P_\bullet=\frac{1}{m}P^\bullet\end{aligned}\right\} \qquad \textbf{(243)}$$

For a *free particle* the force is 0, and we may employ the following reticulation

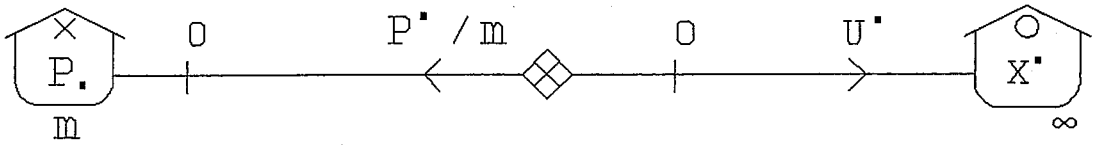


Figure 61. Free particle

We have seen, that the particle's 4-velocity is subject to the restriction (214), that its norm-square *must* have the value $-c^2$. The velocity-4-vector is bound to a hyper-surface in the 4-space, called a *pseudo-sphere* with the equation (214). It is therefore not possible to add 4-velocities by using o-junctions; if one does that, the resulting flow-vector cannot be interpreted as a possible particle's 4-velocity. We have to realize, that multiport o-junctions can't exist in the relativistic particle-dynamics. (This, though, is only true for the four-dimensional vector-bonds. If we reticulate out on lower dimensional bonds, e.g. make a scalar version of a Lorentz-transformer, there will, of course appear o-junctions). On the contrary, the x-junction has an important function in the dynamics, which is suggested by the appearance of a two-port x-junction in figure 61. If the particle is not free, i.e. if a force influences it, this may enter the picture via a third port on the x-junction.

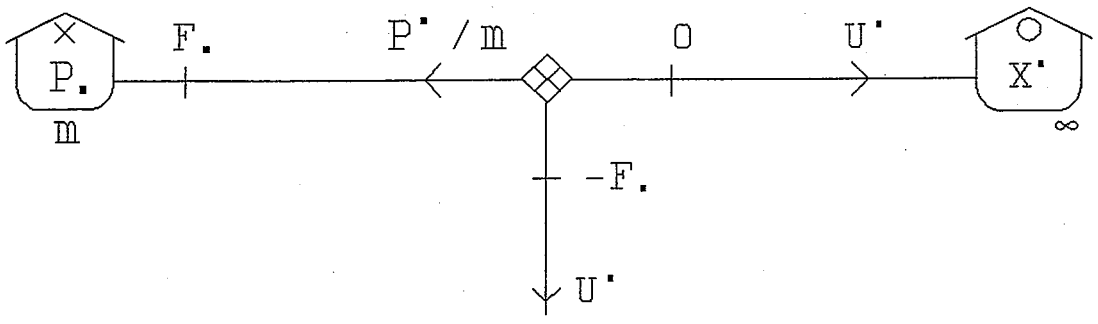


Figure 62. Particle influenced by a force.

From (243) and (214) we find the following expression for the energy-function of the x-storage

$$\varepsilon = \tfrac{1}{2} m U^{\bullet} G_{\bullet\bullet} U^{\bullet} = -\tfrac{1}{2} m c^2 \qquad \textbf{(244)}$$

This storage-function, thus, is something completely different from what we normally understand by the energy of the particle. If the mass of the particle is constant, ε must keep a constant value. Expressing ε by the contravariant 4-momentum, we get

$$\varepsilon = \frac{1}{2m} P^{\bullet} G_{\bullet\bullet} P^{\bullet} = \frac{1}{2m} [\, \vec{p}^{\,2} - (P^4)^2 \,] \qquad \textbf{(245)}$$

From this expression, where we have introduced the 3-momentum, that for small velocities is the same as the Newtonian momentum, we find

$$\left.\begin{aligned} P^4 &= \sqrt{m^2 c^2 + \vec{p}^{\,2}} = \\ mc\sqrt{1 + \gamma^2 \frac{v^2}{c^2}} &= \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mc \end{aligned}\right\} \qquad \textbf{(246)}$$

This quantity can then be identified as the particle's energy E, divided by c. The expression for E and its series-expansion for small velocities give namely

$$E = c\, P^4 = \frac{m c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \approx mc^2 + \tfrac{1}{2} m v^2 \qquad \textbf{(247)}$$

That is the well known Einstein-expression for the particle's rest-energy plus the usual kinetic energy.

As the storage function ε shall be constant (for constant m), the scalar product of flow and effort in the new bond is zero. We have therefore

$$F_4 = -\frac{F_\lambda U^\lambda}{U^4} = -\frac{1}{c} F_\lambda v^\lambda = -\frac{1}{c}\frac{dE}{d\tau} \qquad \textbf{(248)}$$

The rate of work of the force-field is thus

$$\frac{dE}{dt} = \frac{dE}{d\tau}\frac{d\tau}{dt} = \frac{1}{\gamma} F_\lambda v^\lambda \qquad \textbf{(249)}$$

which suggests, that the Newton-force is given by the first three components of the 4-force, divided by the time-dilation-factor γ. For we have also:

$$\frac{dP_\lambda}{d\tau} = m\frac{d(\gamma v_\lambda)}{d\tau} = F_\lambda \Rightarrow m\frac{dv_\lambda}{d\tau} = \frac{F_\lambda}{\gamma} - m v_\lambda \frac{d\gamma}{d\tau} \qquad \textbf{(250)}$$

which shows, that in the momentary rest-system, where v_λ is zero, and where the coordinate-time follows the proper time of the particle. Newton's second law is valid, if the force is F_λ / γ.
Let us calculate the motion of the particle, when it starts from rest at θ=t=0 and is influenced by a constant force, i.e. when it has a constant acceleration relative to the momentary rest-system. We get

$$a_\lambda = \frac{1}{m\gamma}\frac{\mathrm{d}\,p_\lambda}{\mathrm{d}\,\tau} = \frac{\mathrm{d}(\gamma\, v_\lambda)}{\mathrm{dt}} \qquad \textbf{(251)}$$

If the force and the acceleration is in the direction of the x-axis, we get for the velocity in the x-direction:

$$\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} = at \Rightarrow v = \frac{at}{\sqrt{1+\left(\frac{at}{c}\right)^2}} \qquad \textbf{(252)}$$

It is seen, that the velocity goes asymptotically to c for $\theta\ \tau \rightarrow \infty$. by further integration we find x as a function of the proper time:

$$x = \frac{c^2}{a}\int_0^{\frac{at}{c}} \frac{u\ \mathrm{du}}{\sqrt{1+u^2}} = \frac{c^2}{a}\left[\sqrt{1+\left(\frac{at}{c}\right)^2} - 1\right] \qquad \textbf{(253)}$$

The graph of x as a function of t is a hyperbola with the line x=ct as asymptote.

The simplest relativistic force-law corresponds to a linear 1-port connected to the junction in figure 62. As the energy-function for the storage shall be conserved (-½mc^2), this 1-port must be metric anti-hermitean, i.e. it must be a *dynamo,* comp. figure 38. As we shall see, the motion of a charged particle in an electromagnetic field can be described in this way.

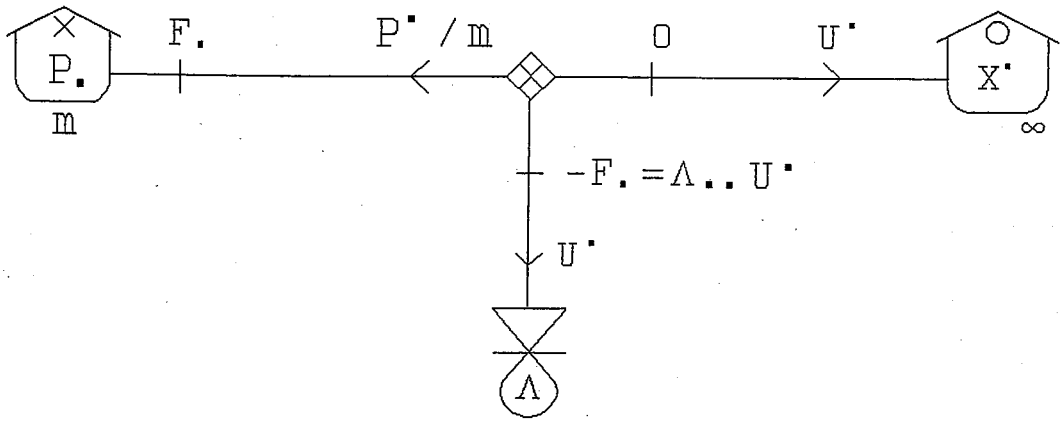


Figure 63. Motion governed by a dynamo.

If the particle has the charge q and is moving in an electromagnetic field $\overline{E}$ and the magnetic induction $\overline{B}$, the anti-hermitean matrix $\Lambda..$ is given by

$$\Lambda.. = q \begin{pmatrix} 0 & -B_z & B_y & -\frac{E_x}{c} \\ B_z & 0 & -B_x & -\frac{E_y}{c} \\ -B_y & B_x & 0 & -\frac{E_z}{c} \\ \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} & 0 \end{pmatrix} \qquad \textbf{(254)}$$

For the force on the particle then becomes

$$K_\kappa = \frac{1}{\gamma} F_\kappa = -\frac{1}{\gamma} \Lambda_{\kappa\bullet} U^\bullet \Rightarrow \vec{K} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \textbf{(255)}$$

which is the well-known Lorentz force. The simple way, this expression enters, suggests, that electrodynamics, so to speak is born relativistic, and this was also

the point of departure for Einstein's article "Zur Elektrodynamik bewegter Körper" from 1905.

However, a field description requires quite another viewpoint than the particle mechanics that is based on our possibility of following the single particle in its motion.

For fields we must use the Euler-viewpoint, which refers to a fixed coordinate system, where the time is the coordinate-time, measured by standard clocks. We can imagine the three-dimensional coordinate system partitioned in small cells, and we must then for every cell keep track of, how field-quantities, like, e.g. the charge density vary as functions of the coordinate time. This time, t, enters as ct in the fourth component of the contravariant coordinate-vector. We can define a four-dimensional differential operator, which acts as a covariant vector in the following way:

$$\partial_{\bullet} = \frac{\partial}{\partial X^{\bullet}} \tag{256}$$

If we have a lot of charged particles, we can describe them by a charge density ρ and a current-density-vector and with these quantities we may form a 4-vector, the contravariant 4-current-density:

$$J^{\bullet} = (i_x, i_y, i_z, c\rho) \tag{257}$$

The continuity equation, expressing charge-conservation

$$\nabla \cdot \vec{i} + \frac{\partial \rho}{\partial t} = 0 \tag{258}$$

may then be expressed on the invariant tensor-form

$$\partial_{\bullet} J^{\bullet} = 0 \tag{259}$$

Looking now at the Maxwell equations for the magnetic field-strength $\overline{H}$ and the electric displacement $\overline{D}$

$$\left.\begin{aligned} \nabla x\vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{i} \\ \nabla \bullet \vec{D} &= \rho \end{aligned}\right\} \tag{260}$$

we may express it in a similar compact way. We introduce a field-tensor whose 3-variant is

$$\Psi^{\bullet\bullet} = \begin{pmatrix} 0 & -H_z & H_y & cD_x \\ H_z & 0 & -H_x & cD_y \\ -H_y & H_x & 0 & cD_z \\ -cD_x & -cD_y & -cD_z & 0 \end{pmatrix} \tag{261}$$

and may then write (260) on the form

$$\partial . \Psi_{..} = J. \tag{262}$$

In vacuum we have

$$\vec{B} = \mu_0 \vec{H} \; ; \; \vec{D} = \varepsilon_0 \vec{E} \tag{263}$$

We see then, that the tensor Ψ (261) and the tensor Λ (254) look very much alike. In fact so much, that they must be proportional, if they are to be tensors at all, i.e. transform by Lorentz transformations, when we change inertial system. This

proportionality does only hold, if the vacuum-constants in (263) stand in a certain relation to "the metric velocity" c:

$$\mu_0 \varepsilon_0 = \frac{1}{c^2} \Rightarrow \Lambda = q\, \mu_0 \Psi \qquad \textbf{(264)}$$

By this we become able to identify c as the velocity of light in vacuum. The two other Maxwell equations

$$\left.\begin{aligned} \nabla x \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \\ \nabla \bullet \vec{B} = 0 \end{aligned}\right\} \qquad \textbf{(265)}$$

can be expressed on a tensor-form like (262) with a 0 on the right side. This, however, we shall not do, but we shall instead introduce the vector-potential and the scalar potential by the definitions

$$\vec{B} = \nabla x \vec{A} \;\; ; \;\; \vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t} \qquad \textbf{(266)}$$

whereby the Maxwell equations (265) are automatically satisfied. We now introduce the 4-vector potential, whose contravariant form is

$$A^{\bullet} = (A_x, A_y, A_z, \frac{\phi}{c}) \qquad \textbf{(267)}$$

The field tensor Ψ can then be expressed on the form

$$\mu_0 \Psi_{ij} = \partial_j A_i - \partial_i A_j \qquad \textbf{(268)}$$

The potential vector A is not uniquely determined, but can be changed by a *gauge transformation*. We shall employ the so called Lorentx gauge, defined by

$$\partial_\bullet A^\bullet = \partial^\bullet A_\bullet = 0 \sim \nabla \bullet \vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \qquad \textbf{(269)}$$

The electromagnetic wave-equation for vacuum then gets its simplest form, when we in (262) with zero on the right side use (268) and (269):

$$\mu_0 \partial^i \Psi_{ij} = \partial^i (\partial_j A_i - \partial_i A_j) = -(\partial^\bullet \partial_\bullet) A_j = 0 \qquad \textbf{(270)}$$

We have here introduced the d'Alembert operator, that 2. order differential operator, that is the Lorentz-metric's generalized edition of the Laplace operator of euclidean metric:

$$\square = \partial^\bullet \partial_\bullet = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \qquad \textbf{(271)}$$

Field-descriptions that involve differential operators, do not straightforwardly let themselves be reticulated out on four-dimensional vector-bonds. The merit of the vector-bonds in this context lies in their definition of the basic metric, that is fundamental to the use of the tensor formalism in the field description. The particle dynamics in vector-bond-formulation is the fundament, that the field description is built upon, because the meaning of the field symbols is defined from the iconic conception of a test-particle as in the diagram figure 63 and the associated formula (254).

When we are going to reticulate field equations out, we must use scalar bonds for building up a dynamic unit cell, which then by repetition in the three-dimensional

euclidean space can reduce the partial differential equations in space and time to a set of ordinary differential equations in time. It won't do to introduce unit cells and lattice-structure for "the four-dimensional space", for there is no four-dimensional geometric space. The merit of the vector-bond-formalism in relativity theory may just be, that it frees us for geometric concepts of a "four-dimensional space-time continuum", where dynamic phenomena are made into geometric patterns of "world-lines" without development. There is a simple reason, why the four-dimensional tensor formalism, based on the Lorentz metric, is so fundamental, and this reason is the structure of the vector-bond, that describes the dynamics of a test-particle. The Euler-viewpoint on fields where we stand still in a coordinate system and keeps watch on the development of a field by means of partial differential equations or periodically repeated energy bond cells is always based on the Lagrange-viewpoint, where we follow the single particle in its motion.
This holds equally well for hydrodynamics and the general relativity theory. In quantum field theory, although, one easily gets the impression, that the field description is the fundamental, that the particle-concept must be based on a field-equation, but in the last instance the quantum mechanical field equations are semantically rooted in classical concepts and the correspondence-principle, as Bohr always maintained.

Let us see how the "born relativistic" electromagnetic field equations for vacuum look like in scalar energy bond formulation. We may start by looking at a situation, where the electric field goes in the x-direction and the magnetic induction in the y-direction. Such a situation can occur when we have a plane polarized wave, propagating in the x-direction. Both fields will then only depend on z and t.
From (260) (without current and charge) and (265) we get:

$$\frac{\partial D_x}{\partial t} = -\frac{\partial H_y}{\partial z} \; ; \; \frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} \qquad \textbf{(272)}$$

This form suggests, that the quantities D and B must be attached to level-variables, while E and H must be attached to rates. As the E-field is proportional to a force on a test-charge, i.e. an effort, E and D must be o-variables, while B and

H are x-variables. We must now express the partial differential equations (272) as ordinary differential equations in time. We therefore introduce *integrated levels*, attached to a cubic cell, that goes from x to x+Δx, from y to y+Δy, and from z to z+Δz. As o-level enters the electric flux $D_x\Delta y\Delta z$ and as x-level the magnetic flux $B_y\Delta x\Delta x$.

We must then think of the side-lengths Δx,Δy, and Δz as small compared to the wavelength, such that the field quantities are approximately constant over the box. We need not think of the box as infinitesimal, but rather as a *finite element*, that has to give the best possible approximation to the partial differential equations for a finite size. If we let the electric flux be defined by the field value in z, we may approximate the spatial differential equation in the best way by the following difference equation

$$\frac{d}{dt}[D_x(z)\Delta y\Delta z]=H_y(z-\tfrac{\Delta z}{2})\Delta y-H_y(z+\tfrac{\Delta z}{2})\Delta y \qquad \textbf{(273)}$$

and a corresponding equation for the level variable $B_y\Delta z\Delta x$.

As the rate variable $H_y\Delta y$ shall be attached to the z-values x-Δz/2 and z+Δz/2, the magnetic levels must also be attached to these points midway between the electric levels. In this way we arrive at the following reticulation:

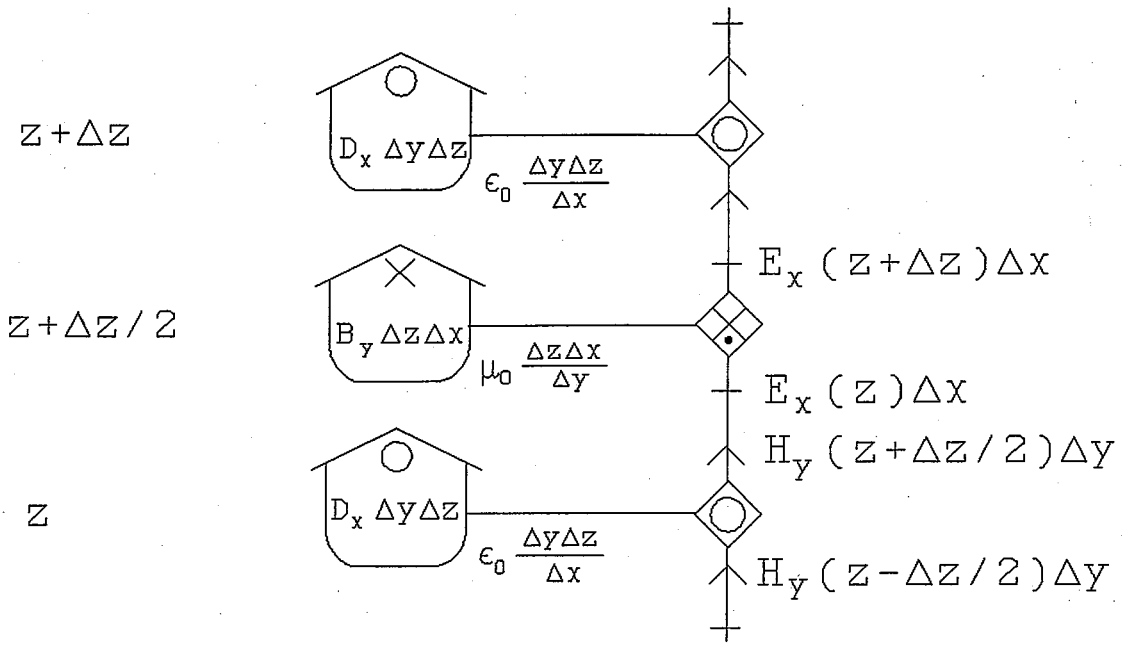


Figure 64. Section of plane wave in the z-direction.

In a similar way we can reticulate the whole set of Maxwell's equations for vacuum as a three-dimensional lattice-structure, where a definite finite element, the unit cell, is repeated in all three dimensions. It is, though, not so easy to draw the unit cell, so on the following figure we have left out the storages and only shown the junctions and the energy bonds. As compensation, the whole is shown in three-dimensional projection, such that every junction is parallel to the plane, witch is perpendicular to the flux-variable in the attached (invisible) storage. The storage capacities all scale proportional to the area of this surface, i.e. as $\Delta x \Delta y$ for a z-flux, and inversely proportional to the length of the cell in the field-direction. In the drawing the Δs are replaced with ds for showing, that the partial differential equations appear in the limit, where the side-lengths are infinitesimal.

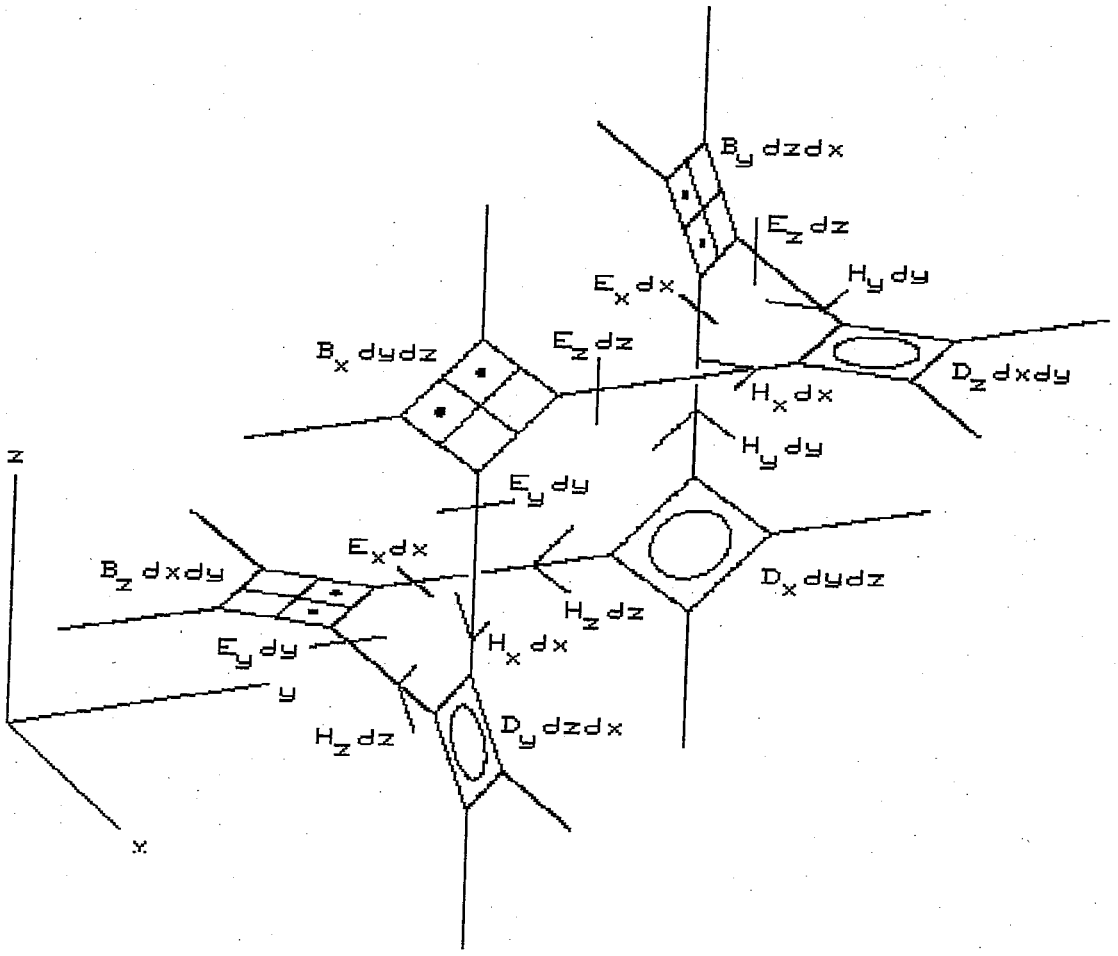


Figure 65. Maxwell's equations for vacuum (unit cell).

$$\vec{S} = \vec{E} \times \vec{H} \qquad \textbf{(274)}$$

The signal-signs for the E- and H- fields in the bonds are also drawn as to show the direction of the relevant field-component. If we look, e.g., at the two bonds that go in the z-direction, one of them (the one shown in figure 64) contains the effort $E_x dx$ and the flow $H_y dy$, and the other has the effort $E_y dy$ and the flow $H_x dx$. The total energy flow in the z-direction is thus

$$(E_x H_y - E_y H_x) dxdy = S_z dxdy \qquad \textbf{(274)}$$

which is the area-element of the unit cell perpendicular to the z-direction times the z-component of the energy-flow-density, the so called *Poynting vector*:

$$\vec{S} = \vec{E} \times \vec{H} \qquad \textbf{(275)}$$

The total energy in the storages of the unit cell is

$$\left(\frac{1}{2\varepsilon_0} \vec{D}^2 + \frac{1}{2\mu_0} \vec{B}^2 \right) dxdydz = u\, dxdydz \qquad \textbf{(276)}$$

where u is the energy density. Energy conservation is expressed by the continuity equation

$$(E_x H_y - E_y H_x) dxdy = S_z dxdy \qquad \textbf{(277)}$$

that corresponds to (258) for the charge. Like in (257) we can then construct a four-vector of Poynting's vector and the energy density, such that the continuity equation can be expressed on tensor-form like (259).

$$\Delta \cdot \vec{S} + \frac{\partial u}{\partial t} = 0 \qquad \textbf{(278)}$$

When one sees a field-reticulation like figure 65, it is not immediately visible, that it is relativistic invariant, i.e. has the same form in all inertial systems, and that it all may be expressed in compact tensor-form. A scalar reticulation like this will in most cases be the final goal for energy-bond-semiotic reflections, as it allows numerical solution of the partial differential equations with standard methods, and one may, of course, reach it without all considerations about vector-bonds. The justification of the vector bond formalism lies in the feature, that it can account for a logic behind equations, and thus give the model a greater theoretical depth. The relativistic particle-mechanics as shown in figure 63 contains the explanation, why there are exactly 6 independent electromagnetic field-quantities, because that is the number of different elements in an anti-hermitean 4*4 matrix (the dynamo Λ).

10. Tellegen, lagrange, and Hamilton

As in the simple formalism we name a certain part of an energy-bond model to be the so called *skeleton diagram.* (comp. text # 8, chapter 5 ff). This is the *topological* part of the model, that accounts for, how the various parts hang together. All the junctions in the model belong to the skeleton diagram, but also transformers and metric transducers ought to be counted within. Characteristic for the components, that belong to the skeleton diagram, is that they all have two or more ports, and that each of them carries with it a pure flow-relation, that is independent of the efforts, and a pure effort-relation, that is independent of the flows. Gyrators do not belong to the skeleton diagram. If two systems have the same skeleton diagram, including the symbolic expressions for the transformers, we say that they are *topologically equivalent.* Looking at the skeleton diagram as a unit, it will be connected to other components(that we disregard at present) through a set of ports, of which some have flow-input and some have effort-input. In the figure below we have placed the flow-ports to the left, numbered by a latin index, while effort-ports are to the right, numbered by greek index. We indicate all ports with orientation towards the skeleton diagram and choose mixed variances with covariant efforts and contravariant flows.

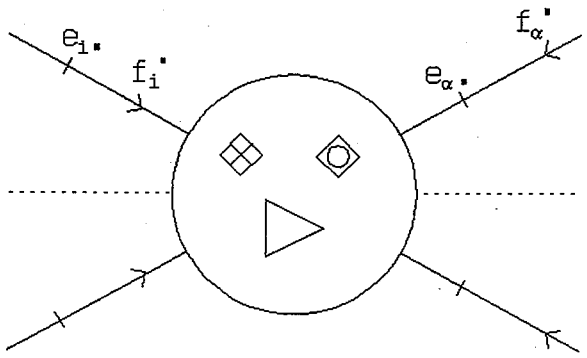


Figure 66. The skeleton diagram.

The bonds in figure 66 need not be of the same type; they can be different with respect to dimension and metric. We can make more bonds fuse together to one, if we just maintain, that flow-relations and effort-relations are not to be mixed. That is, i-bonds may be put together with i-bonds (flow-input) and α-bonds with α-bonds (effort-input) but never an i-bond with a α-bond. In this way we can end

with a situation, where there is only two bonds left, namely the collection of the original i-bonds and the collection of the original α-bonds. The skeleton diagram has then become a transformer:

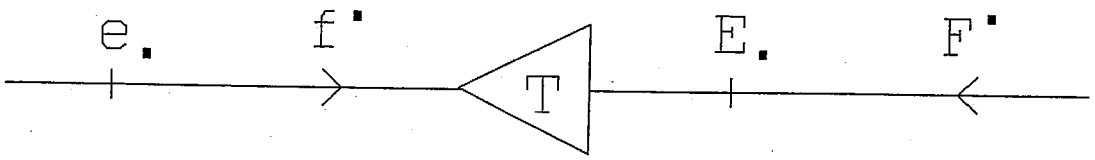


Figure 67. Reduction of skeleton diagram to a single transformer by fusion of bonds.

In this way we get a very broad interpretation of "the topological properties", which the skeleton diagram describes. Likewise, the rule, that gyrators do not belong, has become less severe, for a transformer in a non-euclidean standard metric will include gyrators, when it is reticulated out on scalar bonds. E.g. the figure below shows a transformer-reticulation for a three-dimensional standard metric with 1 in the two first diagonal places and -1 in the third place.

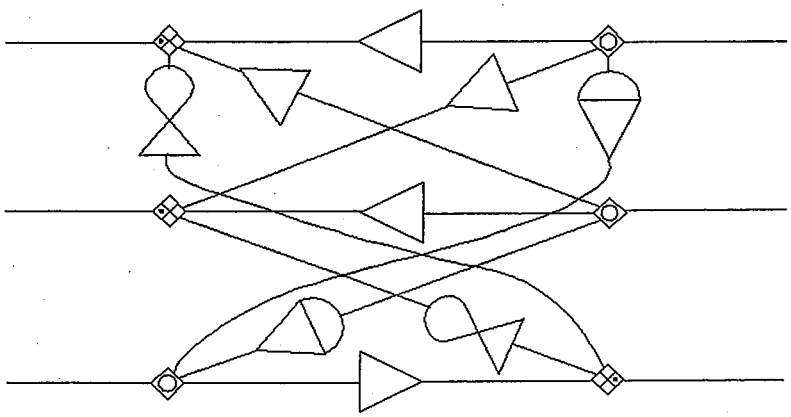


Figure 68. Scalar reticulation of three-dimensional transformer in non-euclidean standard metric.

The two bonds on figure 67 may have different dimension and metric, but in that case the transformer is not a proper transformer, but a kind of transducer. It is not a metric transducer, because flow and effort in the secondary bond are not necessarily produced by the same transformation of the flow and effort in the primary bond, so we shall treat it as a transformer, but must take into account, that its tensor is an exotensor, i.e. involves two different metrics. If the two bonds

have different dimension, the exotensor connected to the transducer will have no reciprocal, so icon-reversal is not possible. Variance-shift and duality of the exotensor then involve two different metric tensors and protensors, as described in chapter 8. If we have the protensor h' on the secondary side, we may define

$$t = h' T \tag{278}$$

and we have then

$$\begin{pmatrix} e_{\bullet} \\ F^{\bullet} \end{pmatrix} = \begin{pmatrix} 0 & t_{\bullet}^{\bullet} \\ -t^{+\bullet}_{\bullet} & 0 \end{pmatrix} \begin{pmatrix} f^{\bullet} \\ E_{\bullet} \end{pmatrix} \tag{279}$$

where the bold 0s stand for quadratic null-matrices, while the ts in general are rectangular matrices with different numbers of rows and columns.
The formula (279) expresses in the most compact way, that effort-relations and flow-relations are completely separated in the skeleton diagram. We may imagine, that the efforts e and E belong to one physical situation (1) and the flows f and F to another situation (2). Maybe (1) and (2) refer to two different instants of time in the development of the same system, or maybe they belong to two entirely different systems, that are topologically equivalent, i.e. have the same skeleton diagram. In all cases we have:

$$\left.\begin{aligned} e^{(1)*}_{\bullet} f^{(2)\bullet} = (t_{\bullet}^{\bullet} E^{(1)\bullet})^{*} f^{(2)\bullet} = E^{(1)*}_{\bullet} t^{+\bullet}_{\bullet} f^{(2)\bullet} \\ E^{(1)*}_{\bullet} F^{(2)\bullet} = - E^{(1)*}_{\bullet} t^{+\bullet}_{\bullet} f^{(2)\bullet} \end{aligned}\right\} \tag{280}$$

and this may be construed as a kind of "work-principle" or energy balance for the skeleton diagram, although it really is not, when (1) and (2) stand for different physical situations:

$$e^{(1)*}_{\bullet} f^{(2)\bullet} + E^{(1)*}_{\bullet} F^{(2)\bullet} = 0 \tag{281}$$

We have here a very condensed version of Tellegen's theorem (comp. text# 8, chapter 8), because all the original bonds with effort-input are collected to one bond and all original bonds with flow-input are collected to one other bond. However, we can easily go the other way and re-separate the collected bonds, so we get the originals back, for every scalar product in (281) is a simple sum of corresponding scalar products for the original bonds. Neither is there in (281) any difference of, how bonds with effort-input and bonds with flow-input appear in that sum of scalar products, which yields zero. We may therefore put up such a sum-accountance for all the original ports of the skeleton diagram on the form

$$\sum_k e^{(1)*}{}_k \cdot f^{(2)}{}_k{}^{\bullet} = 0 \qquad \textbf{(282)}$$

where the summation-index k now includes both the i-bonds and the α-bonds in figure 66. In this way we have got the general formulation of Tellegen's theorem. There are, of course, other formulations with the same generality. We might just as well have used covariant flows and contravariant efforts, but it is important, that the variances are different. WE could also have complex conjugated the flows instead of the efforts, but it is important, that either the flows or the efforts (but not both) appear complex conjugated, if the application requires complex vectors. In the applications within analytical mechanics, that we are soon going to look at, the vectors are real, though, and we need not worry about complex conjugation.

If (1) and (2) refer to the same physical situation, (282) expresses a *work principle* for processes, that are coupled together via the skeleton diagram. Such a principle is used as the foundation for Brønsted's energetics, witch in many ways deviates from the classical thermodynamics, and has much in common with the energy-bond viewpoint. The work principle can be said to express energy conservation, but it is not necessarily energy in the usual sense, that floats in the bonds, which we have seen an example of in the relativistic particle dynamics in the previous chapter.

It is characteristic for the energy-bond-method and for Brønsted's energetics, that one does not need to define in advance, what kind of energy, that runs in the

bonds, whether it is Helmholtz' or Gibbs' free energy or some third possibility. The "freedom" of energy, or the lack of it depends on how much it is possible to make the system perform work on some "work-reservoir" in the surroundings, e.g. a weight that can be raised and lowered, and this possibility is revealed by the complete energy-bond-structure and cannot be discussed for the single bond, independent of the context.

If (1) and (2) are two different situations, (282) has nothing to do with energy conservation. One often expresses Tellegen's theorem by saying, that the flow-vector and the effort-vector lie in orthogonal vector-spaces, but it must be stressed, that it is not possible to define one effort-vector and one flow-vector; the two bonds, that are left in figure 67 must not be collected to one bond.

A number of useful formulations of the theorem are achieved by introducing the concept of *virtual changes*. If we use (282) with situation (2) different from (1) and from that subtract the version, where (2) is identical with (1), we get

$$\sum_k e^{(1)*}{}_k \cdot (f^{(2)}{}_k^{\bullet} - f^{(1)}{}_k^{\bullet}) = 0 \qquad \textbf{(283)}$$

The difference between the two flow-vectors is called a virtual, i.a. an imagined, change of the flow. It is presumed, that the skeleton diagram is the same for the two situations (1) and (2). For a real development of a physical system the skeleton diagram will often contain transformers, that are parametrically dependent of certain level-variables. When the flows are altered due to the system's dynamic, the level variables will also change, and therefore are changes of the flows with fixed skeleton diagram imagined changes that possibly can't take place as a part of a system's temporal evolution. We shall use the symbol δ for a virtual change and can then write (283) in the form

$$\sum_k e^*{}_k \cdot \delta f_k^{\bullet} = 0 \qquad \textbf{(284)}$$

Of course we then also have

$$\sum_k \delta e^*_k \cdot f_k^{\bullet} = 0 \; ; \; \sum_k \delta e^*_k \cdot \delta f_k^{\bullet} = 0 \qquad \textbf{(285)}$$

plus a bunch of other versions, where the variances are shifted, or where the complex conjugation is moved from e to f.

We may also to each bond attach *cyclic levels*, a cyclic momentum (x-level), that is the time-integral of the effort, and a cyclic displacement or coordinate (o-level), that is the time-integral of the flow. We may then make some integrated versions, e.g. corresponding to (284) we get

$$\sum_k e^*_k \cdot \delta q_k^{\bullet} = 0 \qquad \textbf{(286)}$$

As an example of the application of Tellegen's theorem we look at the following problem: If we have a great electrical network with resistances in all branches and we connect an effort-source to two arbitrarily chosen nodes in the net, how will the resulting conductance, seen from the source, vary, when we change one of the conductances of the branches?

In text # 8, chapter 8 it is shown, how the topological system of nodes, branches, and meshes, that form an electric network, may be translated to a skeleton diagram, which only contains junctions. In such a scalar reticulation some of the branch-resistances will get flow-input and others effort-input, and besides, there will be a single scalar bond connecting the skeleton diagram with the effort-source. We can collect all the bonds with effort-input to the branch-resistances to one single vector-bond and all the bonds with flow-input to the branches to another vector-bond, while we keep the bond to the source as a scalar bond. All these three bonds are then euclidean, so we need not distinguish between covariant and contravariant vectors. Just the same, we shall denote the variance in the two vector-bonds, regarding the general formulation. We put the effort from the source to 1, and Y, the resulting conductance of the net, then gives the flow in this bond.

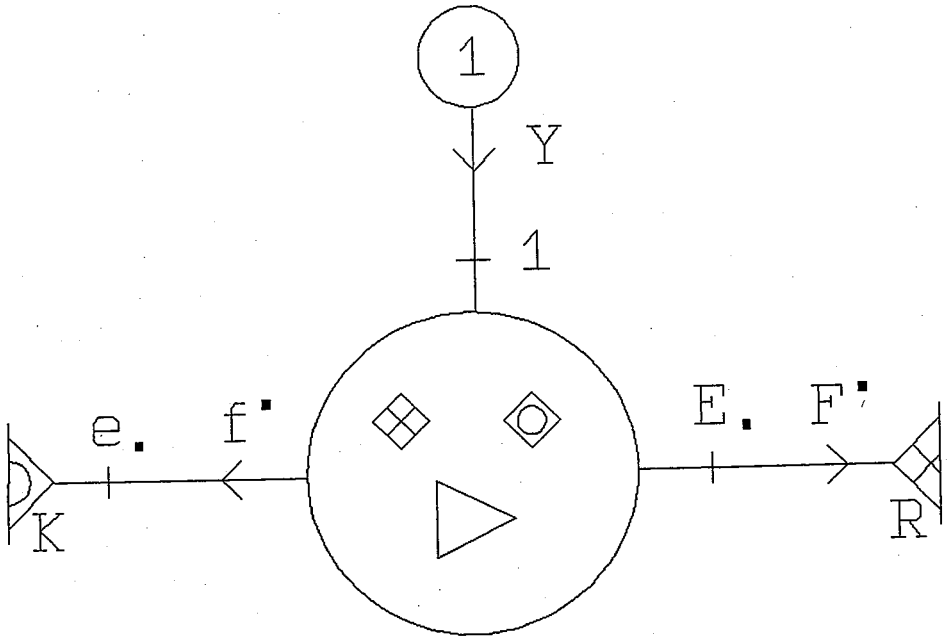


Figure 69. Network with effort-source and branch-resistances.

As the network only contains separate branch-resistances, the conductance-tensor K and the resistance-tensor R will have diagonal matrices with real and positive diagonal elements. We now employ two different versions of Tellegen's theorem, first with virtual flow-changes and then with virtual effort-changes. As the bond to the source has orientation opposite to the two vector-bonds, it will appear with the opposite sign in the sum, and we therefore move the product for this bond to the other side of the equals-sign. In the other version of the theorem this term becomes 0, because the source shall have the constant effort 1.

$$\left.\begin{aligned} E^{*}.\,\delta F^{\bullet} + e^{*}.\,\delta f^{\bullet} &= \delta Y \quad \text{(a)} \\ F^{*\bullet}\,\delta E_{\bullet} + f^{*\bullet}\,\delta e_{\bullet} &= 0 \quad \text{(b)} \end{aligned}\right\} \qquad \textbf{(287)}$$

If the virtual changes of flows and efforts are only due to a change of the conductivity-tensor K, we have:

$$\left.\begin{aligned} e^{*}_{\bullet}.\,\delta f^{\bullet} &= e^{*}_{\bullet}.\,K^{\bullet\bullet}\delta e_{\bullet} + e^{*}_{\bullet}.\,\delta K^{\bullet\bullet} e_{\bullet} \\ f^{*\bullet}\,\delta e_{\bullet} &= e^{*}_{\bullet}.\,K^{\bullet\bullet}\delta e_{\bullet} \\ E^{*}_{\bullet}.\,\delta F^{\bullet} &= F^{*\bullet}\,\delta E_{\bullet} = F^{*\bullet}\,R_{\bullet\bullet}\delta F^{\bullet} \end{aligned}\right\} \qquad \textbf{(288)}$$

We have here used, that the K- and R-matrices are diagonal and real. By subtraction of (287b) from (287a) and use of (288), we then have

$$\delta Y = e^{*}_{\bullet}.\,\delta K^{\bullet\bullet} e_{\bullet} \qquad \textbf{(289)}$$

If there is only one of the branch-conductances, that is changed, K_i, there is only one of the diagonal elements of the δK-matrix, that is different from 0. We then find

$$\frac{\partial Y}{\partial K_i} = |e_i|^2 \qquad \textbf{(290)}$$

This result looks simpler, than it really is. If we in stead of the deep Tellegen's theorem had used the simple work-principle, we could immediately write

$$Y = \sum_i K_i |e_i|^2 + \sum_k R_k |F^k|^2 \qquad \textbf{(291)}$$

and it then looks as if we could get (290) directly by just differentiating Y with respect to K_i. However, this argument does not hold, for if K_i changes, all the efforts and the flows of the branches will also change, and therefore one cannot derive (290) from (291) without further considerations.

We shall now see, how Tellegen's theorem can be used to derivation of fundamental principles for Analytical Mechanics in energy-bond-formulation. In

the following we shall assume, that the mechanical systems are described with real general coordinates (displacements, o-levels) and that all forces, momenta, and velocities are real. So we don't need to take complex conjugation into account in the different formulations of Tellegen's theorem.

The mechanical system is assumed to be *autonomous*, i.e. we don't need sources in the reticulation. We shall also disregard *friction* and other sorts of dissipation, i.e. sinks do not appear, either. The state of the system must be described by a o-level vector Q, that we call the coordinate-vector and an x-level-vector p, that we call the momentum-vector, and there must exist an energy-function of these two vectors. We shall further assume that the coordinate- and the momentum-vector have the *same dimension*, (that all the components of each vector have the same dimension) and that all vector-bonds also have a common dimension, as well as a *common metric* (dimension-less). In general it will not be possible to separate the variables in the energy-function, i.e. write the energy-function as a sum of terms, that each only depends on a single of the dynamic variables. This, however, will sometimes be possible, if one chooses Q and p in a special way, but provisionally we shall presume, that Q and p are chosen by another criterion, e.g. that they seem "natural". General coordinates are usually chosen, such that they in a simple way express the different rigid constraints, that the system is subjected to, such that one in the description has no need for calculating the forces of reaction that ensure, that the constraints are satisfied. If we e.g. have a rigid rod that can rotate about a fixed point then it is natural to choose a turning angle about this point as a generalized coordinate, thus tacitly assuming that the shaft is sufficiently rigid to keep the point of rotation fixed, when the rod rotates. Momenta can be chosen such, that they in a simple way are connected with the generalized velocities, e.g. as a moment of inertia times the angular velocity. The choice of simple or natural coordinates and momenta may have the drawback that they lead to a complicated energy-function and one must then next consider, if it is convenient to re-define the variables in order to get a simpler expression for the energy. We shall assume, however, that the choice of coordinates remains, and that we can do by re-defining momenta, and therefore we name the coordinate-vector with a capital Q and call it *canonical.*

The momentum-vector, that enters the energy-function, has no canonical status, so we name it with a small p. However, we shall require of the momentum-vector, that it together with the coordinate-vector makes possible a *closed* dynamical description, i.e. there must not appear other energy-bond-vectors as velocities and

forces than those, that can be derived from the energy-function. To begin with, we shall not accept a reticulation like figure 63, where the forces from the electromagnetic field are introduced via a dynamo. There must not be gyrators, either, as they reflect an external field, that breaks the time-reversal-symmetry (comp. chapter 5). The entire dynamics then lies in a two-port energy-storage with a p-port and a Q-port and in the skeleton diagram that connects the two ports. We may therefore employ the reticulation below, where there, though, besides the necessary constituents is introduced a *cyclic momentum P,* that will later be identified as the canonical momentum in Hamilton's equations. For now, it is just a tool of calculation, that is not necessary for the dynamics.

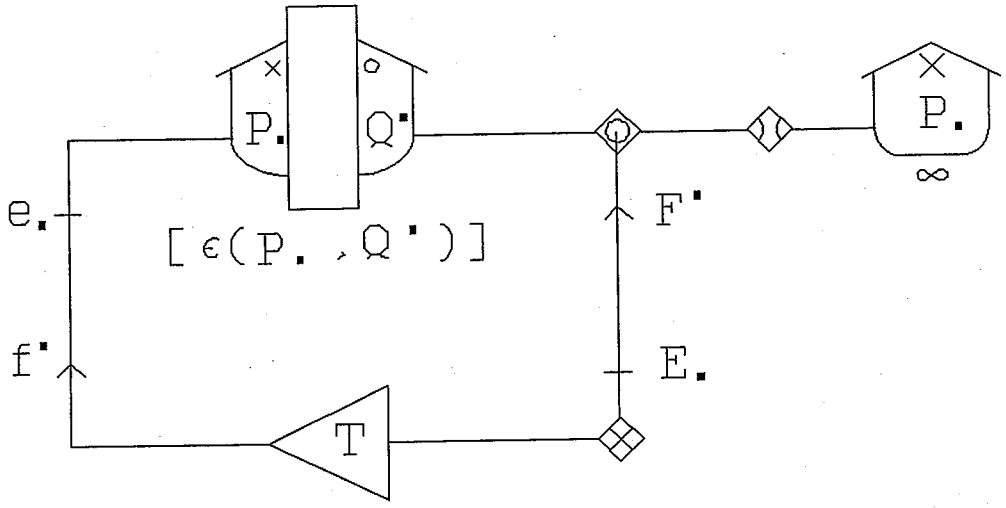


Figure 70. Autonomous mechanical system.

An integrated version of Tellegen's theorem (288) now gives:

$$f^{\bullet}\delta p_{\bullet} = F^{\bullet}\delta P_{\bullet} \qquad \textbf{(292)}$$

We introduce the *Lagrange-function* L by the definition

$$L = P_{\bullet}F^{\bullet} - \varepsilon(p_{\bullet}, Q^{\bullet}) \qquad \textbf{(293)}$$

and then get by using (292)

$$\left.\begin{aligned}\delta L &= P_{\bullet}\delta F^{\bullet} + F^{\bullet}\delta P_{\bullet} - f^{\bullet}\delta p_{\bullet} - E_{\bullet}\delta Q^{\bullet} \\ &= P_{\bullet}\delta F^{\bullet} - E_{\bullet}\delta Q^{\bullet}\end{aligned}\right\} \qquad \textbf{294}$$

The virtual change of the Lagrange-function can thus be expressed by the virtual changes of the coordinate-vector (Q) and the velocity-vector (F). These two vectors, though, are not independent, for the velocity-vector is the time-derivative of the coordinate-vector, but the *virtual* changes in (294) need not respect this dependence. We may therefore define partial derivatives of L as if it were a function of two independent vectors:

$$\left(\frac{\partial L(F^{\bullet},Q^{\bullet})}{\partial F^{\bullet}}\right)_{Q^{\bullet}} = P_{\bullet} \; ; \left(\frac{\partial L(F^{\bullet},Q^{\bullet})}{\partial Q^{\bullet}}\right)_{F^{\bullet}} = -E_{\bullet} \qquad \textbf{(295)}$$

We shall now consider a virtual development, where the virtual coordinate changes start at a time τ_1 and stop at another time τ_2.

$$\delta Q^{\bullet}(\tau) = 0 \text{ for } \tau \leq \tau_1 \text{ og } \tau \geq \tau_2 \qquad \textbf{(296)}$$

We then find by partial integration, using that E. is the time-derivative of P. with opposite sign, and $F^{\bullet}$ is the time-derivative of $Q^{\bullet}$, and that $\delta Q^{\bullet}$ vanishes in the limits τ_1 and τ_2.

$$\int_{\tau_1}^{\tau_2} E_{\bullet}\delta Q^{\bullet}\,\mathrm{d}\tau = -\int_{\tau_1}^{\tau_2} \frac{\mathrm{dP}_{\bullet}}{\mathrm{d}\tau}\delta \mathrm{Q}^{\bullet}\mathrm{d}\tau = \int_{\tau_1}^{\tau_2} \mathrm{P}_{\bullet}\,\delta \mathrm{F}^{\bullet}\mathrm{d}\tau \qquad \textbf{(297)}$$

From this and (294) we can then derive *Hamilton's principle*:

$$\delta\int_{\tau_1}^{\tau_2} L\,\mathrm{d}\tau = 0 \qquad \textbf{(298)}$$

The integral of the Lagrange-function over time is called *the action-integral*, or

just *the action*, and (298) thus expresses, that the action has an extremum for that history of virtual coordinate-changes, that is the real history, as dictated by the dynamics. In (298) it is taken into account, that the virtual change of the velocity-vector is not independent of the virtual coordinate-changes, so the history is written solely by the coordinates as functions of the time τ, the proper time of the system. By variational calculus the extremum-condition leads to the *Euler-Lagrange* equation

$$\int_{\tau_1}^{\tau_2}\left\{\frac{\partial L}{\partial Q^{\bullet}}\delta Q^{\bullet}+\frac{\partial L}{\partial F^{\bullet}}\delta F^{\bullet}\right\}\mathrm{d}\tau=\int_{\tau_1}^{\tau_2}\left\{\frac{\partial \mathrm{L}}{\partial \mathrm{Q}^{\bullet}}-\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial \mathrm{L}}{\partial \mathrm{F}^{\bullet}}\right)\right\}\delta \mathrm{Q}^{\bullet}\mathrm{d}\tau=0 \qquad \textbf{(299)}$$

$$\Rightarrow \frac{\partial L}{\partial Q^{\bullet}}-\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial \mathrm{L}}{\partial \mathrm{F}^{\bullet}}\right)=0$$

The question is now, how the Lagrange-function shall be determined, for the definition (293) involves two sets of momenta, and the cyclic momenta P. has nothing directly to do with the energy. We shall assume, that the system's kinetic energy can be expressed as a homogeneous quadratic form in the velocities:

$$\varepsilon_{kin}=\tfrac{1}{2}F^{\bullet}M_{\bullet\bullet}F^{\bullet}=\tfrac{1}{2}f^{\bullet}m_{\bullet\bullet}f^{\bullet} \qquad \textbf{(300)}$$

This then also means, that the kinetic energy is quadratic in p., and that there is a linear connection between p. and f.:

$$\varepsilon_{kin}=\tfrac{1}{2}p_{\bullet}m^{-1\bullet\bullet}p_{\bullet}\ ;\ p_{\bullet}=m_{\bullet\bullet}f^{\bullet} \qquad \textbf{(301)}$$

As we now also have a linear connection between the two velocity-vectors, given by the transformer T in figure 70:

$$F^{\bullet}=T^{+\bullet}_{\bullet}f^{\bullet}=f^{\bullet}T^{\bullet}_{\bullet} \qquad \textbf{(302)}$$

we have the following connection between the two inertia-tensors in (300):

$$m = TMT^{+} \tag{303}$$

The linear connection between the two velocity-vectors in (302), combined with (292), shows, that there must be a corresponding linear connection between p. and P.:

$$p_{\bullet} = T_{\bullet}^{\bullet} P_{\bullet} \tag{304}$$

such that we also have:

$$\varepsilon_{kin} = \tfrac{1}{2} P_{\bullet} M^{-1\bullet\bullet} P_{\bullet} \; ; \; P_{\bullet} = M_{\bullet\bullet} F^{\bullet} \tag{305}$$

We then get:

$$P_{\bullet} F^{\bullet} = p_{\bullet} f^{\bullet} = 2\varepsilon_{kin} \tag{306}$$

The Lagrange-function (293) can therefore be written in the form

$$L(F^{\bullet}, Q^{\bullet}) = \varepsilon_{kin}(F^{\bullet}, Q^{\bullet}) - \varepsilon_{pot}(Q^{\bullet}) \tag{307}$$

where ε_{pot} is the potential energy, that can only depend on the coordinates, while ε_{kin} through the inertance-tensor M can depend on the coordinates, besides the quadratic dependence on the velocities. The sum of the kinetic and the potential energy is the energy-function ε, which we can express as a function of momenta and coordinates, and instead of the original momenta p. we may substitute *the canonical momentum-vector P.* by means of (304). The energy-function thus made is called *the Hamilton-function* of the system H(P., Q·).
We have by means of (293) and (294):

$$\left.\begin{aligned} \delta H = \delta(P_\bullet F^\bullet - L) = F^\bullet \delta P_\bullet + E_\bullet \delta Q^\bullet \\ \left(\frac{\partial H}{\partial P_\bullet}\right)_{Q^\bullet} = F^\bullet \; ; \; \left(\frac{\partial H}{\partial Q^\bullet}\right)_{P_\bullet} = E_\bullet \end{aligned}\right\} \qquad \textbf{(308)}$$

As F· is the time-derivative of Q· and E. is the time-derivative of P. with opposite sign, we get *Hamilton's equations*:

$$\frac{dQ^\bullet}{d\tau} = \left(\frac{\partial H}{\partial P_\bullet}\right)_{Q^\bullet} \; ; \; \frac{dP_\bullet}{d\tau} = -\left(\frac{\partial H}{\partial Q^\bullet}\right)_{P_\bullet} \qquad \textbf{(309)}$$

Thus, it shows, that the diagram in figure 70 can be reduced to *the canonical form*

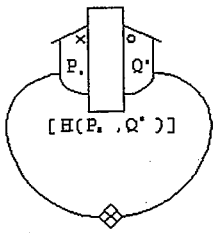


Figure 71. Hamilton's equations.

By the derivation of the Lagrange-equation (299) and Hamilton's equations (309) a number of assumptions have been made, that are not all necessary for the validity of the equations. We may show, e.g. that the relativistic motion of a charged particle in an electromagnetic field, as treated in chapter 9, can be derived from Hamilton's equations, when we use the Hamilton-function

$$H(P_\bullet, X^\bullet) = \frac{1}{2m}(P_\bullet - qA_\bullet)G^{\bullet\bullet}(P_\bullet - qA_\bullet) \qquad \textbf{(310)}$$

Here is X· the particle's coordinate-vector, corresponding to Q· in this chapter, and P. is the canonical momentum, that, however, is not identical with the 4-

momentum from the previous chapter. We get, namely from the first Hamilton-equation:

$$\frac{dX^{\bullet}}{d\tau} \doteq U^{\bullet} = \left(\frac{\partial H}{\partial P_{\bullet\cdot}}\right)_{X^{\bullet}} = \tfrac{1}{m}(P^{\bullet} - qA^{\bullet}) \qquad \textbf{(311)}$$

while the previously used 4-momentum is equal to the mass times the 4-velocity. The Hamilton-function depends on the coordinate-vector through A., so the second Hamilton-equation gives

$$\frac{dP_i}{d\tau} = -\left(\frac{\partial H}{\partial X^i}\right)_{P_{\bullet}} \doteq -\partial_i H = \tfrac{q}{m}(\partial_i A_j)(P^j - qA^j) \qquad \textbf{(312)}$$

We then find by combination of (311) and (312) the following expression for the 4-force F. in figure 63:

$$F_i = m\frac{dU_i}{d\tau} = \frac{dP_i}{d\tau} - q(\partial_j A_i)U^j = -\Lambda_{ij}U^j \qquad \textbf{(313)}$$

where we have used the expressions (264) and (268) for the dynamo in figure 63

We thus see, that the Hamilton-formalism works in this case, even though the presumption, that the kinetic energy is quadratic in the momentum, is not satisfied with the Hamilton-function (310). (The energy is, though, quadratic in the velocity-vector, as for a free particle). The *canonical* momentum P in (310) and (311) is less lucid than the *kinetic* momentum mU in figure 63. The Hamilton-formalism is a *symbolic* theory attached to many disciplines in physics, but when it concerns *iconic* lucidity and direct numerical solvability it lacks something behind both the Lagrange- and the energy-bond-formalism. The Lagrange-formalism requires, that the kinetic energy is expressed as function of the velocities, and it is usually rather easy thus to determine the Lagrange-function, when it can be expressed as in (307). The Hamilton-formalism demands further, that the canonical momentum is determined by (295) expressed by velocities, and

that one then eliminates the velocities for the canonical momenta in the kinetic energy, and this can be rather complicated. The energy-bond-formalism operates with coordinates and momenta o-and x-levels), but does not require, that the momenta shall be canonical. It is therefore possible with energy-bonds to employ the "free space" between the two other formalisms and choose the simplest and most lucid solution in the given situation.

Let us again assume, that the kinetic energy is quadratic in the canonical momentum-vector P., i.e. that it can be expressed by an inertance-tensor M.. as in (301). This tensor must be hermitean and may therefore be diagonalized to a form m.. with real diagonal elements by means of a transformer a:

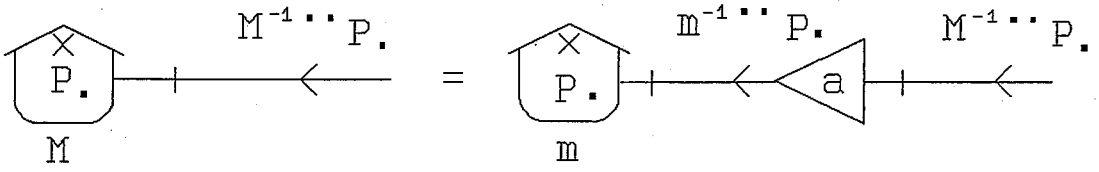


Figure 72. Digitalization of inertance-tensor.

We have then (comp. (303) and (304)):

$$p_{\bullet} = a_{\bullet}^{\bullet} P_{\bullet} \; ; \; m = aM a^{+} \qquad \textbf{(314)}$$

Generally, the original inertance-tensor will depend on the coordinate-vector Q·, but we can by suitable choice of the transformer a achieve, that the diagonal inertance-tensor m is independent of the coordinates. This will then mean, that a becomes coordinate-dependent. As the coordinates change under the system's development, we cannot assume, that the transformed momentum-vector is the time-integral of the transformed effort-vector. Besides, we cannot be sure, that the un-transformed effort-vector is the time-derivative of the canonical momentum, for the partial derivative of the energy-function with respect to the coordinate-vector is something else, when the new momentum p. is kept constant, than it is when the canonical momentum is kept constant. For the new energy-function E we have

$$\left.\begin{aligned} E(p_\bullet, Q^\bullet) &= H(P_\bullet, Q^\bullet) \\ \delta E &= \frac{dQ^\bullet}{d\tau}\delta P_\bullet - \frac{dP_\bullet}{d\tau}\delta Q^\bullet \end{aligned}\right\} \qquad (315)$$

If we call the un-transformed effort e., i.e. the effort on the primary side of the a-transformer, we have

$$\left.\begin{aligned} e_j &= -\left(\frac{\partial E}{\partial Q^j}\right)_{p_\bullet} = \frac{dP_j}{d\tau} - \frac{dQ^k}{d\tau}\left(\frac{\partial P_k}{\partial Q^j}\right)_{p_\bullet} \\ &= \frac{dP_j}{d\tau} - \frac{dQ^k}{d\tau}\frac{\partial a^{-1}{}_k^{\,l}}{\partial Q^j}p_l \end{aligned}\right\} \qquad (316)$$

The time-derivative of the new momentum p. is then

$$\left.\begin{aligned} \frac{dp_i}{d\tau} &= \frac{d}{d\tau}(a_i{}^j P_j) = \\ a_i{}^j &\left(e_j + U^k \frac{\partial a^{-1}{}_k^{\,l}}{\partial Q^j}p_l\right) + P_j \frac{\partial a_i{}^j}{\partial Q^k}U^k \end{aligned}\right\} \qquad (317)$$

where we again have introduced the vector-symbol U for the time-derivative of the coordinate-vector. We use now:

$$P_j = a^{-1}{}_j^{\,l}\, p_l \;;\; a^{-1}{}_j^{\,l}\frac{\partial a_i^{\,j}}{\partial Q^k} = -a_i^{\,j}\frac{\partial a^{-1}{}_j^{\,l}}{\partial Q^k} \qquad (318)$$

and can then write (317) in the form

$$\frac{dp_i}{d\tau} = a_i^{\ j}\left\{ e_j - \left[\frac{\partial a^{-1}{}_j^{\ l}}{\partial Q^k} - \frac{\partial a^{-1}{}_k^{\ l}}{\partial Q^j}\right] p_l U^k \right\} \qquad \textbf{(319)}$$

With this expression we get a recipe for reticulation, that proves very applicable. First, we have got the energy-function separated in a potential energy and a diagonal kinetic energy, and, second, we find a contribution to the force, that can be expressed by a dynamo, like in figure 63, because the factor, that the velocity is multiplied with in (319) is an anti-hermitean tensor:

$$B_{jk} = \left[\frac{\partial a^{-1}{}_j^{\ l}}{\partial Q^k} - \frac{\partial a^{-1}{}_k^{\ l}}{\partial Q^j}\right] p_l \qquad \textbf{(320)}$$

We can therefore use the following diagram:

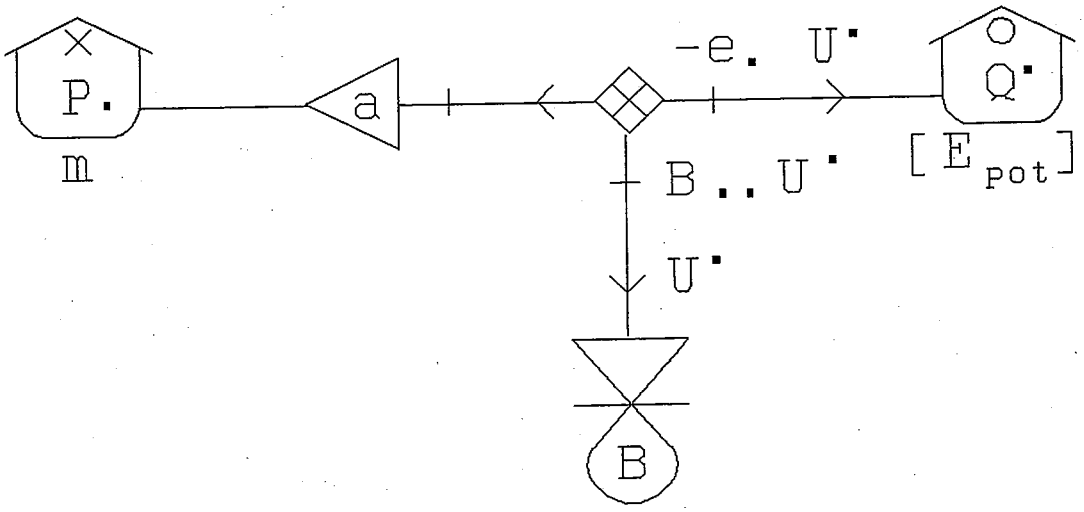


Figure 73. Standard mechanical reticulation.

In the next chapter we shall see, that the method here described has a number of applications from classical analytical mechanics to general relativity.

11. Mechanical reticulations.

We shall in this chapter look at a series of mechanical examples, that are all described with a set of generalized coordinates Q· and a set of momenta p., that are not the canonical momenta belonging to the coordinates, but have the advantage, that they diagonalize the kinetic energy. As we shall see, figure 73 leads to the most applicable general recipe for reticulation, but in certain cases it is convenient to deviate somewhat from it. We shall consider another variant that contains the same diagonalizing transformer a as figure 73, but where there appears a dynamo C, that is transferred to the other side of the transformer.

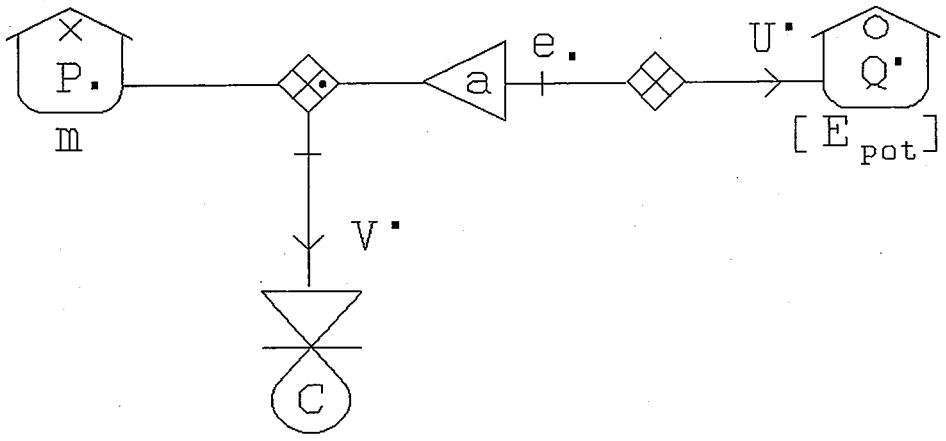


Figure 74. Alternative to figure 73.

As the new dynamo C must give the same effort-input to the p-storage as the earlier B, but has another velocity-input and is on the other side of the transformer, it must have another tensor. We have

$$U^{\bullet} = a^{+\bullet}_{\bullet} V^{\bullet} = V^{\bullet} a^{\bullet}_{\bullet} \; ; \; C_{\bullet\bullet} V^{\bullet} = a^{\bullet}_{\bullet} B_{\bullet\bullet} U^{\bullet} \qquad \textbf{(321)}$$

so we get from (320):

$$C_{ij} = a_i^{\ k} a_j^{\ l} B_{kl} = a_i^{\ k} a_j^{\ l} \left[\frac{\partial a^{-1}{}_k^{\ m}}{\partial Q^l} - \frac{\partial a^{-1}{}_l^{\ m}}{\partial Q^k} \right] p_m \qquad \textbf{(322)}$$

By use of (318) and subsequent exchange of the double indices k and l in one of

the terms we find:

$$C_{ij} = \left[a_i{}^k \frac{\partial a_j{}^l}{\partial Q^k} - a_j{}^k \frac{\partial a_i{}^l}{\partial Q^k} \right] P_l \qquad \textbf{(323)}$$

That is a relatively simple expression in the Hamiltonian canonical variables P and Q. It turns out, that the recipe figure 74 lies close to the Hamiltonian formalism, where figure 73 lies closer to the Lagrange-formalism. In order to see this we introduce the so called *Poisson paranthesis* of two quantities A and B, that are both conceived as functions of the canonical variables

$$[A,B] = \frac{\partial A}{\partial Q^k}\frac{\partial B}{\partial P_k} - \frac{\partial A}{\partial P_k}\frac{\partial B}{\partial Q^k} \qquad \textbf{(324)}$$

By the partial differentiations we tacitly understand, that it is the other *canonical variables* that are held fixed. It is not a random coincidence that we use the square brackets both for Poisson parantheses and for the commutator of two matrices, (comp. (235) and (239)), for in quantum mechanics it turns out, that the commutator of two observable-operators is closely related to the classical Poisson paranthesis. For every function A, that does not depend explicitly on time, but only implicitly through the canonical variables (we then say, that A is a phase-function), we then have from Hamilton's equations:

$$\frac{\mathrm{dA}}{\mathrm{d}\tau} = [\mathrm{A},\ \mathrm{H}] \qquad \textbf{(325)}$$

For the Poisson parantheses of the canonical variables we find:

$$[Q^i, Q^j] = [P_i, P_j] = 0 \ ; \ [Q^i, P_j] = \delta_{ij} \qquad \textbf{(326)}$$

When we introduce the momentum vector p. from figure 73 and 74 instead of the canonical momentum vector P., we find (comp. (314)):

$$\frac{\partial p_i}{\partial P_k} = a_i^{\ k} \; ; \; \frac{\partial p_i}{\partial Q^k} = \frac{\partial a_i^{\ l}}{\partial Q^k} P_l \qquad \textbf{(327)}$$

and thereby

$$\left.\begin{aligned} [\, p_i, Q^j \,] &= - a_i^{\ j} \\ [\, p_i, p_j \,] &= - C_{ij} \end{aligned}\right\} \qquad \textbf{(328)}$$

Thus, we see that all the coefficients of the transformer and the dynamo of figure 74 in a simple and logical way are expressed by Poisson parantheses between precisely those variables they connect. This does not hold for figure 73, that therefore not immediately appears so nicely related to the analytical mechanics (see, e.g. Goldstein). As shown by Jesper Gundermann there is, however, another method, based on *Lagrange parantheses*, that is easier to apply than the Poisson paranthesis reticulation. The Lagrange paranthesis of A and B is defined by

$$\{A,B\} = \frac{\partial Q^k}{\partial A}\frac{\partial P_k}{\partial B} - \frac{\partial P_k}{\partial A}\frac{\partial Q^k}{\partial B} \qquad \textbf{(329)}$$

and for the coefficients in figure 73 one has

$$\left.\begin{aligned} a^{-1}{}_i^{\ j} &= \{ Q^i, p_j \} \\ B_{ij} &= \{ Q^i, Q^j \} \end{aligned}\right\} \qquad \textbf{(330)}$$

The Lagrange parantheses do not look as inviting as the Poisson parantheses, but the use of figure 73 does not really require the use of (329) and (330). We shall

call figure 73 the Lagrange-reticulation and figure 74 the Poisson-reticulation. In order to use the Lagrange reticulation one has to go through the following program:

1) Choose a suitable set of generalized coordinates $Q^{\cdot}$.

2) Express the kinetic energy by the generalized velocities

$$U^{\bullet} = \frac{dQ^{\bullet}}{d\tau}\,;\, E_{kin} = \tfrac{1}{2} U^{\bullet} M(Q^{\bullet})_{\bullet\bullet} U^{\bullet} \qquad \textbf{(331)}$$

3) Determine a linear velocity transformation a^{-1}, that diagonalizes the kinetic energy, such that the transformed inertance-tensor is independent of the coordinates

$$V^{\bullet} = U^{\bullet} a^{-1}(Q^{\bullet})_{\bullet}^{\bullet}\,;\, E_{kin} = \tfrac{1}{2} V^{\bullet} m_{\bullet\bullet} V^{\bullet} \qquad \textbf{(332)}$$

4) Introduce the momentum-vector $p_{\bullet} = m_{\bullet\bullet} V^{\bullet}$ and determine the dynamo $B_{\bullet\bullet}$ by eq. (320).

Using the Poisson-reticulation one also has to go through points 1) to 3) as above, but in addition comes one more point:

3a) Determine the canonical momentum-vector by differentiating the kinetic energy with respect to the velocity-vector $U^{\cdot}$.

Then go through point 4) with the change, that it is now the dynamo $C_{\bullet\bullet}$ that shall be determined by eq. (323). By numerical integration of the model so obtained one has to keep current account of both the diagonalizing momentum-vector and the canonical momentum-vector. All in all it is thus somewhat more complicated to use the Poisson-reticulation, figure 74, than to use the Lagrange-reticulation, figure 73.

There are, however, certain cases, where a gyrator-structure as C in figure 74, which is closely connected with the momentum-vector, seems more inviting. As an example of this we shall look at the *Euler equations* for the motion of a rigid body in three dimensions with an euclidean metric. To such a body is attached a moment-of-inertia-tensor I.., that is diagonalized in one to the body rigidly attached coordinate system after the three principal axes of the body $\vec{n}_i$. The angular momentum vector $\overline{L}$ and the angular velocity $\overline{\omega}$ (both normal three-dimensional vectors) can then be resolved in components after the three principal axes:

$$\left.\begin{aligned} \vec{L} &= L_1\vec{n}_1 + L_2\vec{n}_2 + L_3\vec{n}_3 \\ \vec{\omega} &= \omega_1\vec{n}_1 + \omega_2\vec{n}_2 + \omega_3\vec{n}_3 \end{aligned}\right\} \qquad \textbf{(333)}$$

and the connection between these quantities is then given by the values of the diagonal moment-of-inertia-tensor:

$$L_1 = I_1\omega_1 \; ; \; L_2 = I_2\omega_2 \; ; \; L_3 = I_3\omega_3 \qquad \textbf{(334)}$$

The motion of the body makes the principal axes change direction:

$$\frac{d\,\vec{n}_i}{dt} = \vec{\omega} \times \vec{n}_i \qquad \textbf{(335)}$$

(There is now no reason for distinguishing between the proper time τ and the coordinate time t). If the body is influenced by a torque $\overline{M}$ the rate of change of the angular momentum relative to the inertial system will be given by this torque.

$$\frac{d\vec{L}}{dt} = \vec{M} \qquad \textbf{(336)}$$

and this means that the components of the angular momentum after the three principal axes change for two reasons (335) and (336):

$$\left.\begin{aligned} \frac{dL_i}{dt} &= \frac{d}{dt}(\vec{L}\cdot\vec{n}_i) = \vec{M}\cdot\vec{n}_i + \vec{L}\cdot(\vec{\omega}\times\vec{n}_i) \\ &= [\vec{M} + (\vec{L}\times\vec{\omega})]\cdot\vec{n}_i \end{aligned}\right\} \qquad \textbf{(337)}$$

This *Euler equation* can be expressed by the reticulation below, where we see the dynamo from figure 74 resolved on three scalar gyrators:

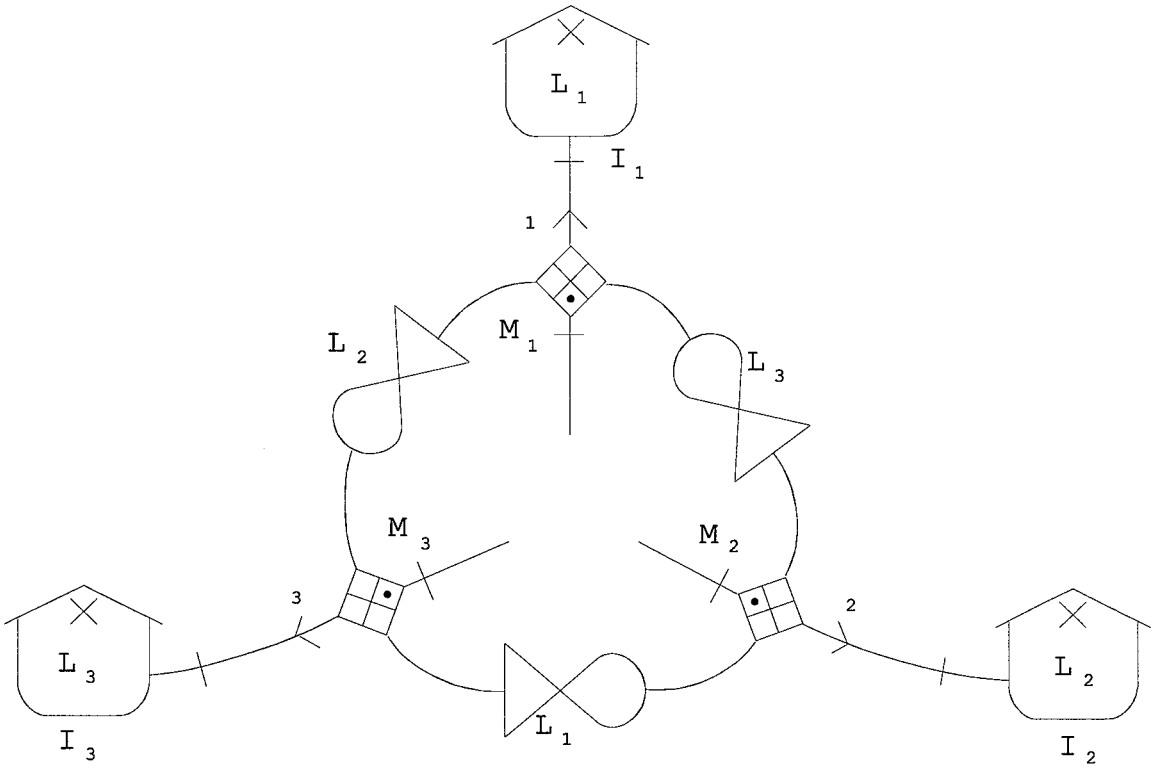


Figure 75. Euler equation for rigid body motion.

The gyrator-structure in figure 75 is thus a fixed ingredient of the motion of every rigid body, but from there and to the completion of the dynamical description there may be a long way to go. The transformer a from figure 74 will in this case transform from the inertial system to the body's system of principal axes and may,

e.g. be expressed by the Euler-angles (see Goldstein). Besides, there may appear kinematic constraints, like rolling on a plane or similar things that we won't go into details with here.

As a non-relativistic example of the use of the Lagrange-reticulation figure 73 we shall look at *the mathematical double pendulum*, that consists of two rigid, massless rods with lengths l_1 and l_2 and with point-masses m_1 and m_2 at the endpoints. The endpoint of the first rod acts as the turning point for the other rod. As generalized coordinates we choose the angles of the two rods with the vertical line, θ_1 and θ_2.

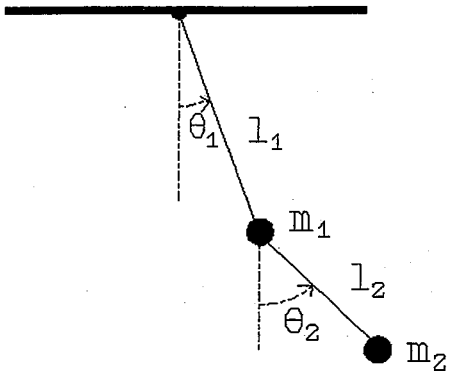


Figure 76. double pendulum.

The potential energy of the double pendulum is

$$\left.\begin{aligned} E_{pot}(\theta_1,\theta_2) &= m_1 g\, l_1(1-\cos\theta_1) \\ &+ m_2 g[l_1(1-\cos\theta_1)+l_2(1-\cos\theta_2)] = \\ &- g[(m_1+m_2)l_1\cos\theta_1 + m_2 l_2\cos\theta_2] + \text{konst.} \end{aligned}\right\} \quad \textbf{(338)}$$

So, the potential energy can be separated out on two simple, but non-linear o-storages.

In order to determine the kinetic energy as function of the velocities, we first find the normal velocity-vectors for each of the two particles:

$$\vec{v}_1 = l_1(\cos\theta_1, \sin\theta_1)\frac{d\theta_1}{dt} \; ; \; \vec{v}_2 = \vec{v}_1 + l_2(\cos\theta_2, \sin\theta_2)\frac{d\theta_2}{dt} \qquad \textbf{(339)}$$

It then turns out that in the kinetic energy appears a product of the two velocities, which means, that we have to enter point 3) in the above program, diagonalization of the kinetic energy through transformation of the velocities.

$$\left.\begin{aligned} E_{kin} = \tfrac{1}{2}m_1\vec{v}_1^{\,2} + \tfrac{1}{2}m_2\vec{v}_2^{\,2} = \tfrac{1}{2}(m_1+m_2)l_1^{\,2}(U^1)^2 \\ + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)U^1U^2 + \tfrac{1}{2}m_2 l_2^{\,2}(U^2)^2 \end{aligned}\right\} \qquad \textbf{(340)}$$

where U^1 and U^2 now, as before, stand for the rates of the coordinates θ_1 and θ_2.

By diagonalization of the kinetic energy it is good to get the a-matrix contain as many zeros as possible, because that will give the simplest reticulation. We choose to let the matrix have *the lower triangular form*, i.e. with zeros in all places above the diagonal. The inertance-tensor M.. in (340) has the coefficients

$$\left.\begin{aligned} M_{11} = (m_1+m_2)l_1^{\,2} \\ M_{12} = M_{21} = m_2 l_1 l_2 \cos(\theta_1 - \theta_2) \\ M_{22} = m_2 l_2^{\,2} \end{aligned}\right\} \qquad \textbf{(341)}$$

We shall then diagonalize M.. to a coordinate-independent form

$$m_{\bullet\bullet} = a_{\bullet}^{\bullet} M_{\bullet\bullet} a^{+\bullet}{}_{\bullet} = \left.\begin{pmatrix} a_1^{\,1} & 0 \\ a_2^{\,1} & a_2^{\,2} \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{pmatrix} \begin{pmatrix} a_1^{\,1} & a_2^{\,1} \\ 0 & a_2^{\,2} \end{pmatrix}\right\} \quad \textbf{(342)}$$

There is then still freedom left in the choice of the a's. We may choose to put $a_1^{\,1} = 1$ and to let the two coefficients m_{11} and m_{22} in the diagonalized inertance-tensor be equal to, respectively, M_{11} and M_{22}, as they are coordinate-independent. Thus:

$$m_{11}=(m_1+m_2)l_1^{\,2} \; ; \; m_{22}=m_2 l_2^{\,2} \quad \textbf{(343)}$$

We then find a and must further determine the reciprocal a^{-1}, that is to be used for determination of the dynamo B.. in figure 73:

$$a_{\bullet}^{\bullet} = \begin{pmatrix} 1 & 0 \\ -\dfrac{l_2 \cos(\theta_1-\theta_2)}{l_1\sqrt{1+\dfrac{m_1}{m_2}}\sqrt{\dfrac{m_1}{m_2}+\sin^2(\theta_1-\theta_2)}} & \sqrt{\dfrac{1+\dfrac{m_1}{m_2}}{\dfrac{m_1}{m_2}+\sin^2(\theta_1-\theta_2)}} \end{pmatrix} \quad \textbf{(344)}$$

and

$$a^{-1}{}_{\bullet}^{\bullet} = \begin{pmatrix} 1 & 0 \\ \dfrac{l_2 \cos(\theta_1-\theta_2)}{l_1\left(1+\dfrac{m_1}{m_2}\right)} & \sqrt{\dfrac{\dfrac{m_1}{m_2}+\sin^2(\theta_1-\theta_2)}{1+\dfrac{m_1}{m_2}}} \end{pmatrix} \quad \textbf{(345)}$$

For the dynamo B, that is antihermitean, there is only one coefficient to be found from (420) and (345):

$$B_{12}=\frac{m_2\sin(\theta_1-\theta_2)}{m_1+m_2}\left[\frac{l_2}{l_1}p_1-\cos(\theta_1-\theta_2)\sqrt{\frac{m_1+m_2}{m_1+m_2\sin^2(\theta_1-\theta_2)}}\,p_2\right] \qquad \textbf{(346)}$$

We then have all parameters for the scalar reticulation of figure 73. By moving transformers and junctions a bit around we may bring the diagram to the form

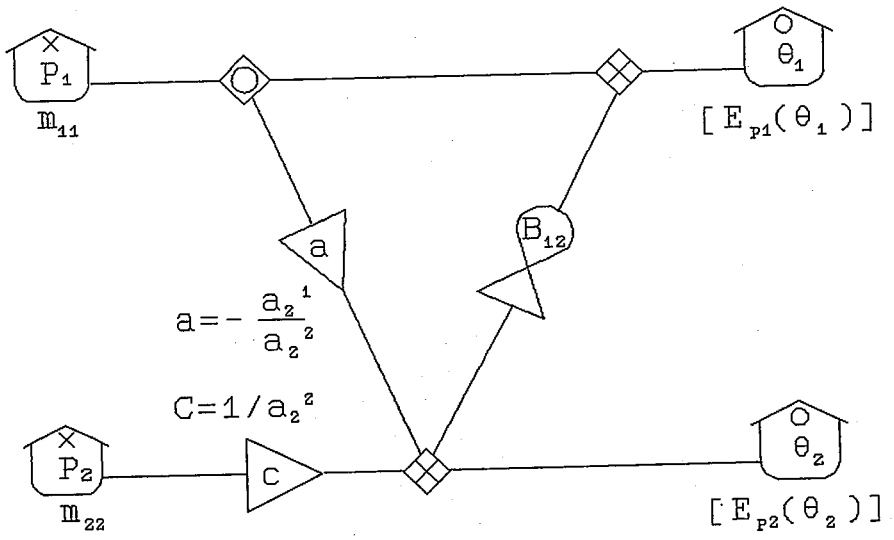


Figure 77. Scalar reticulation of the double pendulum.

Here, $E_{p1}(\theta_1)$ and $E_{p2}(\theta_2)$ are the two separate contributions to the potential energy, that appear in (338).

After this example, that is reasonably representative for the non-relativistic analytical mechanics, we shall return to the discussion from chapter 9 of the relativistic particle-dynamic. We shall see, that the developed standard method, the Lagrange-reticulation in figure 73, works just as well in a relativistic context. The main difference lies in the metric, as the relativistic particle-coordinates

require a non-euclidean standard metric, while the classical analytical mechanics always will have euclidean standard metric, unless one includes x-levels in one's definition of generalized coordinates.

In relativistic particle-dynamics we must also take the precaution, that a potential energy-function of the particle's four-dimensional coordinate-vector cannot occur. Surely, the electric potential may enter the dynamics, but only as a component of the 4-vector-potential, not as a scalar. If force-fields shall be seen as caused by material sources, like distant masses and charges, then the action of the force must be mediated *locally* of a field, wherein disturbances of the sources propagate with a velocity less than the speed of light. This philosophy goes well together with the viewpoint, that the force in figure 63 appears in connection with an anti-hermitean tensor-field, the dynamo, acting locally at the place of the particle, and whose propagation from the sources can be described by local partial differential equations (Maxwell's equations, (262), (270)), that can be formulated in the tensor-formalism, belonging to the metric. On the other hand, it does not suit a scalar, potential energy-function of the coordinate-vector that is set up in advance as parametrically governed by the configuration of the sources. The energy-function of storages must be local, lest it be unphysical, and therefore one should "unfold" it and translate it to that form of structural information, that can be described with junctions, transformers, and gyrators, for when it in this way enters the energy bonds, making directly visible the ways of energy, the locality-principle is made explicit. This has been the main motivation for preferring the Lagrange-reticulation to Hamilton's equations, and when we go on to relativistic particle-dynamics, where the coordinate-vector $Q^{\cdot}$ in figure 73 is the particle's space- and time-vector $X^{\cdot}$, we shall therefore avoid introducing an energy-function $E_{pot}(X^{\cdot})$.

The difference between the special relativity theory, SRT, and the general, GRT, is, that in SRT the metric always has the same form, the standard form (211), whereas in GRT it can be anything, as long as it is hermitean and real (g.. symmetric) and has (211) as standard metric. In SRT distances and durations are defined by standard measuring rods and specially synchronized standard clocks, in GRT the applied measuring rods and clocks may be defined specifically in every single case. The arbitrariness of the metric in GRT cannot be avoided by introducing standard measuring rods and standard clocks, for permanent fields of gravitation will make impossible a *global* coordinate-transformation, which standardizes the metric, although, of course, it may always be standardized *local-*

ly.

Looking at a *free particle* in GRT with a coordinate-dependent metric g..(X·), it will have a Hamiltonian function that is purely kinetic, but none the less place-dependent:

$$H(P_{\bullet},X^{\bullet})=E_{kin}=\frac{1}{2m}P_{\bullet}g(X^{\bullet})^{\bullet\bullet}P_{\bullet} \qquad \textbf{(347)}$$

Hamilton's equations can be reticulated as in figure 71, while we just characterize the mixed storage with the scalar mass m, understanding that the inertance-tensor's 0-variant is mg..

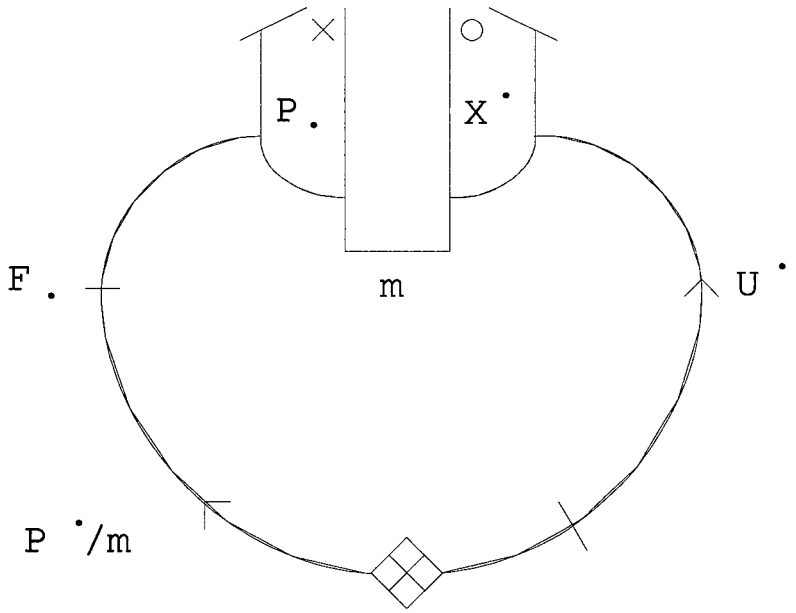


Figure 78. Hamiltonian description of GRT free particle.

Although the particle is "free", a 4-force F. will still govern it, when the metric is place-dependent. From the second Hamilton equation in (309) we get:

$$F_{\bullet} = \frac{dP_{\bullet}}{d\tau} = -\left(\frac{\partial H}{\partial X^{\bullet}}\right)_{P_{\bullet}} \tag{346}$$

The left side can be rewritten to

$$\left.\begin{aligned} \frac{dP_i}{d\tau} &= m\frac{d}{d\tau}\left(g_{ik}\frac{dX^k}{d\tau}\right) = \\ m&\left[g_{ik}\frac{d^2X^k}{d\tau^2} + \frac{\partial g_{ik}}{\partial X^l}\frac{dX^k}{d\tau}\frac{dX^l}{d\tau}\right] \end{aligned}\right\} \tag{347}$$

By exchanging the summed indices k and l in the last term, the expression can be written more symmetrically

$$\frac{dP_i}{d\tau} = m\left[g_{ik}\frac{d^2X^k}{d\tau^2} + \frac{1}{2}\left(\frac{\partial g_{ik}}{\partial X^l} + \frac{\partial g_{il}}{\partial X^k}\right)\frac{dX^k}{d\tau}\frac{dX^l}{d\tau}\right] \tag{348}$$

The right side of (346) becomes

$$\left.\begin{aligned} -\left(\frac{\partial H}{\partial X^i}\right)_{P_{\bullet}} &= -\frac{1}{2m}\frac{\partial g^{rs}}{\partial X^i}P_r P_s = -\frac{1}{2m}\frac{\partial g^{rs}}{\partial X^i}g_{rk}P^k g_{sl}P^l \\ = \frac{1}{2m}g_{rk}g^{rs}\frac{\partial g_{sl}}{\partial X^i}P^k P^l &= \frac{1}{2}m\frac{\partial g_{kl}}{\partial X^i}\frac{dX^k}{d\tau}\frac{dX^l}{d\tau} = \left(\frac{\partial H}{\partial X^i}\right)_{P_{\bullet}} \end{aligned}\right\} \tag{349}$$

We then find, that (346) gives

$$g_{ik}\frac{d^2X^k}{d\tau^2} = -\Gamma_{i,kl}\frac{dX^k}{d\tau}\frac{dX^l}{d\tau} \tag{350}$$

Here we have introduced the *Christoffel-synbol*

$$\Gamma_{i,kl} = \frac{1}{2}\left(\frac{\partial g_{ik}}{\partial X^l} + \frac{\partial g_{il}}{\partial X^k} - \frac{\partial g_{kl}}{\partial X^i} \right) \qquad \textbf{(351)}$$

It can be shown (Møller, IX.103), that Γ does not transform like a tensor, unless the metric transducer is coordinate-independent. Within a given metric we may, however, raise the first index in the normal way and thus define

$$\Gamma^j_{kl} = g^{ji}\Gamma_{i,kl} \qquad \textbf{(352)}$$

And the equation of motion (350) can then be reshaped to the so-called *geodesic equation*:

$$\frac{d^2X^j}{d\tau^2} = -\Gamma^j_{kl}\frac{dX^k}{d\tau}\frac{dX^l}{d\tau} \qquad \textbf{(353)}$$

By a *geodesic curve* is understood that curve between two given points that has the least arc-length. In euclidean geometry the geodesics are straight lines, and the law of inertia can be expressed in the way, that the motion of a free particle follows a geodesic curve. On a spheric surface the geodesics are arcs of great circles, and, correspondingly, one may for every differentiable surface imbedded in the three-dimensional euclidean space determine a metric tensor-field and from there the system of geodesics. Now, the four-dimensional energy-bond-metric, that forms the base of (35#), is not geometry, and when we here speak of arc-length and geodesic curves, it is only an analogy to the Riemannian surface-geometry, not to be taken too literally.

In order to see, that (353) is analogous to a parametric representation of a geodesic curve, we first remark, that the energy of the free particle is quadratic in

the momenta or the velocities, and, as there is no potential energy, the Lagrange-function has the same value as the Hamilton-function:

$$L(U^{\bullet}, X^{\bullet}) = H(P_{\bullet}, X^{\bullet}) = E_{kin} \tag{354}$$

We may regard the proper time τ as a natural parameter for the trajectory of the particle in the four-dimensional coordinate-space. When τ gets a little increment $d\tau$, we have:

$$L\,d\tau^2 = \tfrac{1}{2} m dX^{\bullet} g_{\bullet\bullet} dX^{\bullet} = -\tfrac{1}{2} m ds^2 \tag{355}$$

and the quantity ds is then analogous to the waylength travelled by motion on a Riemannian surface. The negative sign is necessary, if ds^2 is to be positive, when the particle moves slower than light. As L is equal to the energy-function, it is a constant of motion, given by (244), i.e.

$$\frac{ds}{d\tau} = \sqrt{-\frac{2L}{m}} = c \tag{356}$$

If we compare the real motion with all virtual motions from τ_1 to τ_2, we know, that the action-integral (298) assumes an extremum. The waylength for the virtual motions is

$$s(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \sqrt{-\frac{2L}{m}}\, d\tau \tag{357}$$

so, for small virtual changes around the real motion, where (356) is valid, we find

$$\delta s(\tau_1,\tau_2) = -\frac{1}{mc}\delta\int_{\tau_1}^{\tau_2} L\,\mathrm{d}\tau = 0 \qquad \textbf{(358)}$$

It is in this sense, that the motion of the particle is said to follow a geodesic curve in the four-dimensional space-time continuum. The "waylength" s, though, is not *minimal* as for a real geodesic, but *maximal*, which we can see by considering a classical inertial motion in the standard metric, for which one has

$$s^2 = c^2(t_2 - t_1)^2 - D^2 \qquad \textbf{(359)}$$

where D is the distance travelled in the three-dimensional space. As D is minimal for the rectilinear, inertial motion, it follows, that s is maximal. It then also follows from (358), that the action-integral is minimal.

The philosophy of GRT is, that so called fictive forces, that appear in accelerated coordinate systems, not locally can be distinguished from gravitational forces. Proper gravitational forces are just as "fictive" as, e.g., centrifugal forces, as they only enter the dynamics through the derivatives of the metric tensor, as described in the geodesic equation. As the mass does not enter the geodesic equation, this is an expression for the equivalence between heavy and inertial mass. Properly speaking, the time doesn't enter, either, for the proper time τ can be substituted with any other parameter λ, that is a monotonous and differential function of τ, so we get:

$$\frac{\mathrm{d}^2 X^j}{\mathrm{d}\lambda^2} = -\Gamma^j_{kl}\frac{\mathrm{d}X^k}{\mathrm{d}\lambda}\frac{\mathrm{d}X^l}{\mathrm{d}\lambda} \qquad \textbf{(360)}$$

The "free" GRT particle's dynamic can also be described by means of the standard method, the Lagrande-reticulation of figure 73. Here, the point is to diagonalize the kinetic energy, which is the same as diagonalizing the metric tensor's 0-variant g.., because (comp. (345))

$$E_{kin} = \tfrac{1}{2} m U^{\bullet} g_{\bullet\bullet} U^{\bullet} \tag{361}$$

we therefore have to find a velocity-transformation a, such that

$$U^{\bullet} = V^{\bullet} a_{\bullet}^{\bullet} \; ; \; E_{kin} = \tfrac{1}{2} m V^{\bullet} G_{\bullet\bullet} V^{\bullet} \tag{362}$$

where G is the standard metric (211). Normally, such a velocity-transformation will be local, i.e. it cannot be integrated to a global coordinate-transformation. If there exists such a global transformation, such that

$$U^{\bullet} = \frac{dX^{\bullet}}{d\tau} \; ; \; V^{\bullet} = \frac{dX'^{\bullet}}{d\tau} \; ; \; X'^{i} = f^{i}(X^{\bullet}) \tag{363}$$

then we have

$$a^{-1}{}_{j}^{\;i} = \frac{\partial f^{i}}{\partial X^{j}} \tag{364}$$

and this leads to the following integrability-condition, that the a-transformer must satisfy:

$$\frac{\partial a^{-1}{}_{j}^{\;l}}{\partial X^{k}} = \frac{\partial a^{-1}{}_{k}^{\;l}}{\partial X^{j}} \tag{365}$$

If such a global (and possibly non-linear) coordinate-transformation f exists, connecting the given coordinates $X^{\cdot}$ with the coordinates of an inertial system $X'^{\cdot}$, then the corresponding linear velocity-transformation a will diagonalize the kinetic energy. We the see by comparison of (365) and (320), that the dynamo B, belonging to such a transformation, vanishes identically. If there exists a global transformation to an inertial system, it is thus possible entirely to transform away the gravitational forces, that the dynamo describes, i.e. the presence of these

forces is then only due to the fact, that coordinate-system used is *accelerated* relative to an inertial system. If it is not possible to find a global transformation to an inertial system, we must conclude, that the dynamo describes a *permanent gravitational field.* There is, thus, a wide-ranging parallel between the way, the electromagnetic forces appear in SRT, and the way, the permanent gravitational fields appear in GRT. The main task for GRT is then to formulate a theory for the propagation of such fields in the form of tensor-differential-equations, determining those features of the metric, that are invariant to arbitrary coordinate-transformations.

Einstein's field equations give such a theory, but they are not uniquely determined from the requirement, that they must reproduce Newton's theory for small velocities, distances, and mass-densities. It is not at present possible to decide experimentally, which theory is the correct one. Of the three known experimental tests, the deflection of light by a solar eclipse, the red-shift in a gravitational potential, measured by the Mössbauer-effect, and the turning of Mercury's perihelion, it is only the latter, that shows deviations from the classical theory, that cannot be derived entirely from SRT, (and the equivalence-principle), but the actual turning of Mercury's perihelion is much greater than the GRT-effect, due to perturbations from the other planets, so the effect is too much veiled by classical noise for a decision to be reached. Draminsky gives a "heretical" discussion of these circumstances. Finally can be mentioned the possible existence of black holes, predicted by Einstein's theory, but not completely verified.

We shall not here enter the proper gravitational theory, but will restrict ourselves to the discussion of, how a given, space-dependent metric influences the motion of particles. As an example of motion in an accelerated coordinate system we shall resume and continue the discussion of the rotating disc, introduced in chapter 7. It is here natural to use the polar coordinates r and φ, which we can imagine measured by standard measuring rods and painted on the disc, before it is set to rotate. As measure of time we use clocks, that during rotation can be tuned to follow the advance of the synchronized standard clocks in the inertial laboratory-system, that they pass on their way round.

$$X^1 = r \;;\; X^2 = \phi \;;\; X^3 = t \qquad \textbf{(366)}$$

The metric tensor for the system of the disc, that we shall now call g, is then given by the expression (189). In order to connect with the discussion above about global coordinate-transformations we shall call the coordinates of the laboratory-system X', while keeping two cartesian space-coordinates $X'^1 = x$ and $X'^2 = y$ and one time-coordinate $x'^3 = ct$. The global transformation to an inertial system, as given in (363) is then

$$\left.\begin{aligned} X'^1 &= f^1(X^\bullet) = x = r\cos(\phi + \omega t) \\ X'^2 &= f^2(X^\bullet) = y = r\sin(\phi + \omega t) \\ X'^3 &= f^3(X^\bullet) = ct \end{aligned}\right\} \qquad \textbf{(367)}$$

The linear velocity-transformation corresponding to this is then according to (364) given by the matrix

$$a^{-1}{}^\bullet_\bullet = \begin{pmatrix} \cos(\phi + \omega t) & \sin(\phi + \omega t) & 0 \\ -r\sin(\phi + \omega t) & r\cos(\phi + \omega t) & 0 \\ -r\omega\sin(\phi + \omega t) & r\omega\cos(\phi + \omega t) & c \end{pmatrix} \qquad \textbf{(368)}$$

With this choice of the transformer a in figure 73 there will be no use of the dynamo B, as the integrability-condition (365) is fulfilled. There appears no sorts of fictive forces or gravitational forces in the description, that just conducts the entire problem to the laboratory-inertial system. where the particle moves uniformly in a straight line. The method is unsatisfactory, because the transformer a is time-dependent, while the metric tensor in the disc's system (189) is independent of time. We wish to reticulate the problem on the system's own premises, i.e. based on the metric tensor alone, so the elements used must be time-independent.

We shall therefore return to the method, that was used for the double pendulum, i.e. we determine the diagonalizing transformer a, such that its 1-variant has the lower triangular form. We shall then have (comp. (361) and (362)):

$$G_{..} = a_{\bullet}^{\cdot\bullet} g_{..} a^{+\bullet}_{\bullet} \qquad \textbf{(369)}$$

where G.. is the standard metric in (182) and g.. is the matrix (189), and $a_{\bullet}^{\cdot\bullet}$ shall have zeros in all places above the diagonal. The only solution is

$$a_{\bullet}^{\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r} & 0 \\ 0 & -\frac{\omega}{c} & \frac{1}{c} \end{pmatrix} ; \; a^{-1}{}_{\bullet}^{\bullet} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r & 0 \\ 0 & r\omega & c \end{pmatrix} \qquad \textbf{(370)}$$

By scalar reticulation of a transformer in the metric g there can, at most, occur five scalar transformers and four scalar gyrators, as shown in figure 68. With a given by (370) this reduces to a unit-transformer and two non-trivial transformers, and one gyrator ω/c. The dynamo B, that can now be determined from (320) and (370), could in principle contain one scalar gyrator B_{12} and two scalar transformers B_{13} and B_{23}, but it turns out that B_{23} is zero, because a^{-1} does not depend on φ or t, so left is only

$$B_{12} = -p_2 \; ; \; B_{13} = -\omega p_2 \qquad \textbf{(371)}$$

We can then go through the scalar reticulation and arrive at the diagram

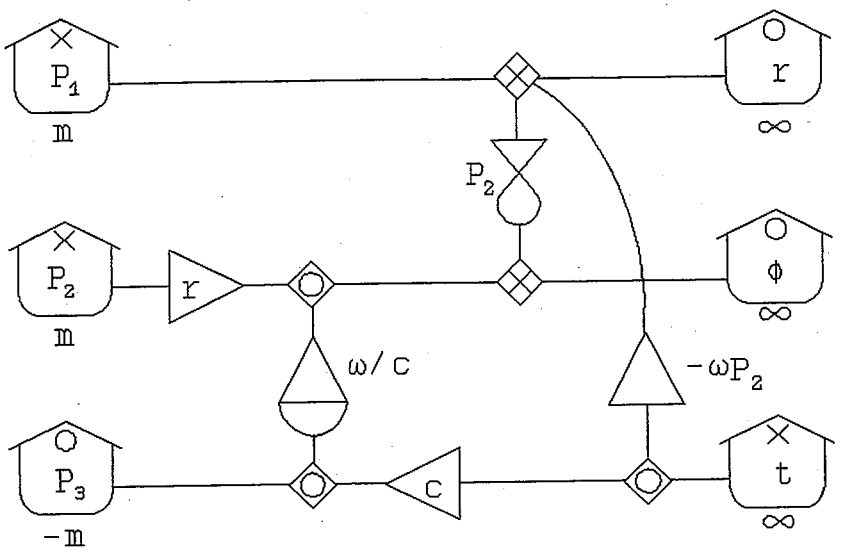


Figure 79. Relativistic particle on rotating disc.

The equations of motion for the three coordinates are

$$\frac{dr}{d\tau}=\frac{p_1}{m}\ ;\ \frac{d\phi}{d\tau}=\frac{p_2}{mr}+\frac{p_3\omega}{mc}\ ;\ \frac{dt}{d\tau}=-\frac{p_3}{mc} \qquad \textbf{(372)}$$

It is now seen, that the two contributions to the flow to the o-storage for p_3, conducted through the gyrator ω/c and the transformer c, exactly cancel each other, i.e. p_3 is *a constant of motion.*

$$p_3=-mc\gamma \qquad \textbf{(373)}$$

where γ is the time-dilation-factor. As the total energy-function of the three x-storages also is a constant, $-\frac{1}{2}mc^2$ (244), and as p_1/m and p_2/m according to (372) and (373) are, respectively the radial and the azimuthal part of the particle's velocity *relative to the laboratory-inertial system*, measured by the particle's proper time, we get

$$\gamma^2 = 1 + \frac{u^2}{c^2} = \frac{1}{1 - \frac{v^2}{c^2}} \qquad \textbf{(374)}$$

where v is the normal velocity of the particle relative to the laboratory, measured with the laboratory's clocks. These expressions only confirm, that it is a SRT free particle, when the motion is described from the laboratory-system, and that the gravitational or fictive forces in the disc's system, that are partly due to the gyrator p_2, can be transformed away.

We shall only look at the fictive forces in the classical limit, v<<c. We can then replace the p_3-storage with an effort-source with the constant effort c. This effort-source is now seen through the gyrator ω/c as a flow-source with the flow ω, which is a more satisfactory reticulation, as c must vanish from the equations of motion in the classical limit. When the effort-source is replaced with the flow-source ω, the transformer $-\omega p_2$ becomes another gyrator p_2. In this way all relativistic features disappear; the t-storage can be left out, as there will be no difference between the coordinate-time t and the proper time τ. We then end up with the following diagram:

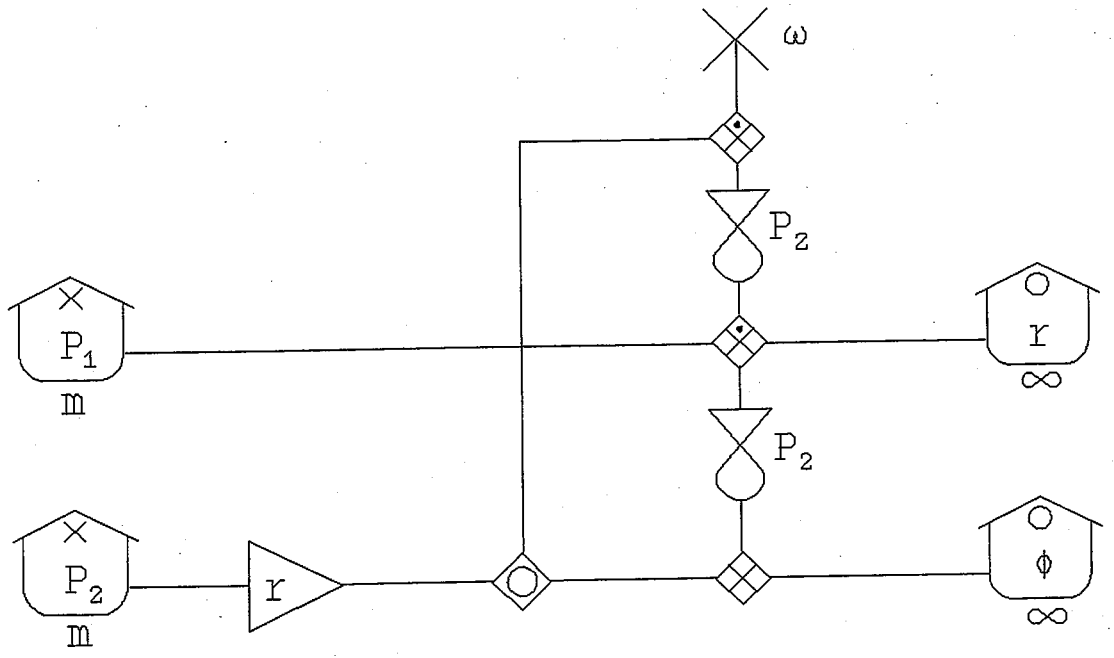


Figure 80. Non-relativistic particle on rotating disc.

The equations of motion for this diagram are

$$\left.\begin{aligned} \frac{dr}{dt} &= \frac{p_1}{m} \; ; \; \frac{d\phi}{dt} = \frac{p_2}{mr} - \omega \\ \frac{dp_1}{dt} &= \left(\omega + \frac{d\phi}{dt}\right) p_2 \; ; \; \frac{dp_2}{dt} = -\frac{p_2}{r}\frac{dr}{dt} \end{aligned}\right\} \qquad \textbf{(375)}$$

By elimination of the two momenta one gets

$$\left.\begin{aligned} \frac{d^2 r}{dt^2} &= r\left(\omega + \frac{d\phi}{dt}\right)^2 \\ \frac{d^2\phi}{dt^2} &= -\frac{2}{r}\frac{dr}{dt}\left(\omega + \frac{d\phi}{dt}\right) \end{aligned}\right\} \qquad \textbf{(376)}$$

In here is found a velocity-independent, radially, externally directed acceleration $r\omega^2$, the centrifugal acceleration, while the others, velocity-dependent terms together give the Coriolis-acceleration. If we introduce unit vectors after increasing radius and increasing azimuth, the velocity-vector of the particle can be written in the form

$$\vec{v} = \frac{dr}{dt}\vec{e}_r + r\frac{d\phi}{dt}\vec{e}_\phi \qquad \textbf{(377)}$$

The unit vectors follow the particle's motion:

$$\frac{d\vec{e}_r}{dt} = \frac{d\phi}{dt}\vec{e}_\phi \; ; \; \frac{d\vec{e}_\phi}{dt} = -\frac{d\phi}{dt}\vec{e}_r \qquad \textbf{(378)}$$

The particle's acceleration is then

$$\vec{a} = \frac{d\vec{v}}{dt} = \left[\frac{d^2 r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2\right]\vec{e}_r + \left[2\frac{dr}{dt}\frac{d\phi}{dt} + r\frac{d^2\phi}{dt^2}\right]\vec{e}_\phi \qquad \textbf{(379)}$$

When we in this enter the expressions (376) we can write the acceleration as a sum of centrifugal- and Coriolis-acceleration, where

$$\left.\begin{aligned} \vec{a}_{cent} &= r\omega^2\vec{e}_r \\ \vec{a}_{Cor} &= 2\vec{v}\times\vec{\omega} = 2r\omega\frac{d\phi}{dt}\vec{e}_r - 2\omega\frac{dr}{dt}\vec{e}_\phi \end{aligned}\right\} \qquad \textbf{(380)}$$

The non-permanent gravitational fields, that characterize accelerated coordinate-systems as different from inertial systems, can, according to Mach's principle be understood as proper gravitational fields, that are due to the motion of the cosmic mass-background relative to the accelerated system. In a similar way that the actual existence of inertial systems, where SRT is valid, is due to this cosmic background. Even the universal value of the speed of light must in a self-consistent way be related to the size and mass-density of the universe. These speculations, that are basic philosophy for GRT, can, however, not without further development find formalistic expression with the concepts of the theory. Maybe the difficulties of gravitation-theory and unified field theories stem from the blind acceptance of the geometrical metaphor and believing that time and evolution could be reduced to patterns of world-lines.

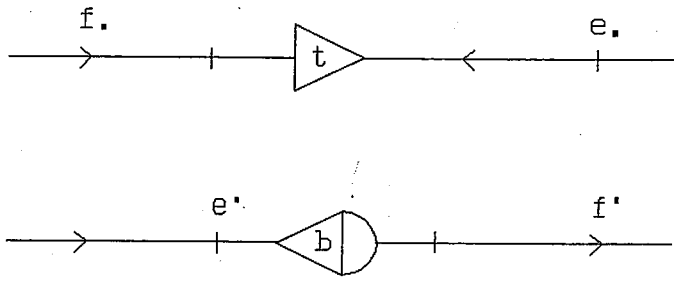
Problems and solutions

a) Problems (mainly for chapters 6 and 7)

1. Show, that the metric unitary transformers form a group.
Show further, that the dual tensor to a metric unitary tensor also is metric unitary. What else can one say about a tensor, that is both metric unitary and metric hermitean? Go through the same questions, with "metric" replaced with "1-".

2. Show, that a self-dual mirroring transformer is icon-symmetric.

3. Fill out the diagrams below in all variants.



4. We consider the two-dimensional σ_z-standard metric

$$G.. = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We let the symbols ξ, η and ζ denote tensors, whose 1-variants are the three Pauli-matrices

$$\xi.^{\bullet} = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \eta.^{\bullet} = \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix};\ \zeta.^{\bullet} = \sigma_z$$

We now consider the set of transformers containing the tensors

$$\pm G, \pm h, \pm \mathrm{i}G, \pm \mathrm{i}h, \pm \xi, \pm \eta, \pm \zeta, \pm \mathrm{i}\xi, \pm \mathrm{i}\eta, \pm \mathrm{i}\zeta$$

a) Among these 20 there are only 16 different, which?

b) Which of these are metric hermitean/ metric unitary/ icon-symmetric/ 1-unitary/ 1-hermitean/ mirrorings (2-cyclic)/ 4-cyclic/ reversible/ anti-reversible ?

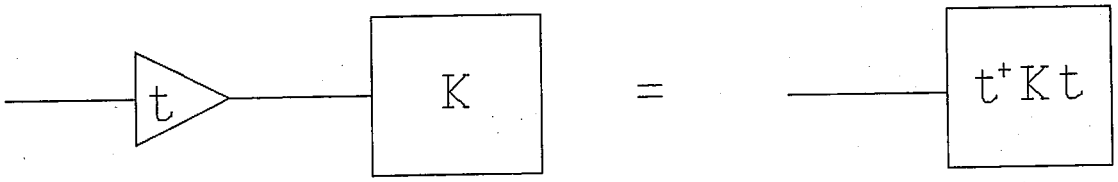
c) Which of these transformers have already been denoted with icons in the text? make suitable icons for the remaining.

d) The reversible elements form a non-commutative sub-group, which is called the dihedral group. Make a multiplication table for this group.

e) Show, that the group considered in d) is isomorphic with the group of movements, that make a square cover itself.

5. Show, that a self-dual gyrator is icon-symmetric and reciprocal (reversible), if it is metric anti-hermitean, and that it is anti-reciprocal if it is metric hermitean.

6. Let K be the response-tensor for a 1-port with effort-input. Show the formula:



Then make a corresponding formula, where the transformer t is replaced with a metric transducer A. Does this formula provide a reason for claiming, that K transforms as a tensor by coordinate-shift?

7 Let t be a tensor and A a metric transducer, so that t in the changed coordinate-system is given by

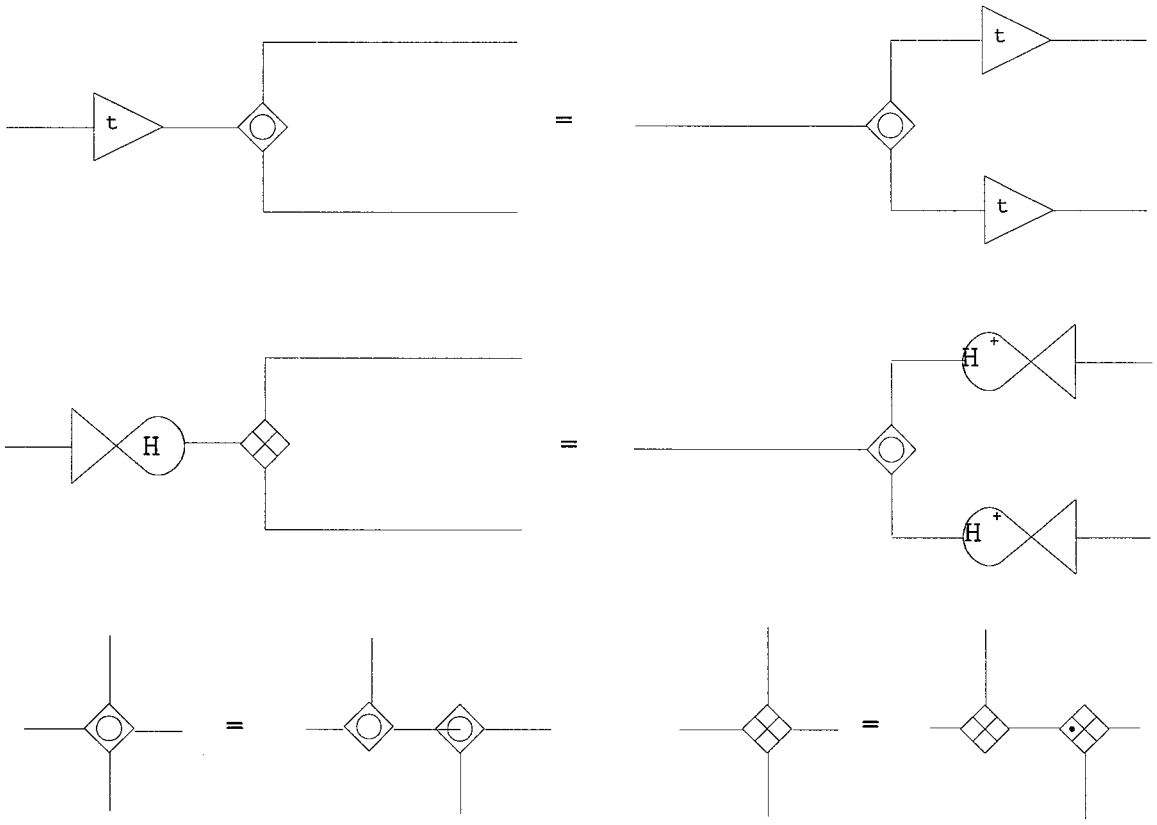
$$t' = A t A^{+}$$

Show, that the dual tensor transforms by the dual transducer.

$$\breve{t}' = \breve{A}\breve{t}\ \breve{A}^{+}$$

8 Correct possible errors in the iconic formula below

9. Determine all variants of the protensors h, H', and Δ for the metric (63).



Determine the protensors h, H', and Δ for this metric.

10 Determine the signature for the metric

$$g_{..} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

Determine the protensors h, H', and Δ for this metric.

Solutions

1. As pointed out in chapter 6, *all* transformers form a group with concatenation as the associative composition rule and with g as the unit element. The metric unitary transformers, t, for which $t^+=t^{-1}$ form a subgroup if the following conditions are satisfied:
a) The product of two metric unitary transformers s and t is also metric unitary. This is true, because when $s^+=s^{-1}$ and $t^+=t^{-1}$ then $(st)^+=t^+s^+=t^{-1}s^{-1}=(st)^{-1}$ (comp. (124)).
b) The set of unitary transformers contains the reciprocal to each element, (which it does, because $t^{-1}=t^+$) and the unit element g. g is metric unitary, because it is metric hermitean, $g=g^+$, and because it is the unit element, and therefore $g^{-1}=g$, thus $g^+=g^{-1}$. So we have shown, that the metric unitary transformers form a group.

The dual to a metric unitary transformer t ist $\breve{t}$ = hth. As h is both self-dual and metric unitary, we have: $(\breve{t}\ )^+=(\ \breve{t}\ )^{-1}$, q.e.d.

When t is both metric hermitean and metric unitary, we have: $t=t^+$, and $t^+=t^{-1}$, hence, $t=t^{-1}$, or $t^2=1$ (g). So t is a mirroring operator.

All the properties of metric unitary transformers, exposed in problem **1** are equally true of 1-unitary transformers, which is easily seen by looking at the 1-matrix-variants of the tensors. That g is the unit element is true, because the 1-variant of g is the unit matrix. The protensor h is 1-unitary and self-dual, so it is also true, that the dual to a 1-unitary tensor t is 1-unitary.

2. When t is a mirroring, we have $t^{-1}=t$. When it is also self-dual: $\breve{t}$ =hth=t, so together we havet $\breve{t}^{-1}=t$, which is the condition (133) for icon-symmetry.

4.
a) In order to see, which of the 20 transformers are different we have to compare the same matrix-variants of them. Choosing the 1-variants we find $\zeta=h$. We can therefore omit $\pm\zeta$ and $\pm i\zeta$ from the list, and the remaining 16 tensors:
$\pm G$, $\pm h$, $\pm iG$, $\pm ih$, $\pm\xi$, $\pm\eta$, $\pm i\xi$, $\pm i\eta$ are all different. They form a group with G as the unit element.

b) The metric hermitean tensors are those whose 0-variants are matrix-hermitean,

i.e. ±G, ±h, ±iξ, ±iη, ±ζ. All of these are equal to their own reciprocal, so they are also metric unitary.
As h=ζ we find that ξ and η are anti-self-dual, i.e. they change sign by the duality-transformation, so according to (128) they are icon-anti-symmetric. The icon-symmetric tensors are therefore: ±G, ±h, ±iξ, ±iη.

1-hermitean are: ±G, ±h ±η, ±ξ,.
Mirrorings are ±G, ±h, ±ξ, ±η,.
1-unitary are: ±G, ±h, ±ξ, ±η.
2-cyclic are: ±G, ±h, ±ξ, ±η.
4-cyclic are: ±iG, ±ih, ±iξ, ±iη.

c) Symmetric icons have already been defined for ±G, ±h, in any metric, so we just need two symmetric icons for iξ and iη (e.g. squares with some mark inside) and asymmetric icons (e.g. triangles) for the rest.

d) The reversibility (reciprocity) condition (138) gives, that the following transformers are reversible: ±G, ±h, ±iξ, and ±η. As ξ, η, and h anticommute, the reversible transformers form a non-commutative group.

The *dihedral group* D_4 is the group of movements that make a rigid plane square cover itself. This group (also called the *octic* group) contains 8 elements and is non-commutative. The square has four symmetry-axes: a horizontal, H, a vertical, V, and two diagonals, D and E, and to each of these correspond a two-cyclic group element, a 180° rotation about the axis. Besides, there is a 4-cyclic element O, that is a 90° rotation about the center of the square, so D_4 contains the 8 elements (H,V,D,E,O,O^2,O^3, and the identity I=O^4). There is only one more non-commutative group of order 8 (see, e.g. W. Ledermann, Introduction to the theory of finite groups, Oliver and Boyd, London, 1957), namely the *quaternion group* consisting of the hypercomplex units±1, ±i,±j, and ±k of the quaternion numbers a+ib+jc+kd, but this group can be excluded, because i, j, and k are all 4-cyclic, whereas the reversible transformers only contains ±iξ, that are 4-cyclic. For D_4 only O and O^3 (-O) are 4-cyclic. Therefore, the group of reversible transformers must be isomorphic to the dihedral group D_4. To work out the two 8*8 multiplication tables and to compare them is a somewhat tedious job. The two lines below suggest a mapping that will bring out the isomorphy:

H	V	D	E	O
-η	η	-h	h	iξ

The multiplication table for the group D_4 is shown below. The product AB, i.e. the result of applying first the transformation B, and then A is found in row A and column B:

I	H	V	D	E	O	O^2	O^3
H	I	O^2	O^3	O	E	V	D
V	O^2	I	O	O^3	D	H	E
D	O	O^3	I	O^2	H	E	V
E	O^3	O	O^2	I	V	D	H
O	D	E	V	H	O^2	O^3	I
O^2	V	H	E	D	O^3	I	O
O^3	E	D	H	V	I	O	O^2

Multiplication table for the group D_4.

5. For a self-dual gyrator G is hGh=G, if G is also anti-hermitean, G^+=G, we have hG^+h=-G, so, according to (135) G is icon-symmetric. The reciprocity-condition (140) hG=-G^+h also follows from self-duality and anti-hermiticity.

7. The formula t'=AtA^+ can be re-written as ht'h=hAh(hth)hA^+h showing that the dual transformer transforms by the dual transducer.

8. The second diagram from above can be changed by altering the x-gyrators to o-gyrators, using figure 39. If we choose contravariant input-efforts $e_1^{\cdot}$ and $e_2^{\cdot}$ in the two bonds to the right and orientations from right to left in all bonds, the output-flow in the left bond will be $H^{-1\cdot}.(e_1^{\cdot}+e_2^{\cdot})$. But the same transformation (x-to o-) of the left gyrator shows that H should be changed to H^+. Thus, we have to hermitean conjugate either the left gyrator or the two right gyrators in order to make the diagram correct.

9. The metric (63) belongs to proper geometry (in polar coordinates) and is therefore quasi-euclidean. Therefore we have h..=g..
The reciprocal matrix $g^{\cdot\cdot}$ is found by taking the reciprocal values of the diagonal elements (and leaving the 0s in place), so we get that the 1-and 2-variants of h are both equal to the unit matrix. As $H.^{\cdot}$ is the square root of g.. found by taking square roots of the diagonal elements of g.. (without changing the 0s). This 1-variant of H is then also the 1-variant of H'. Finally, we have, as always:

$$\Delta^{\cdot}_{\cdot} = \Delta_{\cdot}^{\cdot} = g_{\cdot\cdot}, \quad \Delta_{\cdot\cdot} = (g_{\cdot\cdot})^2.$$

10. $g_{\cdot\cdot}$ has the determinant -1 and the eigenvalues $2 \pm \sqrt{5}$. So, the signature is 4. $g_{\cdot\cdot}$ is one of the metrics that come under the general expression (164). The parameters are:

$$a=1, \quad \varphi=0, \quad b=2.$$

We then get from (169): $\rho=\sqrt{5}$, and then from (171):

$$h_{\cdot\cdot} = \sqrt{5}\delta_{\cdot\cdot} + 2G_{\cdot\cdot}$$

To determine $H_{\cdot\cdot}$ we get from (174): $\sinh\chi=2$, $\cosh\chi=\sqrt{5}$, so

$$\chi/2 = \ln\sqrt{(2+\sqrt{5})}.$$

$H_{\cdot}^{\cdot}$ is then determined from (172).

Appendix

Classical discussion of rotating system

Coriolis- and centrifugal-force

In the last chapter a relativistic reticulation of motion in a rotating system ended by considering the classical limit — a rather long way for obtaining a well-known result. In this appendix we shall proceed more directly to the classical expressions for Coriolis-and centrifugal force. The purpose is to demonstrate how thinking in energy-bond-terms — without being too formalistic — may reveal new aspects and raise new questions in a thoroughly ploughed field.

Let us complete this discussion of rotation by considering motion in a system with a fixed rotation with angular velocity. If we compare the rates of change of a vector in the laboratory-system (L) and the rotating system (R). we have:

$$\left(\frac{d\vec{s}}{dt}\right)_L = \left(\frac{d\vec{s}}{dt}\right)_R + \vec{\omega} \times \vec{s} \qquad (1)$$

Letting now $\bar{s}$ be the velocity of a particle with mass m relative to the laboratory-system:

$$\vec{v}_L = \vec{v}_R + \vec{\omega} \times \vec{r}_R \qquad (2)$$

we find

$$\left(\frac{d\vec{v}_L}{dt}\right)_L = \left(\frac{d\vec{v}_L}{dt}\right)_R + \vec{\omega} \times \vec{v}_L = \left(\frac{d\vec{v}_R}{dt}\right)_R + 2\vec{\omega} \times \vec{v}_R + \vec{\omega} \times (\vec{\omega} \times \vec{r}_R) \qquad (3)$$

We now wish to discuss the motion relative to the rotating system and omit the subscript R. As the proper force on the particle is given by its mass times the acceleration relative to the laboratory system, there will in addition to this force occur fictive forces in the rotating system, so the total effective force on the

$$m\vec{v} = \vec{F} + 2m\,\vec{v}\times\vec{\omega} + m\,(\vec{\omega}\times\vec{r})\times\vec{\omega} \qquad \textbf{(4)}$$

particle appears to be given by:

Here the second term is the *Coriolis-force* and the third term the *centrifugal force.*

There are many ways to reticulate the motion relative to the rotating system, but they are not all equally satisfactory. First, we may observe that the expression for the Coriolis-force looks like the magnetic part of the Lorentz-force (116) on a particle with charge q. If a charged particle is influenced by a homogeneous magnetic field and, besides, is bound to a center of force, then the magnetic force will be cancelled by the Coriolis-force in a system that rotates with the angular velocity

$$\vec{\omega}_L = -\frac{q}{2m}\vec{B} \qquad (5)$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \qquad (6)$$

In the rotating system there will be, though, a centrifugal force that has no electromagnetic counterpart, but this will be of second order in the magnetic field and held at bay by the binding to the force-center. For weak magnetic fields it will therefore be valid, that the situation in the system that rotates with the *Larmor frequency* $_L$ is the same, as it was in the laboratory-system before the magnetic field was applied. This is *Larmor's theorem.*

The equivalence between the Coriolis- and the Lorentz-force makes possible a Lagrangian or Hamiltonian description of the particle's motion in the rotating system, analogous to (119) and (122), but it is unsatisfactory to make the centrifugal force enter the picture as a conservative force to be derived from a potential, so we shall point out a reticulation, that more clearly shows, what goes on.

$$L(\vec{r},\vec{v})=\tfrac{1}{2}mv^2-q\phi+q\vec{v}\cdot\vec{A} \tag{7}$$

If the particle in some way (e.g. by friction) interacts with the rotating system, then the engine that keeps the rotation going, deliver some work in order to keep the angular velocity constant. We shall therefore introduce a flow-source to describe the action of the drive. The work that the drive must perform, when the particle moves the distance $d\vec{r}$ in the rotating system, is:

$$\left.\begin{aligned} dw_d=d\left(\tfrac{1}{2}m\vec{v}_L^{\,2}\right)&=m(\vec{v}+\vec{\omega}\times\vec{r})\cdot d(\vec{\omega}\times\vec{r}) \\ &=\vec{p}'\cdot(\vec{\omega}\times d\vec{r})=(\vec{p}'\times\vec{\omega})\cdot d\vec{r} \end{aligned}\right\} \tag{8}$$

where we have introduced the vector ' =m(+ ×), that is the momentum of the particle relative to *the laboratory system* as expressed in the rotating system. If the particle lies still in the rotating system, ' is constant, but $_L$, the momentum in the laboratory system, is not constant. The drive shall therefore provide energy with the rate

$$\frac{dw_d}{dt}=(\vec{p}'\times\vec{\omega})\cdot\vec{v} \tag{9}$$

This amounts to, that the flow-source $\overline{\omega}$ acts through a gyrator, that on the secondary side has the velocity $\overline{v}$ as input-flow. The effort in the secondary port is then

$$\vec{p}' \times \vec{\omega} = H..\omega. \tag{10}$$

where H.. is the gyrator's tensor, which thus must be

So, it is an anti-hermitean gyrator, i.e. it is icon-symmetric, comp. figure 58.

$$H.. = \begin{pmatrix} 0 & -p'_z & p'_y \\ p'_z & 0 & -p'_x \\ -p'_y & p'_x & 0 \end{pmatrix} \tag{11}$$

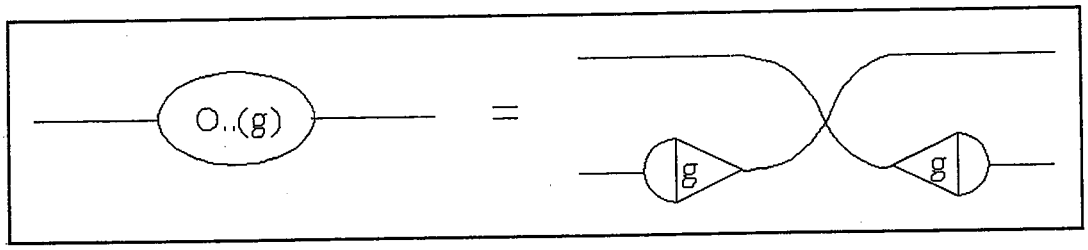


Figure 1. Icon-symmetric gyrator for d=2.

The output-effort from this gyrator is:

$$\vec{p}' \times \vec{\omega} = m(\vec{v} + \vec{\omega} \times \vec{r}\,) \times \vec{\omega} = m\vec{v} \times \vec{\omega} + m(\vec{\omega} \times \vec{r}) \times \vec{\omega} \tag{12}$$

I.e. it accounts for *half* the Coriolis-force and *the whole* centrifugal force. The other half of the Coriolis-force is provided by a dynamo with the tensor

$$C..=\mathrm{m}\begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \tag{13}$$

In stead of showing the tensors of the symmetric gyrator and the dynamo in figure 68 we shall denote the two vectors $\overline{p}'$ and $\mathrm{m}\overline{\omega}$ that vectorially multiply the input-flow. In three dimensions there will always be a vector connected with these two icons, as it can be shown, that when their anti-hermitean matrix transforms like a tensor by coordinate-shifts, then the three matrix-elements in the places 32, 13, and 21 will transform like the x-, y-, and z- coordinate of a vector, i.e. according to (148). If we expand the coordinate-shifts to such that change the "handedness" of the system, i.e. unitary transformations with the determinant -1, in stead of +1, that all transformations that can be generated continuously from the unit-matrix have, one distinguishes between *polar* or *proper* vectors that are conserved by such a transformation, and *axial* or *pseudo-* vectors, that are reversed. Here $\overline{p}'$ is polar and $\overline{\omega}$ axial. The vector-product of two polars is axial, but if one factor is axial, the product will be polar.

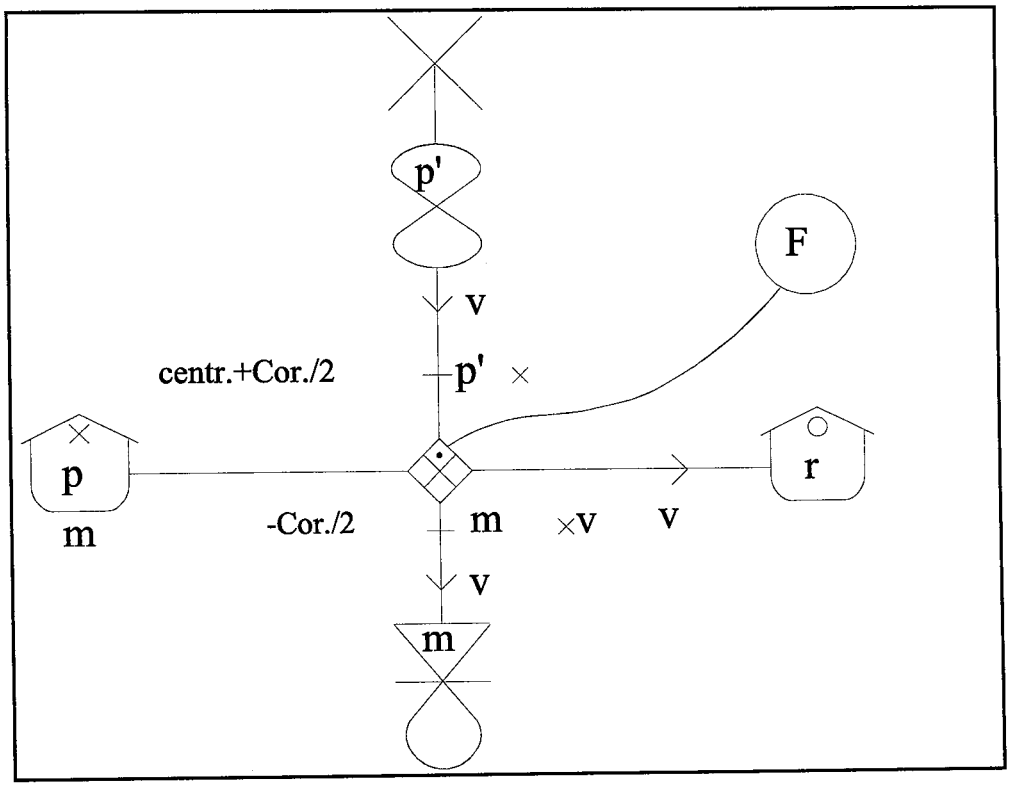


Figure 2. Particle in a rotating system.

The explanation of the flow-source, that it describes the work, that the drive must provide to keep the angular velocity constant, is only valid, if the particle can be kept fixed and be governed in a controlled way in the rotating system. If we look at a *free* particle that does not in any way interact with the rotating system (perhaps this system is a fictive construction), the drive, in fact, shall provide no work. Still, the model in figure 68 says that there from the source flows energy into the system with a rate, that is the scalar product of the centrifugal force and the velocity of the particle. This fictive work becomes kinetic energy of the particle in the rotating system, but if one cannot decelerate the particle relative to this system, then we cannot get hold of this energy, so we may imagine, that it "floats back to the source", so that this in reality shall not perform any work. If one in the rotating system can get hold of the particle and bring it to rest, one may convert its kinetic energy to something else, e.g. to heat, and then the source must

deliver the work in reality, that has been fictive during the free motion of the particle. The forces in the rotating system, that are used to decelerate the particle, cause a torque on the drive, which, thus, must deliver a work to maintain the angular velocity.

PART III, QUANTIZING

the Energy Bond Graph Approach to Quantum Semiotics

1. The quantized Sign Relation.

Quantum Semiotics (QS) is an attempt to rationalize the basic philosophy of Quantum Mechanics (QM) through EB-reticulations of the sign relations, including the measurement-process.

The Dirac-formalism in QM is taken for granted in QS, but we wish to take a closer look on the various epistemological and ontological questions that have divided physicists into different camps or "schools" of interpretation, such as the Copenhagen-interpretation and the "many-worlds"-interpretation. The purpose of this discussion is to remove the veil of paradoxicality from QM and expose it as a rational and coherent world-view. My philosophical attitude rests on Peirce's semiotic realism (that sign-processes take place in the physical world outside our consciousness), his doctrine of *tychism* (that chance or randomness exists as a real element in evolution) and his doctrine of *synechism* (that space and time are continuous and continuously related to mind). Furthermore, I find that *realism*, *tychism*, and *synechism* are well connected to Quantum-Philosophy through Peirce's *logic of vagueness*. Thus, I find no difficulty in accepting, that the wave-function of QM is a real, iconic sign, but that its meaning is vague, because the experimental context is open and the detection governed by real chance.

The fundamental concepts of the EBG-formalism are all defined by their transformation-properties with respect to certain operations, like time-and orientation-reversal, coordinate transformations, etc. This is in accordance with a modern view on the basics of mathematics as expressed by the Erlanger program and the *category-theory*. In these programs the theory of sets is considered less important than the theory of transformations. In Category-theory the fundamental concepts are not the mathematical individuals like points and numbers, but rather their way of relating to each others by transformations — homomorphisms, called *morphs* or *arrows*. EBG-theory-arrows, like transducers are generally belonging to Lie-groups, continuously connected with the identity, as synechism would require (*synechi* means to hang together). Every point in space-time is continuously connected with other points through translations. An energy bond associated with a point can be considered a generator of infinitesimal translations and thus a basic physical category. From a given point P two kinds of bonds may emerge: effort-bonds with effort-input to P and flow-bonds with flow-input to P. These bonds are attached to junctions with *weak causality*, the effort bonds to a common x-junction and the flow bonds to a common o-junction. Thus, the effort-bonds have flow-output to the external world and the flow-bonds have effort-output. These output-variables then define the *properties* of P, and the two junctions its *structure*. This view of a point is illustrated in figure 1.

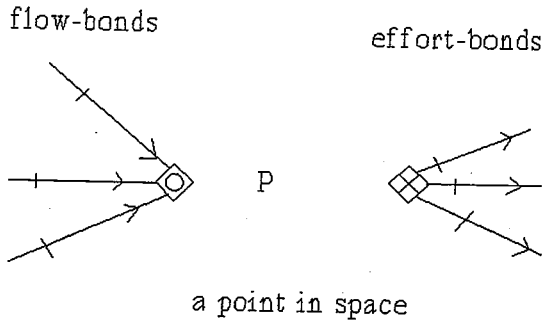


Figure 1. Structure and properties of a point.

If one of the bonds in figure 1 were to assume *strong causality* over its junction, this would disrupt both the structure and the properties of the point.

The disruption of signals around a junction that results, when a new bond takes dominance (strong causality) is the quantum-semiotic paradigm of *an elementary quantum-process*. It explains why there is quantization in the world. Furthermore, the phenomenon is *non-local*, because all the weak bonds that are suddenly "taken over" may belong to widely separated points in space. We have here the clue to quantum entanglement and EPR-non-locality. Still, it must be emphasized, that the EB-formalism is *explicitly local*. The bonds are generators of space and all connections through bonds are through space. There is no preestablished harmony or synchronicity. The "spooky action at a distance" that Einstein worried about seems to be real, but it is not spooky, unless the quantized process of assuming dominance over a junction is an instantaneous event. This then has to be investigated: We must develop a model of the elementary quantum process, although S. Kierkegaard stressed that "the qualitative jump" can not be analyzed in detail. There is one other discrete process in the EB-formalism: the forcing of a storage to assume differential causality. Actually, the domination of a junction is the same as the domination of a storage, since the storage always contains a hidden junction of the same type (x or o) as itself. All bonds that connect to the storage are normally weak bonds of the hidden junction dominated by a single bond from the storage (see figure 19 in sect. I.3), but when one of the previously weak bonds takes dominance, the storage assumes differential causality. As a storage contains a hidden junction, we might also expect that a junction contains a hidden storage. A node in an electric network has a small capacity and a mesh a small self-inductance, and the ideal conservative Kirchhoff-relations are only obtained in the limit when these capacities are neglected, although we know, that they do exist.

This would introduce a time-delay in the quantum process and presumably make superluminal signals impossible. In this way the EB-formalism (and QS in general) may support a *pragmatic locality* while maintaining a theoretical non-locality (entanglement). We shall later (chapter 7) look at a superconducting device *the resistively shunted Josephson diode* as providing a dynamical model in continuous time for the quantum process. We shall need, however, the existence of a constant of nature — $\hbar$ — the quantum of action — just as we needed c in the relativistic derivations in part II.

When we speak of an *object* as something having certain *properties* or attributes we are in fact outlining a sign-relation with only two factors, like I — O, where I is the property of the object O. This is the classical conception, in which the interpretation of I as a sign of O in no way disturbs O or the other properties it might have. An object is simply the sum of its properties, according to the positivists, and in the european tradition of semiotics, founded by the swiss linguist Ferdinand de Saussure, we find the same conception in considering the sign as a two-factor-relation between the *aignifiant*, the sign I and the *signifié, the object O*. This type of semiotics (or *semiology,* as Saussure called it) is clearly too narrow even in everyday-situations, that have nothing to do with QM. If, e.g. I ask a person "which number are you thinking of now?" and get the answer "17" it would be rather stupid of me to regard the number 17 as a property of the person in question. Rather, I would have to say, that the answer expresses a *propensity* of the person to produce that answer when asked that question. I could not be sure, that he would answer the same, if I asked him again. Thinking of a number is not a property of any person, but just a response that is *provoked* by the question. Perhaps the person was *really* thinking "what are we going to have for dinner to-night?" and the number 17 only appeared in his head when he heard the question. There is no reason to assume, that the problems of QS regarding properties and signs of atomic objects can be treated as a two-factor sign relation, that is so clearly incomplete in everyday-life as the example above shows. So we need, at least, a three-factor-relation, where what I called *the propensity* of the person has a place. Such a three-link sign-relation we find in the semiotics of Charles S. Peirce. Peirce's sign-relation may be depicted diagrammatically as I — R — O, where O is the object, R the *representamen* or sign-vehicle, and I the *interpretant.* In the example O is the person or the thoughts in his head, R is some *propensity* to produce a certain type of answer, and I is the actual answer. Similarly in QS we may let O represent an atomic object, R a propensity (in QM represented by the wave-function or state-vector), and I the result of some measurement. It is clear, then, that the peircean sign-relation may be arbitrarily expanded to the left in the diagram, thus K — J — I — R — O, where J is a mental interpretant of reading the measurement-result and K some thoughts about the significance of the result I. However, in QS we shall consider the establishment of a sign as a purely

physical process, and we therefore don't need the J and the K in the diagram above. We shall assume, that the physical process of measurement is the same, irrespective of, whether the observer reads the result on the meter or not. Normally, a measurement today is performed as a fully automatic process, and while it takes place the physicist is sitting elsewhere, drinking coffee, and she only reads the result some hours later. We shall assume, that the later act of reading can have no influence on the earlier measurement, i.e. we disregard "backwards causality" in measurement. Likewise, we shall consider it irrelevant, whether the physicist is interested in the result, or not, so we also disregard the possibility that the observer's consciousness might have any role to play in the physical process of measurement. We shall therefore only use the three-factor diagram I — R — O in our description of the measurement process as a physically linked chain from the object O to the measurement apparatus I. The mediating factor R then only represents the object in a *vague* sense. It is like "a smoky dragon" (to use a metaphor by J.A. Wheeler), so it is not "a property" of the object O and only a *propensity* for the production of the result I. And the reading of the result, thinking about it, and publishing it, is considered irrelevant to QS. The sign-diagram I — R — O thus has three factors, connected by two *links*. These links represent real energetic processes and may thus be conceived as energy bonds. The R — O-link is called *the preparation link* and the I — R-link *the detection link.*

As is normally done in physical applications of Peirce's semiotics (comp. part II.1) we classify the links (and thereby the sign) by Peirce's three phenomenological categories:

1. The *firstness category* stands for *the potential*, perhaps not yet actualized link.

2. The *secondness category* stands for *the actual*, perhaps not yet generalized link.

3. The *thirdness category* stands for the *general* or *habitual* link.

These categories are *inclusive* such that thirdness contains and presumes secondness, which again contains and presumes firstness.

We may regard the sign-diagram I — R — O as describing a signal, proceeding from O to I (the reason why we prefer the backwards or retrograde reading from right to left was discussed in part II). It is then clear, that the category of the detection-link I — R cannot exceed that of the preparation-link R — O. By this *selection rule* we obtain the result, that there are exactly 6 classes of signs in QS. (For a chain of n links there will be $\frac{1}{2}(n+1)(n+2)$ classes, as discussed in part

II.1). In figure 2 below these six classes are shown with their categories (io, where i is the detection- and o the preparation-category) also shown are Peircean names of the classes and corresponding signs of the Dirac-formalism.

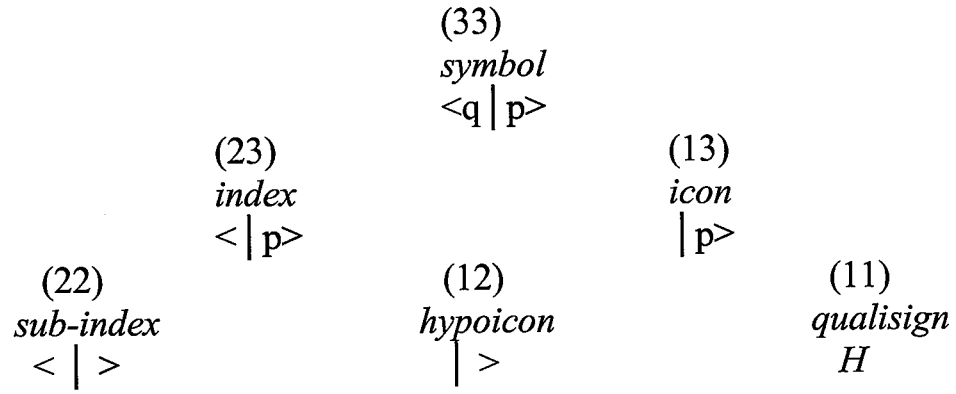


Figure 2. The six sign-classes of QS.

The signs of QS support the view of fundamental particles as *quanta of a field.* The "invisible" field lives in the R of the I — R — O-relation. It is like Wheeler's "smoky dragon" — Intangible, but with two "handles" to the phenomenal world — the preparation-link R — O and the detection-link I — R. In the philosophy of QS it is not just a coincidence, that the basic Fermion building blocks of matter are sixfold: six leptons (e, μ, τ, ν_e, ν_μ, ν_τ) and six quarks (u, d, s, c, t, b).

We shall now through some verbal comments try to relate the six sign-classes to the quantum world:

The *qualisign* (11) is the *Hilbert space H* the empty precursor of all signs. It is like the white paper before any words are written on it. *H* is a vector-space of potentially infinite dimensionality. It is a space of *complex* vectors, as the general EB-formalism requires.

The *hypoicon* (12) is related to Dirac's *empty ket* | >. Some unidentified particle enters from outer space — potentially detectable — but not registered in any way.

The *sub-index* (22). Here the empty ket is combined with an empty bra: < | >. The unknown particle is somehow registered, but we don't know, what the registration means — perhaps we are holding an empty can and hear a sound "pling" indicating that the can was hit by something.

The *icon* (13) deserves a symbol within the ket |p> because the particle is

prepared in a general way, such that we know, that it has a certain momentum p.

The *index* (23) or the Dirac-sign < | p>. The particle with a known momentum is somehow registered, but not in a general way, so we can't relate what was measured.

The *symbol* (33) is the full-fledged Dirac bra-ket <q | p>. It is the *amplitude* (whose absolute square is the probability) of a general detection of the generally prepared | p>-ket particle, in a specific location, described by the bra <q | .

The two links of the QS-sign-relation I — R — O describe elementary quantum processes and are therefore associated with assuming dominance over junctions as described earlier. It seems natural to let the emission of particles be represented with a flow-source, which will then dominate an x-junction connected to it. The various detectors may then be connected to a common o-junction, so that when one of them assumes dominance it will prevent the others from doing so. In this way the structure of the QS-sign-relation shown in figure 3 is able to account for the collapse of the wave-function.

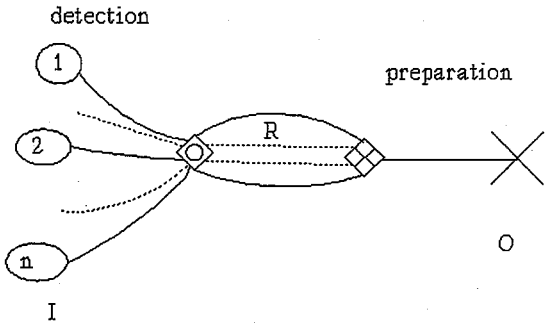


Figure 3. Structure of the QS-sign-relation.

2. EPR-entanglement.

The structure on figure 3 is much too crude to cover all applications. It cannot be true in general, that all detectors are coupled to the same o-junction, for it means that the action of one detector will exclude all the others. This is reasonable when the detections really are exclusive as, e.g. the detection of position on a photographic plate, where the detectors may represent the different sensitive grains in the photographic emulsion. Really incompatible detections like position and momentum of the same particle must have their detectors connected to different o-junctions. It is possible, however, that compatible detectors may connect to the

same junction if they are separated in space or time. As an example we can consider the famous EPR-experiment in D. Bohm's version, where two spin ½-particles in a singlet state are separated. We know in this case that a detection of particle 1 in a spin-up-state will not exclude the detection of particle 2 in a spin-down-state, but rather make the second detection redundant. Both these detections will project the two-particle-wave-function to the state $|1\uparrow 2\downarrow>$ and the same projection will result from a coincidence-detection of the two particles simultaneously with opposite spins. The three detectors $|1\uparrow>$, $|2\downarrow>$, and $|1\uparrow 2\downarrow>$ may thus be connected to the same o-junction, meaning that they are not exclusive, but that any of them will result in the same projection of the state-vector. As another example of this we shall consider the two-photon-experiments as conducted by Aspect et al. In this case the two photons are detected behind polarizers, so that if the polarizer-direction is the x-direction the detection of the photon means that it is x-polarized, and the non-detection means that it is y-polarized. The two photons are emitted in a state, where they have the same polarization, both x or both y:

$$|12>=\frac{1}{\sqrt{2}}(|1x2x>+|1y2y>) \qquad \textbf{(1)}$$

We shall now consider an experiment, where the polarizer for photon 1 is in the x-direction and the 2-polarizer is turned an angle ϕ from the x-direction If photon 1 is detected behind its polarizer, we may conclude that it is in the state $|1+>=|1x>$, and if it is undetected it is in the state $|1->=|1y>$. As in the second of Aspect's experiments there may be detectors to register the photons that do not pass the polarizers, such that all the four states $|1+>$, $|1->$, $|2+>$, and $|2->$ may be realized. As the photon is a spin 1 or vector-particle, we may find its components after the new states by simple projection, thus:

$$\begin{aligned} |2+>&=\cos\phi|2x>=\sin\phi|2y> \\ |2->&=-\sin\phi|2x>=\cos\phi|2y> \end{aligned} \qquad \textbf{(2)}$$

and therefore:

$$\begin{aligned} |2x>&=\cos\phi|2+>-\sin\phi|2-> \\ |2y>&=\sin\phi|2+>+\cos\phi|2-> \end{aligned} \qquad \textbf{(3)}$$

We can then expand the original state (1) on the new basis-states:

$$|12>=\frac{1}{\sqrt{2}}(\cos\phi(|1+2+>+|1-2->)+\sin\phi(|1-2+>-|1+2->) \qquad (4)$$

We can now reticulate both the superposition (4) and the logic of detectors. In figure 4 we use the abbreviations q=√2, s=sinφ, and c=cosφ. The x-junctions and transformers shows the expansion (4). The dot in the lower x-junction takes care of the minus-sign in the second term. Two detectors are shown at each of the four o-junctions and optional coincidence counters may be added, e.g. (1+2+) at the junction with the single-detectors (1+) and (2+). Only one detector at each o-junction is sufficient to produce the projection (collapse) of the wave-function, the others (including the optional coincidence-counters) are redundant.

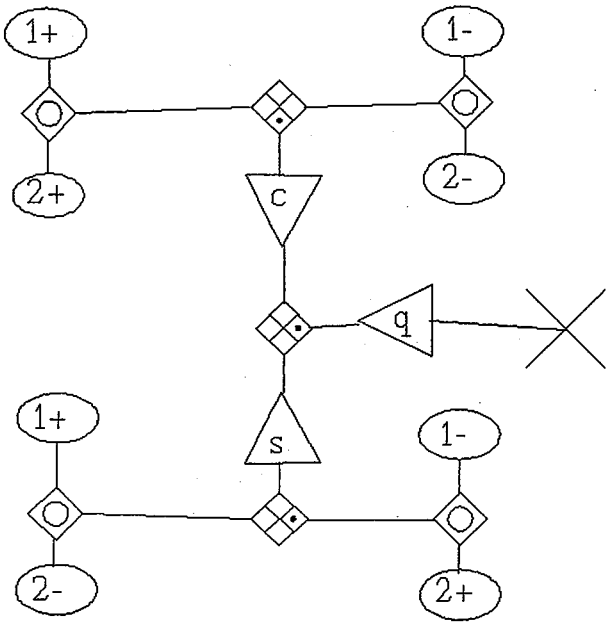


Figure 4. EB-reticulation of two-photon EPR-experiment.

We recognize in figure 4 in basic structure of figure 3 as well as the necessary modifications.

The property of *quantum entanglement* is traditionally associated with a violation of *Bell's inequalities*. These inequalities may be formulated in many different ways, but we shall consider a version that uses probabilities of coincidence-

measurements. We consider a situation where the polarizers at particle 1 and 2 each have two different settings: the angles a_1 and b_1 for particle 1 and a_2 and b_2 for particle 2. We now consider the following sum of coincidence-probabilities:

$$B_c = p(a_1 \wedge a_2) + p(a_1 \wedge b_2) + p(b_1 \wedge a_2) - p(b_1 \wedge b_2) \quad (5)$$

Bell's inequalities then states, that

$$0 \leq B_c \leq 1 \quad (6)$$

Now, the coincidence probabilities are all given by $\frac{1}{2}\cdot\cos^2 u$, where u is the difference between the two angles involved. For the case already considered in (4) and figure 4 we have:

$$a_1 = 0;\ b_1 = \pi/2;\ a_2 = \phi;\ b_2 = \pi/2 + \phi. \quad (7)$$

so we get:

$$B_c = \tfrac{1}{2}\cos^2\phi + \tfrac{1}{2}\sin^2\phi + \tfrac{1}{2}\sin^2\phi - \tfrac{1}{2}\cos^2\phi = \sin^2\phi \quad (7)$$

So in this case the inequalities (6) are obviously satisfied and no direct sign of entanglement is obtained this way, but it lies inherent in the probabilities, which may be demonstrated, if we put:

$$a_1 = 0;\ a_2 = \phi\ \ b_1 = b_2 = -\phi \quad (8)$$

In this case we get:

$$B_c = 2\cos^2\phi - 1 \quad (9)$$

Which violates the left inequality (6) for $\phi > \pi/4$.

Historically, the violation of Bell's inequality after their formulation in 1960 has been the great challenge to experimentalists and especially Aspect's results have nourished a lot of rather weird speculations of an antirealistic flavour about how mysterious and counter-intuitive the world according to QM really is. The Copenhagen-interpretation has been firm in its orthodox faith in the quantum-

formalism, but has also supported an anti-realistic view on the wave-function and its collapse. The outcome of the many discussions has been a flourishing of different "schools" of interpretation of which "the many-worlds-interpretation" (MWI) seems to be winning in terms of number of adherents and weirdness of a totally unfalsifiable philosophy. The philosophy of QS differs from both Copenhagen and MWI by taking a realistic view on the wave-function and in believing that its collapse is a real event.

As we have seen, Quantum entanglement is inherent in the superposition of the state-vector (4) and its connection with the logic of detection as in figure 4. The non-locality of QS is reflected in the properties of junctions in the quantized sign-relation and has really not much to do with the violation of Bell's inequalities. A lot of experimental settings will satisfy the inequalities without proving the locality-principle or disproving quantum-entanglement.

One of the questions that has been settled recently is the question whether the actual use of coincidence-counters is vital to the production of entangled probabilities. Experiments performed by A. Zeilinger et al. in Innsbruck (1998) proved that this is not the case. The reticulation in figure 4 is in accordance with this result by showing that the coincidence counters are optional, but redundant. Detection of single particles is sufficient to take dominance over the relevant o-junction and thus produce the projection of the wave-function to one of its entangled rays.

3. Dimensioning bonds.

As the energy bond and its related sign-system is a very general concept we need some ways of distinguishing between the different types of bonds. One way of characterizing a bond is by its *metric*, another way is by its *dimension*. The latter is defined by the exponents, M, K, S, and A, by which the four fundamental units of the SI-system, the *meter*, m, the *kilogram*, k, the *second*, s, and the *Ampère,* a, define the unit of the variable in question. (which is thus $m^M k^K s^S a^A$).We shall consider four dynamic variables, the o-level, q, the x-level, p, the flow, f, and the effort e, and three response-properties, the compliance, C, the inertance, L, and the resistance, R. The table below shows how the M,K,S,A-exponents of these 7 quantities are related:

o-level q	M	K	S	A
x-level p	2-M	1-K	-1-S	-A
x-rate f	M	K	S-1	A
o-rate e	2-M	1-K	-2-S	-A
Compliance C	2M-2	2K-1	2S+2	2A
Inertance L	2-2M	1-2K	-2S	-2A
Resistance R	2-2M	1-2K	-1-2S	-2A

Table 1. The general case.

From this table follows dimensional relations, that are valid for all types of bonds, such as: pq has always the dimension *action* (m^2ks^{-1}) and CR^2/L is always *dimensionless*.
From the "spread-sheet" appearance of table 1 follows that all dimensions are given by any row, so we may define the type from the dimension of q alone. In the following tables we have added one more column giving names of the units. First, we consider *electric bonds*:

	M	K	S	A	
q	0	0	1	1	Coulomb
p	2	1	-2	-1	Weber
f	0	0	0	1	Ampère
e	2	1	-3	-1	Volt
C	-2	-1	4	2	Farad
L	2	1	-2	-2	Henry
R	2	1	-3	-2	Ohm

Table 2. Electric bonds.

The case when q is dimensionless like an angle and p an action may be called *angle-action bonds* or just *angle bonds* is shown in the next table:

	M	K	S	A	
q	0	0	0	0	angle
p	2	1	-1	0	action
f	0	0	-1	0	frequency
e	2	1	-2	0	energy
C	-2	-1	2	0	
L	2	1	0	0	
R	2	1	-1	0	action

Table 3. Angle-bonds.

Finally, we consider *neutral bonds* where x- and o-variables have the same dimension:

	M	K	S	A	
q	1	½	-½	0	
p	1	½	-½	0	
f	1	½	-3/2	0	
e	1	½	-3/2	0	
C	0	0	1	0	
L	0	0	1	0	
R	0	0	0	0	

Table 4. Neutral bonds.

We shall in the following proceed with the assumption, that angle-bonds belong to a classical level of description, while neutral bonds are elementary bonds of a quantum level. The fact that p and R of the angle bonds are actions suggests that they may be quantized. The angle q is also naturally quantized to the number of full rotations or the number of particles absorbed. The angular momentum p is known to be quantized in units of $\hbar$, and quantization of R is also known from nano-bridges or the quantum-Hall-effect. The quantized sign-relation figure 3 makes it natural to associate quantized values of p and q with classical counting numbers of the elementary quantum processes of taking dominance at the o- and x-junctions, respectively. The distinction between o- and x-variables by the time-

reversal-criterion certainly belongs to a classical description, so a fundamental quantum level must have an unbroken x-o-symmetry and thus be described with neutral bonds. Different types of bonds can be related through a *dimensional transducer*, which is just a transformer with dimension, but without numerical value. Thus, we go from a general bond to an angle-bond through a transducer q which is just the q-dimension in the general bond. From the angle-bond we may go to a neutral bond by a transducer, that is the square root of an action, say $\sqrt{\hbar}$ (as we can't distinguish between h and $\hbar$ in a dimensional transducer. These relations (from table 1 to 3 to 4) is shown on figure 5.

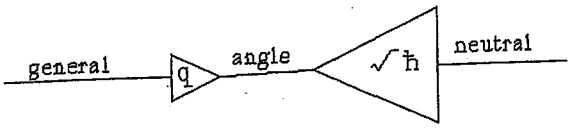


Figure 5. Dimensional transducers between bond-types.

4. Spinor-bonds.

The idea of a fundamental quantum-level where the x-o-symmetry is unbroken not only means that such bonds are neutral, that flows and efforts have the same dimension, but also that they must be indistinguishable by the time-reversal-criterion. This requires, that the bonds are at least two-dimensional, for, as we have seen in part II there are only two types of one-dimensional bonds, whether they are real or complex, namely *scalar* and *pseudoscalar* bonds, and for both these types time-reversal applies (though in different ways). Two-dimensional real bonds may have, e.g., a σ_z-metric, and in this case flows and efforts mix, such that the second component of the effort-vector is a flow (and the second component of the flow is an effort), but flows and efforts are still different. For one-dimensional complex bonds there is also a kind of mixing, as the imaginary part of the effort is a flow (although in a pseudoscalar bond) and vice versa. It is therefore natural to look for a complete x-o-symmetry in two-dimensional complex bonds, i.e. the *spinor bonds* of standard QM. In a spinor-bond the covariant effort-vector will have the form:

$$e. = \begin{pmatrix} a \\ b \end{pmatrix} \qquad \textbf{(10)}$$

where both a and b are complex numbers. Such spinors, that are normalized by the condition $|a|^2+|b|^2=1$ all indicate a direction in space given by the unit-vector $=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$ in the sense that the spinor is an eigenvector with eigenvalue +1 or -1 to the Pauli-spin-operator

$$\sigma(\theta,\phi)=\sigma_x n_x+\sigma_y n_y+\sigma_z n_z=\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix} \qquad (11)$$

The eigenvector to $\sigma_{\bar{n}}$ with eigenvalue +1 is then the spinor:

$$|+1,\theta,\phi>=\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix} \qquad (12)$$

and the eigenvector with eigenvalue -1 is

$$|-1,\theta,\phi>=\begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2}e^{i\phi} \end{pmatrix} \qquad (13)$$

The vector with the direction opposite to $\bar{n}$, i.e. $-\bar{n}$ is obtained by changing the angles θ and ϕ to $\theta'=\pi-\theta$ and $\phi'=\pi+\phi$. We see then by comparing (12) and (13) that:

$$|-1,\theta,\phi>=|+1,\theta',\phi'> \qquad (14)$$

So the same physical situation can be described with both the original and the opposite direction of $\bar{n}$ if we, when changing "orientation" also change the sign of the spin-eigenvalue. This rule is similar to the flow-orientation-rule, but for the

spinor-bonds we can't identify the spin-eigenvalue as a flow, because flows and efforts are supposed to be indistinguishable. However, the fact that every spinor (10) uniquely determines a direction $\overline{n}$ for which the spinor is an eigenvector suggests that the spinor be associated with a directed bond. The reversal of $\overline{n}$ then corresponds to an orientation-shift.

An orientation-shift is also a reversal of *spatial parity*. In view of (14) a parity-reversal is also a sign-change of the spin-eigenvalue, so we could try to define the parity as a linear operator P' by the condition

$$P'|+1,\theta,\phi> = |-1,\theta,\phi> \qquad \textbf{(15)}$$

In this way we find:

$$P' = \begin{pmatrix} 0 & e^{-i\phi} \\ -e^{i\phi} & 0 \end{pmatrix} \qquad \textbf{(16)}$$

However, parity-shift must be a mirroring-operator, but (16) gives $P'^2=-1$, and the actual parity-operator P must satisfy $P^2=1$, so we define instead:

$$P = iP' = \begin{pmatrix} 0 & ie^{-i\phi} \\ -ie^{i\phi} & 0 \end{pmatrix} \qquad \textbf{(17)}$$

We may then determine eigen-spinors to P with eigenvalues +1 and -1. We find for $a=\pm 1$:

$$|a,P> = \frac{1}{\sqrt{2}} \begin{pmatrix} a \\ -iae^{i\phi} \end{pmatrix} \qquad \textbf{(18)}$$

So, given an arbitrary spinor of the form (10) there are uniquely defined angles of direction, θ and ϕ, such that the spinor is $|+1,\theta,\ \phi>$, given by (12), and for this direction there is a parity-even spinor $|+1,P>$ and a parity-odd spinor $|-1,P>$, given by (18).

We might get the idea, that P acts as a time-reversal operator, because it changes the sign of the spin-eigenvalue, but, as we saw in chapter II.4, the time-reversal

operator T is *antilinear* because it involves the complex-conjugation-operator K, but P is linear. A possible expression for T, satisfying the mirror-condition $T^2=1$, is the operator

$$T=-ie^{i\phi}\sigma_y K=\begin{pmatrix} 0 & -ie^{i\phi}K \\ -ie^{i\phi}K & 0 \end{pmatrix} \qquad \textbf{(19)}$$

By comparing this expression with the general form of time-reversal in chapter II.4 we see, that a spinor-bond must have genus $\gamma=ie^{i\varphi}$ and metric $g_{..}=\sigma_x$. Earlier we expected $\gamma=\pm i$ for spinor- or neutral bonds because this would make time-reversal look the same for flows and efforts ($-\gamma^{*}=\gamma$).

It seems that not all spinor-bonds are equally neutral, so we are led to consider the special case

$$\theta=\frac{\pi}{2};\varphi=0 \qquad \textbf{(20)}$$

As defining *neutral spinor-bonds*. In this case the spinor will be an eigenvector to the metric $g_{..}=\sigma_x$, the parity-operator is $P=-\sigma_y$, and the time-reversal operator:

$$T=-i\sigma_x K=\begin{pmatrix} 0 & -iK \\ -iK & 0 \end{pmatrix} \qquad \textbf{(21)}$$

This indicates a σ_x-metric and genus $\gamma=-i$.

As the parity (σ_y)-eigenvectors $|+1,P\rangle$ and $|-1,P\rangle$ (18) with respect to orientation-shift behave similarly to efforts and flows in scalar bonds, we shall see how they behave by time-reversal (21). We find:

$$T|+1,P\rangle=e^{-i\varphi}|+1,P\rangle;T|-1,P\rangle=|+1,P\rangle \qquad \textbf{(22)}$$

Having defined parity P and time-reversal T as mirroring operators, we would like to identify a third mirroring, viz. *charge-conjugation C*, because these three operators are connected by the famous *CPT*-theorem of quantum field theory. So

we define

$$C = PT = -\sigma_y \sigma_x K = i\sigma_z K = \begin{pmatrix} iK & 0 \\ 0 & -iK \end{pmatrix} \qquad \textbf{(23)}$$

and we verify easily, that C satisfies the mirroring-condition $C^2 = 1$, which may also be expressed as CPT=1.

The significance of charge-conjugation cannot be fully appreciated yet, because it is rooted in Dirac's relativistic theory, that needs the concept of *antiparticles* and an algebra of 4-dimensional *double-spinors*, so what we have formulated above can only be a shadow of the CPT-theorem.

A general spinor-bond may have a metric belonging to the equivalence-class $\sigma(\theta, \phi)$. This whole class is generated from the standard metric σ_z by 1-unitary transducers $U(\theta,\phi)$:

$$\sigma(\theta,\phi) = U(\theta,\phi)\sigma_z U^{-1}(\theta,\phi) \qquad \textbf{(24)}$$

with

$$U(\theta,\phi) = \cos\frac{\theta}{2} + i\sin\frac{\theta}{2}(\sin\phi\,\sigma_x - \cos\phi\,\sigma_y) \qquad \textbf{(25)}$$

For the neutral spinors with σ_x-metric we have $\theta = \pi/2$, $\varphi = 0$, so (25) gives:

$$U = \frac{1}{\sqrt{2}}(1 - i\sigma_y);\; U^{-1} = \frac{1}{\sqrt{2}}(1 + i\sigma_y) \qquad \textbf{(26)}$$

5. The Schrödinger equation.

The QS-description of the quantized sign-relation I — R — O in figure 3 shows the way from preparation (O) to detection (I) of a particle through an R-field that must be regarded as a maze of energy-bonds connected through junctions and transformers. This field may also contain x- and o-storages to represent kinetic and potential energy of the field, but all the relations embedded in the R-field are *linear*, so the solution to the equations of motion must be *wave-like*.

The state-vector $|p\rangle$ of a generally prepared particle was described in chapter 1 as an *icon* (figure 2). This means, that its symbolic interpretant (e.g. a set of

numbers) only exists as a possibility — a Peircean firstness. We may develop an *indexical* notion of the state as a *wave-function* — a set of o-levels attached to bonds in every spatial point of the R-field, but the whole mesh of interconnected simultaneous feedbacks, including unstable loops of gain-factor 1, makes every attempt to assign symbolic values to these levels self-denying. It is like trying to assign truth-value to a paradoxical self-referring statement, like the liar-paradox "this sentence is not true" — if it is true, it is false, and if it is false, it is true.

The most common view on the ontological question of how the quantum world really is based on the particle-concept: A particle is *localized* — it must be at some definite place at any time. However, trying to describe a localized state in the R-field, e.g. by assigning unit value to the o-level of one particular point and zero to all the others, one will quickly discover, that such a state cannot be stable. Such a "particle" will jump erratically around from one storage to the others and will not accept localization. The ontology of QS must therefore be built on the wave-concept — not the wave itself, but the underlying structure of energy bonds that sustains the wave and defines the equation it must obey.

For every point in space there is an energy bond with a o-level Ψ — a dimensionless *wave-amplitude.* All the amplitudes form together a wave-function or state-vector Ψ. The vector-bond containing the dimensionless o-level Ψ is then an *angle-bond* as defined in chapter 3. To every angle-bond we may then associate a dimensionally *neutral* bond by the dimensional transducer $\sqrt{\hbar}$ as shown in figure 5. We shall at first think of the neutral bonds as defined by their dimension alone (table 4). As associated to the scalar angle-bonds by a simple dimensional transducer they are also scalar bonds — we shall not regard them as spinors or double spinors, which would require a metric transducer as well. So for every neutral bond we shall now assume that we can distinguish efforts from flows. If the o-level-vector of the angle-bonds Ψ is reckoned contravariant ($\Psi^{\cdot}$) all the flows together form a contravariant flow-vector $F^{\cdot}$ and all the efforts form a covariant effort-vector E. E. In the angle-bonds E. will have the dimension energy and be related to $\Psi^{\cdot}$ through a linear operator H — the Hamiltonian operator, representing the energy of the system:

$$E_{\cdot} = H_{\cdot\cdot} F^{\cdot} \qquad (27)$$

The dynamics of the system can then be described in a *symplectic* form by a relation between the flows and the efforts of the neutral bonds. The most general prescription for this relation is *Tellegen's theorem* stating that the effort and the flow belong to *orthogonal subspaces* (chapter II.10). The flow is therefore given

by the effort through an *antihermitean 1-port*, i.e. a *dynamo* with effort-input. For scalar bonds, as the neutral bonds are here considered, there exists only one such dynamo, given by the imaginary unit i (or -i). Thus, we get the symplectic dynamics of figure 6 below:

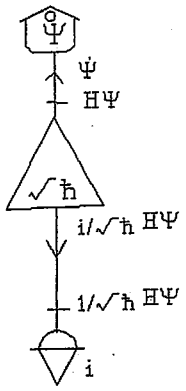


Figure 6. Symplectic dynamics.

Figure 6 then gives the symplectic form of the Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \qquad \textbf{(28)}$$

and the reason for the occurrence of the imaginary unit i has been shown to follow from Tellegen's theorem. For comparison we have the symplectic formulation of Hamilton's equations of classical mechanics:

$$\begin{pmatrix} \frac{dq}{dt} \\ \frac{dp}{dt} \end{pmatrix} = i\sigma_y \begin{pmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{pmatrix} \qquad \textbf{(29)}$$

The antihermitean operator $i\sigma_y$ in (29) may be used as the dynamo of figure 6 if the neutral bonds are spinor-bonds with metric σ_y instead of scalar bonds.

The symplectic reticulation of figure 6 is incomplete, because it does not show the structure of the Hamiltonian operator H. In order to do this we must be more explicit about the dynamical model. We shall therefore consider the simple case of a free particle with mass *m* moving in one dimension *x*. The Hamiltonian is then given by:

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \tag{30}$$

The imaginary unit i in (28) causes the scalar wave-function to be complex: $\Psi(x) = a(x) + i\, b(x)$, and the reticulation in figure 7 shows that if a is a o-level, then b must be a x-level. The figure shows a segment of length Δx to be repeated along the x-axis, and the following abbreviations are used:

$$\Delta b = \Delta x \cdot \frac{\partial b}{\partial x};\ \Delta a = \Delta x \cdot \frac{\partial a}{\partial x};\ K = \frac{2m\Delta x}{\hbar} \tag{31}$$

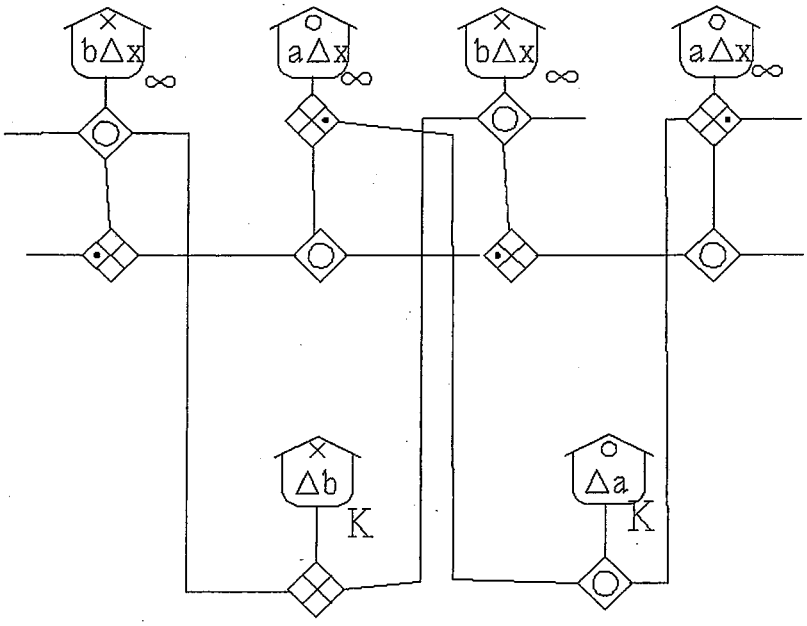


Figure **7.** One-dimensional Schrödinger equation.

The storages above in the figure are shown as *cyclic* (capacity ∞) because the particle is free. If it moves in a potential V(x) the cyclic storages must be replaced with active storages with output V(x).

For a *free* particle there is only kinetic energy in the imaginary Δb storage but also a kind of elastic potential energy in the Δa storages. The waves are thus similar to elastic waves on a flexible string.

Although the imaginary unit *i* does not appear explicitly in figure 7 the general

vector-reticulation figure 6 shows that it is present implicitly through the dynamo-representation of Tellegen's theorem. A material implementation of all the EB-icons of a graphic model will obey the Schrödinger-equation and thus make the model a *quantum analog computer*. Such an implementation can be made of *superconducting* electric wires. According to the Ginzburg-Landau theory the state of a superconducting wire is described by two fields: a *scalar* field $\psi(x,t)$ that is the common wave-function of the particles in the super-condensate (the Cooper-pairs) and a vector-field A(x,t) that is the magnetic vector-potential. In one dimension the x-component of A can be expressed as

$$A = L'_M \cdot I \tag{32}$$

Where L'_M is the magnetic self-inductance per unit length and I is the current. The complex scalar field ψ is conveniently expressed by two real fields f and ϕ as

$$\Psi(x,t) = f(x,t)e^{i\phi(x,t)} \tag{33}$$

This representation has the advantage that f and ϕ are readily interpreted in terms of particle-properties: while the square $f^2 = n(x,t)$ is the density of particles, the phase ϕ is related to their momentum p by the relation

$$p = \hbar\frac{\partial\phi}{\partial x} \tag{34}$$

Although Heisenberg's uncertainty-relation in general forbids us to assign simultaneous definite values to the position and the momentum of a particle it is allowed for superconductors, because the uncertainty, so to speak, is diluted over all the condensate-particles. We may then for a wire with cross section area σ introduce the velocity v of the particles by the relations:

$$\hbar\frac{\partial\phi}{\partial x} = Mv + QA = Mv + \sigma L'_M \cdot Q f^2 v \tag{35}$$

Where M is the mass and Q the charge of a condensate-particle (M=2m and Q=2e for Cooper pairs) and where we have used equations (34) and (32).

Eq. (35) may be rewritten in a form that suggests a reticulation by two self-

inductances connected in series, i.e. by an x-junction:

$$I = \frac{\Delta\phi}{\frac{M\Delta x}{\hbar\sigma Q f^2} + L'_M \Delta x} = \frac{\frac{\partial\phi}{\partial x}\Delta x}{I' + L_M} \tag{36}$$

Where I' is the *inertance* and L_M the magnetic inductance of a section with length Δx. This section may then be described as containing an x-level that is the sum of $\Delta\phi$ and a magnetic flux F as shown in figure 8:

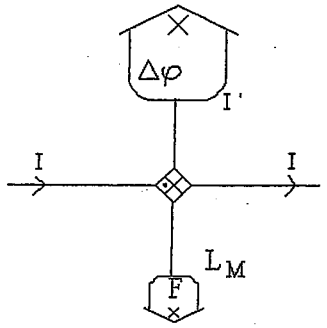


Figure 8. x-storage for Δx-section.

We have seen that the interpretation of f^2 as a density of particles and of the ϕ-gradient as the momentum of these particles is allowable when there is a condensate of many particles as in the Ginzburg-Landau theory of superconductivity. A similar interpretation of the wave-function of one particle is wrong, because it violates the Heisenberg uncertainty-relation. Still, it gives a clue to a possible hidden variable-interpretation, and as such it is used in the theory of D. Bohm. We shall try to reticulate this theory, because it has been suggested as a possible explanation of Quantum-non-locality. By introducing the expression (33) in the Schrödinger-equation (28) with the Hamiltonian (30), we get two equations, one by collecting the real terms and one by collecting the imaginary terms. The real terms give:

$$\frac{\partial\phi}{\partial t} = \frac{\hbar}{2M}\left(\frac{1}{f}\frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial\phi}{\partial x}\right)^2\right) \tag{37}$$

And the imaginary terms give:

$$\frac{\partial f}{\partial t} = -\frac{\hbar}{2M}\left(f\frac{\partial^2 \phi}{\partial x^2} + 2\frac{\partial f}{\partial x}\cdot\frac{\partial \phi}{\partial x}\right) \qquad \textbf{(38)}$$

Of these two equations the last one, eq. (38) is readily interpreted as a continuity equation. We have seen that the particle-density is f^2, and the current-density of particles is

$$i = \frac{I}{Q\sigma} = f^2 v = f^2 \frac{\hbar}{M}\frac{\partial \phi}{\partial x} \qquad \textbf{(39)}$$

So the continuity equation

$$\frac{\partial f^2}{\partial t} + \frac{\partial i}{\partial x} = 0 \qquad \textbf{(40)}$$

is seen to be identical with eq. (38).

This may then be reticulated with figure 9:

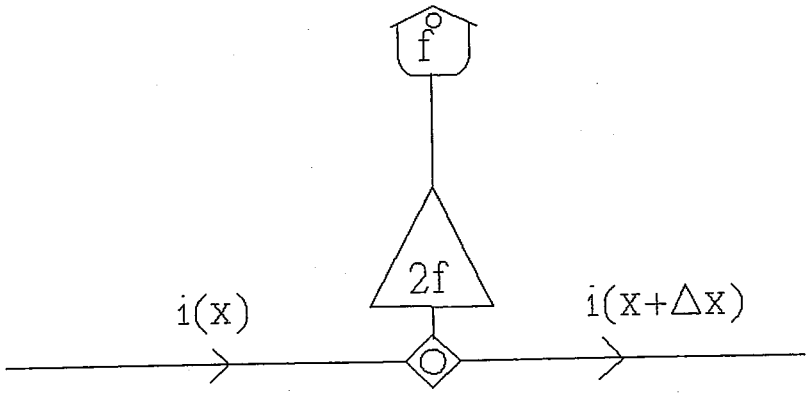


Figure 9. Reticulation of eq. (38).

For a final reticulation we prefer to replace the current density i with the current iσ. (σ is the cross-section area). We shall neglect the charge Q and the magnetic inductance L_M of figure 8. The inertance I' must then be replaced by

$$I'' = \frac{M\Delta x}{\hbar\sigma f^2} \tag{41}$$

As the o-level f is dimensionless, we are dealing with angle-bonds, so x-levels must have dimension action. We therefore introduce $\hbar\phi$ and $\hbar\Delta\phi$ as x-levels.

Although the f-storage in figure 9 may be regarded as cyclic for a free particle, the effort-output from it is not zero, because the kinetic energy of the x-storage in figure 8 depends on f. We therefore get a "Bernoulli" output effort from the f-storage, given by:

$$V_B = (\frac{\partial}{\partial f})\frac{(\Delta\Phi)^2}{2I''} = f\hbar\sigma\Delta x\left(\frac{\partial\Phi}{\partial x}\right)^2 \tag{42}$$

This effort is recognized in the second term in eq. (37). The first term is D. Bohm's "quantum-potential" describing how the wave-function f acts as a "guide-wave" governing the motion of the particle in a non-local way.

The quantum potential may be included in a reticulation if the particle-density-gradient Δf is associated with an "elastic" potential energy like Δa in figure 7. Regarding f in figure 9 as belonging to the point x we may associate Δf- storages in $x-\frac{1}{2}\Delta x$ and $x+\frac{1}{2}\Delta x$. If these storages have an elastic compliance C, the quantum potential arises as the gradient of the effort-outputs $\Delta f/C$:

$$\Delta(\frac{\Delta f}{C}) = \Delta x\cdot\frac{\partial}{\partial x}(\frac{\Delta x}{C}\frac{\partial f}{\partial x} = \frac{\Delta x^2}{C}\cdot\frac{\partial^2 f}{\partial x^2} \tag{43}$$

We then have the following reticulation:

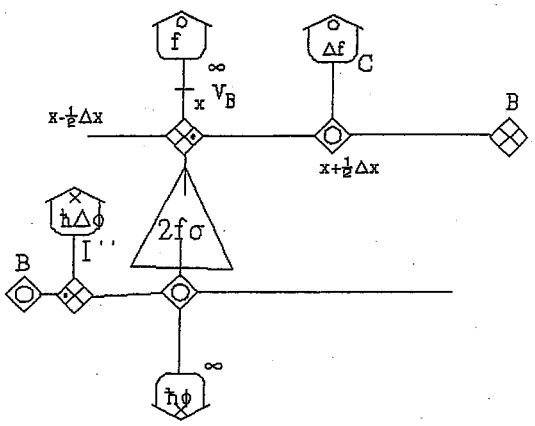


Figure 10. Including the quantum potential.

The correct value of the quantum potential V_Q is obtained for

$$C = \frac{M\Delta x}{\sigma \hbar^2} \tag{44}$$

In figure 10 the phase ϕ is represented as an x-level conjugate or complementary to the number-density f as these two levels are connected through a two-port x-junction like the coordinate q and the momentum p in the Hamiltonian equations.

The boundary-conditions $\Delta\phi=0$ and $\Delta f=0$ are represented by the two one-port-junctions, marked B.

The quantum potential is described as non-local in Bohm's theory, but the reticulation in figure 10 exhibits it as entirely local and due to the elastic energy of the Δf-storages. For *two* particles with coordinates x_1 and x_2 the quantum potential for particle 1 will depend on x_2 in a "parametric" way, and this looks like a kind of non-locality, but considering that the coordinate x_2 enters the reticulation just like the second coordinate y_1 of particle 1 and that these two coordinates are connected to the x_1-dimension of figure 10 at the o-junction the whole "ghost of non-locality" associated with the quantum-potential seems to disappear.

6. Decoherence and collapse.

The realistic view of QS on the wave function also forces us to regard the collapse or reduction of the wave function as an objective process. The Copenhagen interpretation, in contradistinction, takes a *nominalistic* view in regarding the wave function as a kind of probability distribution and its collapse as something that takes place in our mind rather than in the external world. However, the phenomena of quantum interference show that the wave function is *not* a probability (but a probability *amplitude*) until a measurement transforms it into a probability. If a measurement projects the state into one of two possible eigenstates $|1\rangle$ or $|2\rangle$ of the measured observable, the state before measurement is described as a *superposition*:

$$|\Psi\rangle = a|1\rangle + b|2\rangle \qquad (45)$$

where a and b are complex numbers. The measurement then projects $|\psi\rangle$ to $|1\rangle$ with probability $|a|^2$ and to $|2\rangle$ with probability $|b|^2$. By the same process the pointer of the measurement apparatus will be brought to show one of the eigenvalues λ_1 or λ_2 with the same probabilities. So the pointer is not in a superposition after the measurement. However, if we try to describe the development during the measurement process by the Schrödinger equation for the quantum system coupled to a measurement apparatus we shall always find, that the pointer ends in a superposition of pointing to λ_1 *and* λ_2. This is in fact the problem presented by Schrödinger's cat. Orthodox Copenhagen logic would deem the cat to the fate of being dead *and* alive at the same time (i.e. a superposition), but we know full well, that it is described by the classical logic of being *either* dead *or* alive with the two possibilities weighted by real probabilities ($|a|^2$ and $|b|^2$) , not by complex amplitudes (a and b). Somewhere in the measurement process a transition takes place that necessitates a change from quantum to classical description, but QM has proved unable to place the "cut" between the quantum and the classical world. Niels Bohr tried to describe the "cut" by saying that we enter the classical world in the limit of large quantum numbers but couldn't say how large they should be. Recent experiments with macroscopic quantum effects in superfluids and double-slit-interference with particles as large as Fullerene (C_{60}) or hemoglobin molecules show that the "cut" extends to much larger quantum numbers than could be imagined in Bohr's time. In QS we shall try to be more precise by identifying the "classical" description as one where thermodynamic *dissipative* processes take place. Every measurement must lead to a permanent mark of registration, so a measurement is necessarily an irreversible

and dissipative process. We must here emphasize that, when speaking of "the measurement process" we mean a whole cycle of measurement, including the necessary amplification and the final act of bringing the apparatus back to "the null state", so it is ready to perform a new measurement. In this way we can be sure that every measurement is associated with entropy production. Entropy production is necessary in order for the measurement to provide information. According to physical information theory, as developed by L. Brillouin an information I (measured in bits) must be accompanied by an entropy production ΔS limited by

$$\Delta S \geq I \cdot k \ln 2 \qquad \textbf{(46)}$$

Where k is Boltzmann's constant.
There is one more important reason why the collapse and the "cut" must be associated with the occurrence of dissipation, and this reason is the Fluctuation-Dissipation (FD)-theorem of Callen and Welton. According to this important theorem (to be discussed in detail in later chapters) a dissipative process is always accompanied by *random noise*, so the FD-theorem is able to explain, why chance and probability enters the quantum description in connection with the dissipative measurement process. This explanation also makes clear, that a measurement is not the only way for chance to enter the picture. Other dissipative phenomena, like friction and diffusion may also serve to change amplitudes to probabilities, or to introduce *decoherence*, i.e. the destruction of quantum interference effects.

The transition from a pure state of superposition like (45) to a statistical mixture of the same basic states $|1\rangle$ and $|2\rangle$ is exhibited more clearly by representing the state as a *density matrix* which for a pure state will be the *projection operator*:

$$P = |\Psi\rangle\langle\Psi| = \begin{pmatrix} |a|^2 & ab^* \\ ba^* & |b|^2 \end{pmatrix} \qquad \textbf{(47)}$$

For a density matrix in general the diagonal elements will be probabilities, whereas the non-diagonal elements are complex numbers that describe quantum interference between the base-states. The collapse — or decoherence-process — is then simply a process that *diagonalizes* the density matrix. We shall see that diagonalization can be the result of the system's natural development, when it is coupled to the environment or to a measuring device.

For a Hamiltonian H the state $|\psi\rangle$ will develop by the Schrödinger equation (28)

and the density matrix P by the von Neumann equation:

$$\frac{dP}{dt} = \frac{1}{i\hbar}(HP - PH) \qquad \textbf{(48)}$$

Assuming that the states $|1\rangle$ and $|2\rangle$ are eigenstates of the Hamiltonian with energies E_1 and E_2 we find that the two diagonal elements of P will be constant under the development by (48), whereas the two interference terms P_{12} and P_{21} will oscillate:

$$P_{12} = a b^* \propto e^{\frac{i}{\hbar}(E_2 - E_1)t} \; ; \; P_{21} = P_{12}^* \qquad \textbf{(49)}$$

This regular oscillation, however, will not cause decoherence, because it maintains a strict correlation between the phases of the matrix elements. But if the system is coupled to the environment or a measuring device we may assume that the energy-difference $E_2 - E_1$ becomes a randomly varying function of time, U(t), and the development of the interference-terms is then given by:

$$P_{12} \propto \exp\frac{i}{\hbar}\int_0^t U(t')\,dt' \qquad \textbf{(50)}$$

The system in case may be a spin ½ particle in a randomly varying magnetic field or a molecule with electric dipole moment, like an ammonia- molecule in a randomly varying electric field. The diffusive phase-motion described by (50) will then gradually make the phase-coherence get lost, whereby interference-effects (like the maser-action of ammonia molecules) disappear.

The time average of the non-diagonal elements of the density matrix will tend towards zero and this is interpreted by proponents of the decoherence picture as a proof for the collapse to have happened "FAPP" (i.e. "for all practical purposes").

However, in QS we shall not accept the FAPP-argument, because it fails in accounting for "the qualitative jump", (as described by S. Kierkegaard and accepted by N. Bohr) in the transition from an interference-term being small to its being exactly zero.

We shall insist, that all quantum interference has vanished after the collapse, so the continuous decoherence-effect, described by (50) is not sufficient as an explanation of objective collapse. We have to look for "the qualitative jump"

elsewhere, and we shall find it in the FD-theorem of dissipative noise, interpreted as giving waiting-time-distributions for discrete events.

7. Quantum propagation

We have earlier described the preparation of a particle in a certain state $|\psi>$ as an elementary quantum process whereby a flow-source assumes dominance over an x-junction (figure 3). If the state is superposition like (45) the particle or quantum proceeds through the x-junction to one of the weak bonds of the superposition, shown in figure 11:

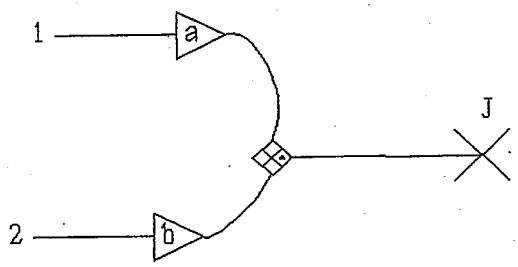


Figure 11. Quantum entering a superposition.

Despite the conviction of Kierkegaard and Bohr, that "the qualitative jump" cannot be analyzed in infinitesimal details, we shall try to do just that. As remarked earlier, the attainment of dominance over an x-junction is analogous to imposing differential causality over an x-storage. A model of this phenomenon is found in the superconducting Josephson-diode, controlled by a flow-source. We have seen that the current of free particles, described by a wave-function $\psi=fe^{i\varphi}$ is related to the phase-gradient $\Delta\phi$ by the relation

$$j=\frac{\hbar}{m\Delta x}f^2\Delta\phi \qquad \textbf{(51)}$$

This expression, however, is only valid for an infinitesimal Δx and $\Delta\phi$. In order to generalize it to a finite phase-difference we consider a wave-function in a superposition like (45) where the base-vectors $|1>$ and $|2>$ are situated at x_1 and x_2 a finite distance apart. The state-vector and the Hamiltonian in this base can then be expressed as:

$$|\Psi>=\begin{pmatrix} a e^{i\phi_1} \\ b e^{i\phi_2} \end{pmatrix} : H=\begin{pmatrix} E & U e^{i\alpha} \\ U e^{-i\alpha} & F \end{pmatrix} \qquad \textbf{(52)}$$

By using the Hamiltonian H in the Schrödinger-equation the current from x_1 to x_2 can then be expressed as:

$$j=\tfrac{1}{2}\frac{d}{dt}(b^2-a^2)=2\frac{Uab}{\hbar}\sin(\phi_2-\phi_1+\alpha) \qquad \textbf{(53)}$$

Disregarding the α and substituting ϕ for the phase-difference ϕ_2-ϕ_1 we then arrive at the *dc-Josephson-equation* for the supercurrent tunnelling from x_1 to x_2:

$$I_s=I_c\sin\phi \qquad \textbf{(54)}$$

where I_c is the *critical current.*
As the particles in the super-condensate are Cooper-pairs with charge Q=-2e, the presence of a voltage V over the diode will cause a time-dependence of the phase ϕ, given by *the ac-Josephson-equation*:

$$\frac{d\phi}{dt}=\frac{-2eV}{\hbar} \qquad \textbf{(55)}$$

We can combine eq.s (54) and (55) by a transducer

$$T=\frac{-2e}{\hbar} \qquad \textbf{(56)}$$

and an x-storage with ϕ as x-level and the energy-function

$$E(\phi)=\frac{\hbar I_c}{2e}\cos\phi \qquad \textbf{(57)}$$

As we are trying to describe the transition to differential causality of this x-storage as a gradual process we cannot connect a flow-source directly to it through the transducer, but we have to introduce a "causal buffer", which is most simply described as a resistance R, shunted over the diode, i.e. connected with it by an o-junction. The significance of this resistance is the resistance of the normal electrons. In general there will also be a capacitance C of the diode, i.e. a linear o-

storage connected with the o-junction, but this will complicate matters by creating an oscillating plasmon-mode, so we shall disregard the capacity, and in this way we arrive at the model of the resistively shunted current-controlled Josephson diode, shown in figure 12.

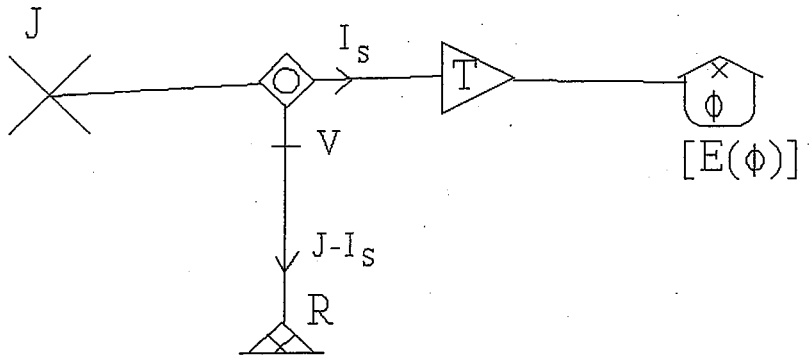


Figure 12. Resistively shunted Josephson diode.

The equation of motion for this system is:

$$\frac{d\phi}{dt} = -\frac{2e}{\hbar} R(J - I_c \sin \phi) \qquad \textbf{(58)}$$

This equation can be solved analytically (by quadrature) and the solution ϕ (t) (or rather t(ϕ)) expressed as an elliptic integral.

There is a simple mechanical analogue to figure 12. By exchanging x and o we obtain a model of a torque-driven pendulum, shown in figure 13:

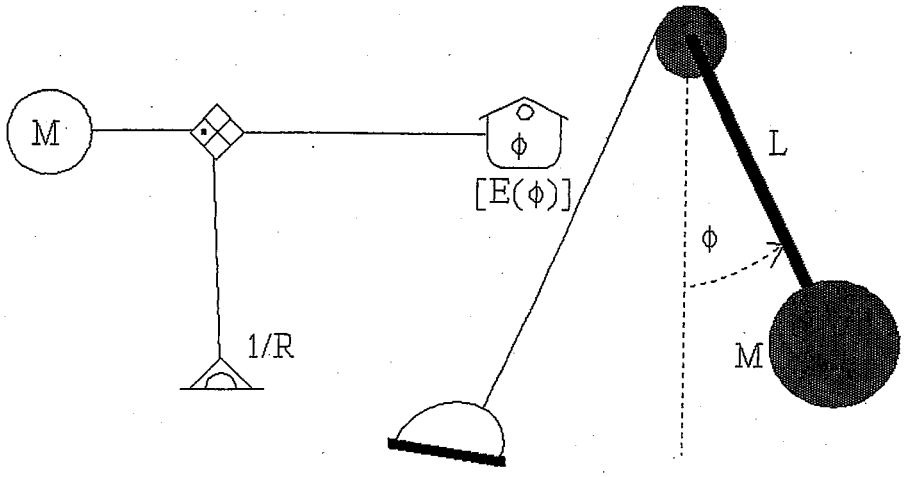


Figure 13. Torque-driven pendulum, dual analogue to figure 12.

The Josephson-specific transducer T has here been omitted, and the energy-function is:

$$E(\phi) = MgL \cdot (1 - \cos\phi) \qquad \textbf{(59)}$$

While figure 13 as such is dimensioned by angle-bonds, figure 12 is not, unless we re-dimension the x-level ϕ to $\hbar\,\phi$ with dimension of an action. A whole turn (2π) of ϕ then corresponds to Planck's constant h — the quantum of action.

A numerical solution (CTS) of eq. (58) shows on figure 14 the voltage V as a function of time for J=1.01 I_c.

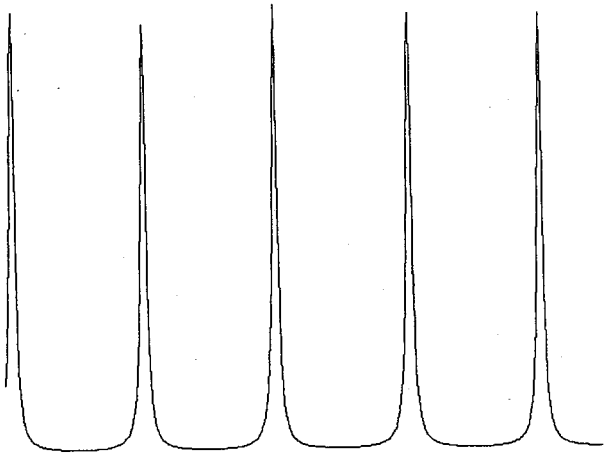


Figure 14. The voltage V(t) for J=1.01I_c

Although the model is continuous and differentiable the quantized nature of the qualitative jump shows itself in the voltage, looking like a series of delta-function-pulses. To each of these pulses corresponds a 2π-increase in ϕ. From eq. (55) we then see, that a pulse centered at time t_i is described by the expression

$$V_i(t) = \frac{h}{2e}\delta(t - t_i) \qquad \textbf{(60)}$$

So, although the condensate is quantized by the charge 2e, the quantum reveals itself in eq. (60) as the quantity h/2e, which is *the quantum of magnetic flux*. Each voltage-pulse can be seen as caused by magnetic induction when a flux-quantum moves across the Josephson-diode. If we had included a capacitor shunted over the diode, the charge on that would be quantized in units of 2e, so we both have a *o-quantum 2e* and a *x-quantum h/2e* and the product of these is Planck's constant h. Similarly, in figure 13, we could have included an x-storage connected to the x-junction and with an x-level that is the angular momentum of the pendulum — quantized in units of $\hbar$, while — again — the ϕ-quantum is 2π and the product of x-and o-quantum equal to h.

The time-averaged voltage <V> will asymptotically approach the normal characteristic <V> = R·J when J>>I_c and all the current is forced through the normal resistance R. In this limit the number of flux-quanta passing in the time-interval t will be given by:

$$N(t) = \frac{2e}{h}RJt \qquad \textbf{(61)}$$

The total charge emitted by the flow-source during the same time is 2eJt, so we see from eq. (61) that the fraction R/h of the charge-quanta are propagating as flux-quanta through the Josephson diode. Note, that this fraction R/h will be dimensionless, when the model of figure 12 is re-dimensioned by angle-bonds (comp. table 3).

Returning now to the general case of figure 11, where a flow-source J emits quanta in a state of superposition (45) we can imagine that each of the quanta will have to choose between the two rays $|1\rangle$ or $|2\rangle$ of the superposition. We could then follow its way of propagation by inserting resistively shunted Josephson-diodes, RSJ1 and RSJ2 at the ends of each of the weak bonds and in this way decide, where a quantum-event takes place. This is shown in figure 15.

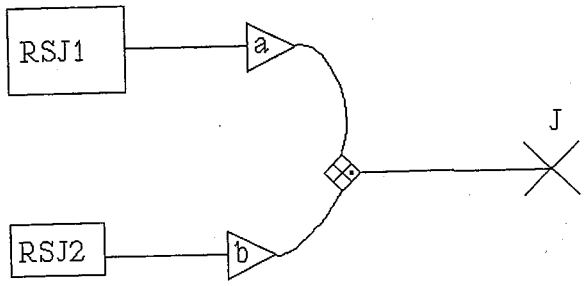


Figure 15. Detecting the way of a quantum.

Now, RSJ1 will include a resistance R_1 and RSJ2 a resistance R_2 by figure 12. We can then let the transformers a and b pass through the o-junction and "get swallowed" by the resistances, which thereby change their values to

$$R'_1 = R_1 \cdot |a|^2 \; ; R'_2 = R_2 \cdot |b|^2 \qquad \textbf{(62)}$$

From the interpretation of eq. (61) it is then clear, that the fraction of quanta passing through rays 1 and 2 will be $|a|^2$ and $|b|^2$, respectively. In this way the QS-view on quantization supports the probability-interpretation of the wave-function.

8. The Fluctuation-Dissipation Theorem.

The Fluctuation-Dissipation- (FD-) theorem, as derived by Callen and Welton in 1951, plays a very important role in QS. The reason is, that a measurement (a full measurement-cycle) is a thermodynamically irreversible — or *dissipative* process, because it must leave a permanent mark of registration. The FD-theorem tells us, that a dissipative process is always accompanied by random noise, and this explains, why chance and probability enters the quantum-formalism in connection with the measurement-process, but not with the reversible and deterministic evolution by the Schrödinger-equation.

We shall start by considering a quantum "black box" Y, perturbed by a classical field E, as shown in figure 16:

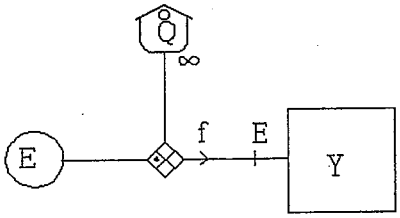


Figure 16. Quantum system Y perturbed by classical field E.

The Hamiltonian of the system is composed of an unperturbed part H_0 and a perturbation H_1 from the field E:

$$H = H_0 + H_1 = H_0 - E(t) \cdot Q \tag{63}$$

We shall now imagine, that the system, hitherto undisturbed, is hit by a particle with momentum p_0 at time 0:

$$E(t) = p_0 \delta(t) \tag{64}$$

The subsequent behaviour of the cyclic coordinate Q in the linear limit then defines the time-dependent mobility-function Y(t):

$$\langle Q(t) \rangle = p_0 Y(t) \tag{65}$$

The unperturbed system is described by the density-matrix

$$\rho_0 = \frac{1}{Z} e^{-\beta H_0} \tag{66}$$

where β is the reciprocal temperature and Z the partition-function

$$\beta = \frac{1}{kT}; Z = Tr(e^{-\beta H_0}) \tag{67}$$

In the Schrödinger-picture Q = Q(0) is a fixed operator, while the density-matrix is time-dependent:

$$\rho(t) = \rho_0 + \rho_1(t) \tag{68}$$

and may be determined from the Liouville-v. Neumann-equation:

$$\frac{d\rho}{dt} = \frac{i}{\hbar}[\rho, H] = \frac{i}{\hbar}(\rho H - H\rho) \tag{69}$$

To first order in the perturbation we then have:

$$\frac{d\rho}{dt} = \frac{d\rho_1}{dt} = \frac{i}{\hbar}([\rho_0, H_1] + [\rho_1, H_0]) \tag{70}$$

As ρ_0 and H_0 commute. Here, the first commutator will contain a delta-function, due to (64), whereas the second does not. In order to determine ρ_1 just after time 0 we may therefore do with the first term:

$$\rho_1(0+) = \frac{i}{\hbar}\int_{0-}^{0+}[\rho_0, -p_0\delta(t)Q] = \frac{1}{i\hbar}p_0[\rho_0, Q] \tag{71}$$

After time 0 the Hamiltonian is again identical with H_0, so for t>0 we have:

$$\rho_1(t) = e^{-\frac{i}{\hbar}H_0 t}\frac{p_0}{i\hbar}[\rho_0, Q]\ e^{\frac{i}{\hbar}H_0 t} \tag{72}$$

AS we have defined, by (65): <Q(t)>= 0 for t<0, we find for t>0:

$$Y(t)=\frac{1}{p_0}<Q(t)>=\frac{1}{p_0}Tr(\rho_1(t)Q) \tag{73}$$

which then, by (72) gives:

$$Y(t)=\frac{1}{i\hbar}Tr(\rho_0 Q e^{\frac{i}{\hbar}H_0 t} Q e^{-\frac{i}{\hbar}H_0 t} - e^{\frac{i}{\hbar}H_0 t} Q e^{-\frac{i}{\hbar}H_0 t} Q \rho_0) \tag{74}$$

where we have used, that the trace is invariant to the exchange of two factors: Tr(AB) = Tr(BA). We now introduce the Heisenberg-operator

$$Q(t) = e^{\frac{i}{\hbar}H_0 t} Q e^{-\frac{i}{\hbar}H_0 t} \tag{75}$$

and get from here the *Kubo-formula*:

$$Y(t)=\frac{1}{i\hbar}Tr\{\rho_0[Q(0),Q(t)]\}=\frac{1}{i\hbar}<[Q(0),Q(t)]>_0 \tag{76}$$

where the denotation $<>_0$ means, that we by statistical averaging use the density-matrix of the unperturbed system. This will be understood without mentioning in the following. We may then assume that

$$<Q(t)>=<Q(0)>=0\,;<Q(t)^2>=<Q(0)^2>=<Q^2> \tag{77}$$

As Q(0) and Q(t) do not commute, we may define the *autocorrelation function* for Q(t) as a symmetrized product:

$$\Phi_Q(t)=\tfrac{1}{2}<Q(t)Q(0)+Q(0)Q(t)> \tag{78}$$

This function is real, and $\Phi_Q(0) = <Q^2>$. The *real width-function* is given by:

$$\Gamma(t)=<(Q(t)-Q(0))^2>=2\{<Q^2>-\Phi_Q(t)\} \quad \textbf{(79)}$$

While *the complex width-function* is given by:

$$B(t)=<Q(0)^2+Q(t)^2-2Q(0)Q(t)> \quad \textbf{(80)}$$

From (76) and (79) we then get:

$$B(t)=\Gamma(t)-i\hbar Y(t) \quad \textbf{(81)}$$

The significance of B(t) stems from its relation to van Hove's incoherent scattering function

$$I_s(\kappa,t)=<e^{-i\kappa Q(0)}\,e^{i\kappa Q(t)}> \quad \textbf{(82)}$$

which for a homogeneous system can be written as:

$$I_s(\kappa,t)=\exp(-\frac{\kappa^2}{2}B(t)) \quad \textbf{(83)}$$

By differentiation of (79) we get:

$$\frac{d\Gamma(t)}{dt}=-<f(t)Q(0)+Q(0)f(t)>=-<f(0)Q(-t)+Q(-t)f(0)> \quad \textbf{(84)}$$

so, one further differentiation gives:

$$\frac{d^2\Gamma(t)}{dt^2}=<f(t)\,f(0)+f(0)\,f(t)>=2\Phi_f(t) \quad \textbf{(85)}$$

where $\Phi_f(t)$ is the symmetrized autocorrelation function for the velocity f. This function can, according to *the*

WienerKhinchin-theorem be expressed by the power spectrum $\Phi_f(\omega)$:

$$\Phi_f(t) = \int_0^\infty \Phi_f(\omega)\cos\omega t dt = \tfrac{1}{2}\int_{-\infty}^{\infty} \Phi_f(\omega) e^{-i\omega\cdot t} d\omega \qquad \textbf{(86)}$$

by defining $\Phi_f(-\omega) = \Phi_f(\omega)$. From (85) we then get by double integration:

$$\Gamma(t) = 2\int_0^\infty \Phi_f(\omega)\frac{1-\cos\omega t}{\omega^2}\,\mathrm{d}\omega \qquad \textbf{(87)}$$

while:

$$\Phi_Q(t) = \int_0^\infty \Phi_f(\omega)\frac{\cos\omega t}{\omega^2}\,\mathrm{d}\omega \qquad \textbf{(88)}$$

Both $\Phi_Q(t)$ and $\Gamma(t)$ are even functions of time, but Y(t) is odd, so B(t)in (80) and (81) is neither even nor odd.

We have:

$$B(-t) = B^*(t) = 2(< Q^2 > - < Q(t)Q(0) >) \qquad \textbf{(89)}$$

We may now, quite formally, write:

$$< Q(t)Q(0) > = Tr\{ e^{-\beta H_0} e^{\frac{i}{\hbar}H_0 t} Q(0) e^{-\frac{i}{\hbar}H_0 t} Q(0)\} = < Q(0)Q(t + i\hbar\beta) > \textbf{(90)})$$

So, we have from (89) and (90) and (81):

$$B(-t)=B^{*}(t)=B(t+i\hbar\beta)=\Gamma(t)+i\hbar Y(t) \qquad \textbf{(91)}$$

Comparing (81) and (91) we then get *the time-dependent FD-theorem*:

$$\Gamma(t)-\Gamma(t+i\hbar\beta)=-i\hbar[Y(t)+Y(t+i\hbar\beta)] \qquad \textbf{(92)}$$

We have here assumed that the response-function Y(t) has a unique analytical continuation to the complex plane — a normal assumption in linear response theory.
In *the classical limit* we may regard $i\hbar\beta$ as small, so to first order in this quantity we get from (92):

$$-i\hbar\beta\frac{d\Gamma(t)}{dt}=-2i\hbar Y(t)\Leftrightarrow\Gamma(t)=2kTJ(t) \qquad \textbf{(93)}$$

where J(t) — the time-integral of Y(t) — is the time-dependent *compliance-function*. A well known example of this relation is found in the case of *Brownian motion* of a particle in a liquid with time-independent mobility μ, where the width-function (in one dimension) is given by the diffusion constant D as $\Gamma(t) = 2$ D t. Eq. (93) (with $J(t)=\mu t$) then gives, that $D = \mu$ k T — the *Nernst-Einstein relation* which — historically — is the first known example of the FD-theorem.

For a quantum mechanical free particle with mass m we have:

$$Y(t)=\frac{t}{m};\ J(t)=\frac{t^2}{2m};\ \Gamma(t)=\frac{kT}{m}t^2 \qquad \textbf{(94)}$$

The purely classical result for the real width-function $\Gamma(t)$ follows from the fact that a free particle has a continuous energy-spectrum.

so the complex width function (81) is:

$$B(t) = \frac{kT}{m} t^2 - i\hbar \frac{t}{m} \tag{95}$$

By inserting this in (83) we determine the scattering function $I_s(\kappa,t)$ which is the Fourier-transformed of the frequency-dependent *Structure-function* $S_s(\kappa, \omega)$:

$$I_s(\kappa,t) = \int_{-\infty}^{\infty} S_s(\kappa,\omega)\, e^{-i\omega t}\, \mathrm{d}\omega \tag{96}$$

The structure-function gives the scattering cross section for a wave, that changes its wave-vector by the amount κ. Performing the reverse Fourier-transformation with (95) in (83) we get:

$$S_s(\kappa,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} I_s(\kappa,t)\, e^{i\omega t}\, \mathrm{dt} \tag{97}$$

with the result

$$S_s(\kappa,\omega) = \sqrt{\frac{2\pi m}{\kappa^2 kT}} \cdot \exp - \frac{(\omega - \frac{\hbar\kappa^2}{2m})^2}{\frac{2}{m}\kappa^2 kT} \tag{98}$$

So we see that the scattering cross- section is a normal distribution in ω. While the width of this function is a "Doppler-broadening" of purely classical origin, we see that the absorption-energy $\hbar\omega$ is centered about the *recoil energy*:

$$(\hbar\omega)_r = \frac{\hbar^2\kappa^2}{2m} \tag{99}$$

showing that a quantized momentum $\hbar\kappa$ has been transferred to the particle. We see thus, that the quantum mechanical modification of the classical FD-theorem in the imaginary part of the width-function B(t) (81) leads to a quantization of the wave-concept. The appearance of the recoil energy (99) is a genuine quantum-effect and very important, e.g., for the understanding of the Mössbauer-effect.

9. The Power spectrum of dissipative noise.

The time-dependent FD-theorem (92), where the temperature enters as an imaginary time shall now be used for deriving the frequency-dependent *power spectrum*, as defined in (86). Again, we shall start by deriving the *classical* expression, starting from (93) in order to appreciate how quantization modifies the classical picture.

From (85) and (93) we get:

$$\frac{d^2\Gamma(t)}{dt^2} = \frac{d^2}{dt^2} 2kTJ(t) = 2kTF(t) \qquad \textbf{(100)}$$

Where F(t) is the time-dependent susceptibility-, or lightness-function. Eq. (85) and (100) then immediately give the classical FD-theorem for the velocity- auto-correlation-function $\Phi_f(t)$:

$$\Phi_f(t) = kTF(t) \qquad \textbf{(101)}$$

Now, the susceptibility-function F(t) is related to the frequency-dependent mobility-function Y(s) (s=-iω) by a Laplace-transformation:

$$Y(s) = \int_0^\infty F(t)\ e^{-st}\, dt \qquad \textbf{(102)}$$

And, conversely, F(t) is given by the real part of Y(ω), $Y_1(\omega)$ by a cosine-transform:

$$F(t) = \frac{2}{\pi}\int_0^\infty Y_1(\omega)\cos\omega t\, d\omega \qquad \textbf{(103)}$$

By comparing (86) and (103) we then find the following expression for the powerspectrum of the velocity-noise:

$$\Phi_f(\omega)=\frac{2}{\pi}kT\,Y_1(\omega) \qquad \textbf{(104)}$$

Because $Y_1(\omega)$ is the *dissipative* part of the mobility response function eq. (104) expresses clearly the intimate connection between fluctuations and dissipation, which is the core-insight of the FD-theorem.

Going now to the QM-version for the power spectrum, we proceed from (92): The functions Γ and Y of the complex time can be defined by analytical continuation. First, we get from (87):

$$\Gamma)(t) = \int_{-\infty}^{\infty} \frac{\Phi_f(\omega)}{\omega^2}(1 - e^{-i\omega t})\,\mathrm{d}\,\omega \qquad \textbf{(105)}$$

and by analytical continuation:

$$\Gamma(t) - \Gamma(t + i\hbar\beta) = \int_{-\infty}^{\infty} \frac{\Phi_f(\omega)}{\omega^2} e^{-i\omega\cdot t}(e^{\beta\cdot\hbar\omega} - 1)\,\mathrm{d}\,\omega \qquad \textbf{(106)}$$

Next, by inverse Laplace-transformation of (103), substitution s = -i ω and integrating F(t) over time to Y(t) we find:

$$Y(t) = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{Y_1}{\omega} e^{-i\omega t}\,\mathrm{d}\,\omega \qquad \textbf{(107)}$$

giving:

$$Y(t)+Y(t+i\hbar\beta)=\frac{i}{\pi}\int_{-\infty}^{\infty}\frac{Y_1}{\omega}e^{-i\omega\cdot t}(1+e^{\beta\cdot\hbar\omega})\,\mathrm{d}\,\omega \qquad \textbf{(108)}$$

Finally, by inserting (106) and (108) in the FD-theorem (92) we find:

$$\Phi_f(\omega) = \frac{2}{\pi} Y_1(\omega) \frac{\hbar\omega}{2} \coth \frac{\beta\hbar\omega}{2} \qquad \textbf{(109)}$$

By comparing the classical expression (104) with the quantum expression (109) we see that both can be written as:

$$\Phi_f(\omega) = \frac{2}{\pi} Y_1(\omega)\theta(\omega,T) \qquad \textbf{(110)}$$

where θ(ω,T) is the energy of a harmonic oscillator with resonance frequency ω in thermal equilibrium with temperature T = beta/k:

$$\theta(\omega,T) = kT \text{ for } \hbar\omega << kT \qquad \textbf{(111)}$$

and:

$$\theta(\omega,T) = \tfrac{1}{2}\hbar\omega \coth \frac{\beta\hbar\omega}{2} = \tfrac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} \text{ for } \hbar\omega >> kT \qquad \textbf{(112)}$$

The derivation of the quantum-mechanical expression (109) can in this way also be seen as a derivation of Planck's expression for the thermal oscillator-energy (second term in (112)) as well as the fact that the oscillator at zero temperature has the "zero point energy" ½ħω. This term is a basic sign of quantum-uncertainty — analogous to Heisenberg's uncertainty-relations and will prove important in QS for the discussion of the measurement-process in following chapters.

10. Quantum mechanical diffusion.

As a special example of the general system on figure 16 we shall consider a particle of mass m performing one-dimensional Brownian, or diffusive motion in a medium with mobility μ=1/R and temperature T.

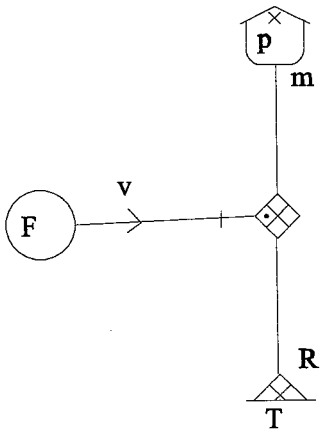


Figure 17. Brownian motion of particle.

We shall assume that the driving force F is absent, such that the particle only moves under the influence of the random noise from the sink R at temperature T. We have already seen that the classical thermal noise produces a diffusive motion with the width-function $\Gamma(t) = 2Dt$, where $D = kT/R$ is the diffusion constant (comp. (93)). The question is now, how the quantum-modifications of the FD-theorem, especially the presence of zero-point-noise changes the diffusion-process.

There is a characteristic time in the problem — the *relaxation time* $\tau = m/R$, which we shall use in the expressions for the relevant response-functions:

$$F(t) = \frac{1}{m} e^{-\frac{t}{\tau}} ; Y(s) = \frac{1}{R + ms} \qquad \textbf{(113)}$$

From Y(s) ($s=-i\omega$) we then find the dissipative response-function $Y_1(\omega)$:

$$Y_1(\omega) = \frac{1}{R} \cdot \frac{1}{1+(\omega\tau)^2} \qquad \textbf{(114)}$$

For the classical case when the velocity-autocorrelation-function $\phi_v(t)$ (we now use 'v' instead of 'f' for the velocity) is given by (101), the mean square velocity in thermal equilibrium is:

$$<v^2> = kTF(0) = \frac{kT}{m} \qquad \textbf{(115)}$$

we recognize the law of equipartition, $<½mv^2> = ½ kT$ of classical statistical mechanics. Now, let us suppress all thermal motion, putting T=0, $\beta=\infty$. What happens then?

The zero-point-oscillator-energy $½\hbar\omega$ will still be present in the noise-spectrum, so we get from (109):

$$\Phi_v(\omega) = \frac{\hbar\omega}{\pi R}\frac{1}{1+\omega^2\tau^2} \qquad \textbf{(116)}$$

and this then, by (86) gives the time-dependent autocorrelation

$$\Phi_v(t) = \int_0^\infty \frac{\hbar\omega}{\pi R}\frac{\cos\ \omega t}{1+\omega^2\tau^2}\,d\omega \qquad \textbf{(117)}$$

This integral is divergent for t=0, so the quantum value of $\phi_v(0)$ — the mean square velocity — is infinite — there is an "ultraviolet catastrophe" in the zero-point-motion. This, however, can be "renormalized" away by integrating the velocity-autocorrelation (117) twice over time, in which way we get the *width-function* $\Gamma(t)$:

We shall express the width-function of reduced time $\Gamma(t)=G(t/\tau)$ where the function G(x) is given by:

$$G(x) = \frac{\hbar}{\pi R}\int_0^\infty \frac{1-\cos\ ux}{u(1+u^2)}\,du \qquad \textbf{(118)}$$

This integral can be expressed analytically as:

$$g(x)=\frac{\pi R}{\hbar}G(x)=\tfrac{1}{2}(e^{x}E_{1}(x)-e^{-x}Ei(x))+\gamma+\ln x \qquad \textbf{(119)}$$

where E_1 (x) and Ei(x) are exponential integrals and γ=0.57721 is Euler's constant. Figure 18 shows the function g(x) for x from 0 to 1:

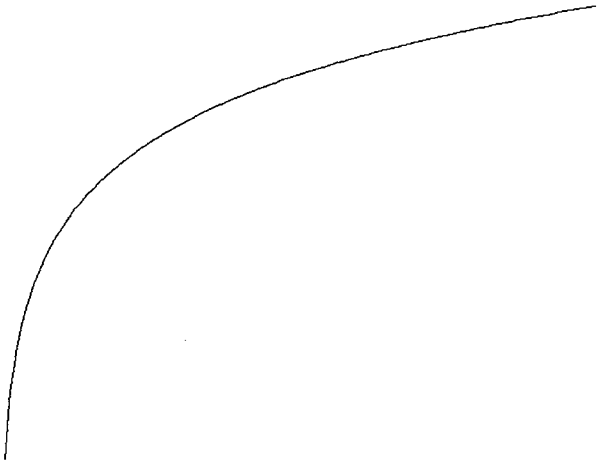


Figure 18. the width function g(x).

In the two limits x<<1 and x>>1 we have the asymptotic expressions

$$g(x)\approx\tfrac{1}{2}x^{2}(\frac{3}{2}-\gamma-\ln x) \quad for \quad x<<1 \qquad \textbf{(120)}$$

and

$$g(x)\approx\gamma+\ln x \quad for \quad x>>1 \qquad \textbf{(121)}$$

If the particle moves by jumping a distance 1 forward or backwards on a one-dimensional lattice, then the mean square displacement Γ(t) will be l^2 N(t), where N(t) is the number of jumps during the time t. We see thus, that for classical diffusion, where Γ(t) = 2Dt,d N(t)/dt = $2D/l^2$ — the process goes on with a constant speed — while for quantum-diffusion N(t) $\approx$ ln t/τ — a strongly decelerating process.

We have now performed a renormalization of the quantum noise by finding a finite expression for the width-function $\Gamma(t)$. Still, the problem remains that the mean square velocity of the particle is infinite. This would indicate that the kinetic energy is infinite, which it clearly can't be. A standard statistical mechanical calculation of the thermal energy will provide a finite value for all temperatures, but would miss the point, that the FD-theorem provides us with a sufficient foundation of Statistical Mechanics. The dissipative noise is the real cause of statistical fluctuations, so the FD-theorem alone should be able to replace the standard calculation of the partition function and its derivatives. We shall illustrate this point by considering the diffusing particle as the kinetic part of a harmonic oscillator:

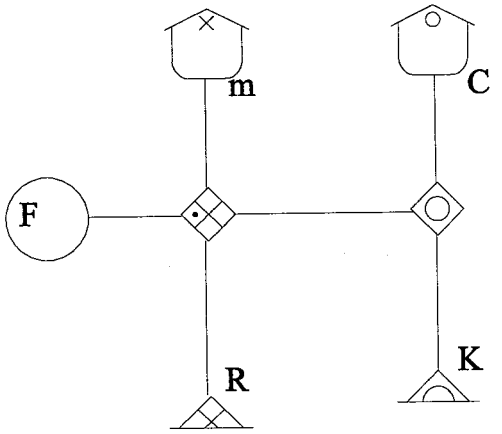


Figure 19. Harmonic oscillator.

In the undamped case R=K=0 The dissipative response-function, seen from the source, is:

$$Y_1(\omega)=\frac{\pi}{2m}\{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\} \qquad \textbf{(122)}$$

where

$$\omega_0=\frac{1}{\sqrt{mC}} \qquad \textbf{(123)}$$

is the resonance-frequency of the undamped oscillator.

Inserting (123) in (109) and (86) we find:

$$\Phi_v(0) = < v^2 > = \frac{1}{m}\cdot \tfrac{1}{2}\hbar\omega_0 \coth\frac{\hbar\omega_0}{2kT} \qquad \textbf{(124)}$$

and from this the kinetic energy:

$$<\tfrac{1}{2}m v^2 > = \tfrac{1}{2}\cdot\tfrac{1}{2}\hbar\omega_0 \coth\frac{\hbar\omega_0}{2kT} \qquad \textbf{(125)}$$

As the potential energy is the same, we have from the FD-theorem proved, that the expression (112) is the thermal energy of the harmonic oscillator, and that the kinetic energy of the diffusing particle at T=0 is finite:

$$< \tfrac{1}{2} m v^2 > = \frac{1}{4}\hbar\,|\,\omega\,| \qquad \textbf{(126)}$$

We may determine ω_0 for the particle by the consideration, that it is located in a potential box of width l. The energy-difference from the qround state to the first excited state is then:

$$\Delta E = \frac{h^2}{2m\ l^2} \qquad \textbf{(127)}$$

By equating this to the energy-difference $\hbar\omega_0$ for the harmonic oscillator we find,

$$<\tfrac{1}{2}m v^2> = \frac{\Delta p^2}{2m} = where\ \ \Delta p = \frac{h}{2l} = \frac{h}{2\Delta x} \qquad \textbf{(128)}$$

that the kinetic energy (126) is:

In this way we see, that the zero-point-noise can be regarded as the the source of the quantum-uncertainty, described by Heisenberg's uncertainty-relation $\Delta x\ \Delta p = h$

.

Although this argument is somewhat vague with respect to the numerical factor in the uncertainty-relation its qualitative content is clear: If the zero-point-noise were not present, even the basic quantum-uncertainty would be damped out by the sink in figure 17. (this argument was first presented by Senitzky).

11. Zero point noise as discrete events.

We saw in chapter 7, that a quantum-event is associated with a delta-function-pulse from a sink. Now, what we usually describe with the sink-icon — e.g. an ohmic resistance — is seldom a pure sink, but is "dressed" with other icons. A resistance consisting of a metallic wire is rather like the model of figure 17, where the x-.storage describes the inertance of electrons, and this gives the impedance-function a frequency-dependence, which is absent for a pure x-sink. However, the FD-theorem allows us to treat a dissipative system as a whole and the frequency-dependent dissipative response-function as the source of noise, so there is no reason to believe that the quantum-events are only associated with pure sinks. For the diffusion-model of figure 17, where the particle makes jumps of ±l on a one-dimensional discrete lattice, the output-velocity is given by:

$$v(t) = \sum_{t_i=-\infty}^{t_i=\infty} l \cdot \eta_i \delta(t - t_i) \qquad \textbf{(129)}$$

where η_i is a random number, either +1 or -1, associated with the jump at time t_i. We may assume that the η_is are statistically independent, so the width-function is given by:

$$\Gamma(t) = l^2 \cdot \sum_{t_i=0}^{t_i=t} <\eta_i^2> = l^2 \cdot N(t) \qquad \textbf{(130)}$$

Where N(t) is the number of events between time 0 and time t.

We may therefore infer, that the number-function N(t) — just like the width-function Γ(t) is determined by the dissipative response-function $Y_1(\omega)$ for the whole system (114) in connection with the zero-point oscillator-energy $\frac{1}{2}\hbar\omega$.

We shall now — gradually — turn to the problem of *detection*, assuming that the collapse, or projection of the wave-function onto an eigenstate of the detector, can be described as the qualitative jump, that occurs as a result of a delta-function-noise-pulse from the dissipative measuring apparatus. As a first model of a detector we shall use the system on figure 20 — the dual to figure 18, because we have seen that the number- (or width-) function for this system has a finite value for the zero-point-noise:

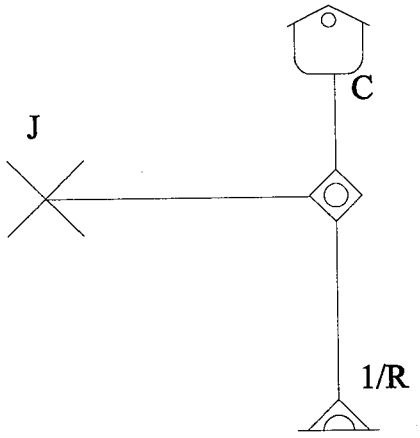


Figure 20. First model of a detector.

The dissipative part of the impedance-function for this system is analogous to (114):

$$Z_1(\omega) = \frac{R}{1+\omega^2\tau^2} \qquad \textbf{(131)}$$

Where $\tau = RC$ is the relaxation time.

Assuming that the o-level is dimensionless, the zero point noise will give the output-effort:

$$e(t) = \sum_{t_i=-\infty}^{t_i=\infty} h\,\eta_i\,\delta(t - t_i) \qquad \textbf{(132)}$$

Reconstructing the calculation of the width-function $\Gamma(t)$ for the dual model of figure 20 (which is now concerning *momentum*-diffusion) and dividing it by h^2 we find again, that the number-function N(t) can be expressed by the dimensionless function $g(t/\tau)$ of eq. (119):

$$N(t) = \rho g(\frac{t}{\tau}) = where\,\rho = \frac{R}{4\pi^3\hbar} \qquad \textbf{(133)}$$

We shall now see that the knowledge of the number-function N(t) allows us to calculate waiting time distributions for any number of events as well as probability-distrubutions for the number of events in a certain time-interval.

We have from (132):

$$N(t) = <\frac{1}{h}\int_{0+}^{t} |e(t')|\,dt'> = \sum_{i,t_i<t} <|\eta_i^2|> \qquad \textbf{(134)}$$

So, N is the average over a sub-ensemble starting at t=0 of an integer-variable N', that can be interpreted as the number of coin-tosses (heads or tails), where η_i is the gain (+1 for heads, -1 for tails) of the ith throw. If n throws have given a positive gain, there has been N'-n with a negative gain, so the total gain will be

$$g = 2n - N' \qquad \textbf{(135)}$$

Now, n has a *binomial* probability-distribution:

$$p_n = 2^{-N'} \frac{N'!}{n!\,(N'-n)!} \qquad \textbf{(136)}$$

It follows that g for every N' has the mean value 0 and the standard deviation $\langle g^2 \rangle = N'$, and the average over the sub-ensemble of this quantity will be the number function $N(t) = \rho\, g(t/\tau)$ with g given by (119). Laplace-transformation of this function gives:

$$N_L(s) = \int_0^\infty N(t)\, e^{-st}\, dt = \rho \frac{\ln \frac{1}{s\tau}}{s \cdot (1 - s^2\tau^2)} \qquad \textbf{(137)}$$

This then represents the zero-point noise as filtered by the o-storage on figure 20. For comparison we consider the unfiltered classical noise from a frequency-independent ohmic resistance R. In this case the dual version of the FD-theorem (93) gives:

$$N(t) = \frac{2RkT}{h^2} \cdot t \;;\; N_L(s) = \frac{2RkT}{s^2 h^2} \qquad \textbf{(138)}$$

We shall now see how the waiting time distributions can be calculated from $N_L(s)$. Let $P_n(t)$ be the probability that there has been n events between time 0+ and time t. The probability density for one event happening between t and t + dt is then:

$$p_1(t) = -\frac{d}{dt} P_0(t) \qquad \textbf{(139)}$$

It is necessary here to make a small digression for discussing the nature of the time-scale involved in eqs. (134)-(140). The FD-theory of waiting times for discrete events is based on the concept of the width-function, eq. (79), which presupposes the existence of a fixed zero of time. The waiting-time-scale is a *ratio-scale*, like the scales of mass and absolute temperature, but it is not an *interval-scale* like mechanical time and conventional temperature, where the zero-point can be chosen arbitrarily. Still, the FD-theory of waiting times is *time-homogeneous* in

the sense, that any new jump-event marks a new zero on the time axis, and the absolute location of this zero does not enter the theory explicitly. In this sense the waiting-time theory resembles Kierkegaard's and Bohr's idea of the qualitative jump, that there is no sense in discussing the time *during* a jump. The discussion of the Josephson-diode and the pendulum in chapter 7 was exceptional in making a differential analysis of the jump possible, but this was only because we had a mechanical model to analyze, so the time involved was mechanical time, that belongs to an interval-scale. For the diffusing particle the application of the FD-theorem presupposes, that the particle is in thermodynamic equilibrium at time zero, and this can not be the case *during* a jump, but only just after a jump has taken place. So, all expressions for waiting-time-distributions etc. in the following, involving the symbol 't' presuppose, that a jump has occurred at time 0-. The time dependent response-functions, like Y(t) in eq. (65) also presume, that the stimulus was applied at time 0. The method of Laplace-transforms always presumes that time is defined on a ratio-scale with a fixed zero-point.

As $P_0(0)=1$ we get from (139) by Laplace-transformation and partial integration:

$$P_{1L}(s) = \int_0^\infty p_1(t)\, e^{-st}\, \mathrm{dt} = 1 - \mathrm{s}\, P_{0L}(\mathrm{s}) \qquad \textbf{(140)}$$

The property of time-homogeneity, discussed above, implies, that we have the recursion-formula:

$$P_{n+1}(t) = \int_0^t p_1(t')\, P_n(t - t')\, \mathrm{dt}' \qquad \textbf{(141)}$$

which, by the convolution theorem for Laplace-transforms can be translated to:

$$P_{n+1,L}(s) = p_{1L}(s) P_{nL}(s) \qquad \textbf{(142)}$$

From (140) we then get:

$$P_{nL}(s) = [1 - s\, P_{0L}(s)]^n\, P_{0L}(s) \qquad \textbf{(143)}$$

The number-function is given by:

$$N(t) = \sum_{n=1}^{\infty} n\, P_n(t) \qquad \textbf{(144)}$$

which, by Laplace-transformation and use of (143) gives:

$$N_L(s) = P_{0L}(s) \sum_{n=1}^{\infty} n[1 - s\, P_{0L}(s)\,]^n = \frac{1}{s}\left[\frac{1}{s\, P_{0L}(s)} - 1\right] \qquad \textbf{(145)}$$

Conversely, we then have:

$$P_{0L}(s) = \frac{1}{s[1 + s\, N_L(s)]} \qquad \textbf{(146)}$$

From this and (143) we can express all the functions $P_{nL}(s)$ by the single function $N_L(s)$.

If there exists a *mean waiting time*, $\langle t_1 \rangle$ for the first event, it is given by:

$$\langle t_1 \rangle = \int_0^{\infty} t\, p_1(t)\, dt = -\int_0^{\infty} t P'_0 dt = \int_0^{\infty} P_0(t)\, dt = P_{0L}(0) \qquad \textbf{(147)}$$

The *intensity* of the process is defined by

$$\lambda(t) = -\frac{d}{dt} \ln P_0(t) = \frac{p_1(t)}{P_0(t)} \qquad \textbf{(148)}$$

As an example we look at the family of *scale invariant processes*:

$$N(t) = N_0(\frac{t}{\tau})^{\alpha} \tag{149}$$

where we get:

$$N_L(s) = \frac{N_0}{s} \cdot \frac{\alpha!}{(s\tau)^{\alpha}} \tag{150}$$

and

$$P_{0L}(s) = \frac{1}{s + \alpha! N_0 \tau^{-\alpha} s^{1-\alpha}} \tag{151}$$

It is seen that $<t_1>$ is finite for $\alpha=1$, infinite for $\alpha<1$ and apparently 0 for $\alpha>1$. The last makes no sense, so we keep to the interval $0 \leq \alpha \leq 1$.

An important special case is *the Markov-process* $\alpha=1$, where we get:

$$N_L(s) = \frac{1}{s^2 \tau}; \; P_{0L} = \frac{1}{s + \frac{1}{\tau}} \tag{152}$$

and:

$$<t_1> = \tau; \; P_0(t) = e^{-\frac{t}{\tau}}; \; p_1(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}}; \; \lambda(t) = \frac{1}{\tau} \tag{153}$$

So the Markov process has a constant intensity without memory of the previous history. In this case the choice of time zero is completely arbitrary, and waiting times are defined both on a ratio-scale and an interval-scale, but this is the only case, where it is so.

An example of a Markov-process is the classical noise, described by (138).

However, since scale-invariant processes have no characteristic time-scale, the time τ is not necessarily the relaxation time RC. For classical noise we have:

$$\tau = \frac{h^2}{2RkT} \qquad \textbf{(154)}$$

The occurrence of h in this formula makes it look not entirely classical, but we must remember, that R in the denominator also has the dimension 'action', so τ in (154) is a dimensionless number R/h times the fundamental 'quantum time' h/(kT), which is the lower limit for resolution on the time scale for the classical FD-theorem.

The higher waiting-time-distributions for the Markov process are the *Gamma-distributions*:

$$p_n(t) = \frac{1}{n!}\left(\frac{t}{\tau}\right)^n e^{-\frac{t}{\tau}} \qquad \textbf{(155)}$$

So for a fixed time-interval, t, the probability distribution for the number of events is the Poisson-distribution:

$$P_n = \frac{1}{n!} m^n e^{-m} \qquad \textbf{(156)}$$

where $m=t/\tau$ is the mean number of events.

Another important special case is *the geometric process*, $\alpha=0$, where N(t) is the constant N_0 for $t>0$. So for this process all events come in a bunch at time 0+, and then the process comes to a halt. For this case we get from (149)-(151):

$$N_L(s)=\frac{N_0}{s}\,;\; P_n(t)=\frac{1}{1+N_0}\cdot\left(\frac{N_0}{1+N_0}\right)^n \qquad \textbf{(157)}$$

We have here for $t>0+$ a geometric distribution over n with mean value N_0.

If $\alpha<<1$, or, if we can just say that N(t) is approximately constant over many orders of magnitude of t/τ, we can use the time-dependent geometric distribution:

$$P_n(t) = \frac{1}{1+N(t)}\left[\frac{N(t)}{1+N(t)}\right]^n \qquad \textbf{(158)}$$

This will be the case for the zero-point-noise (133) in the long-time-limit $t >> \tau$, where $N(t) \approx \rho \ln(t/\tau)$ and therefore:

$$P_0(t) \approx \frac{1}{1+\rho \ln \frac{t}{\tau}} \quad for\ t >>> \tau \qquad \textbf{(159)}$$

The exact expression for $P_0(t)$ can be derived by inverse Laplace-transformation of eq. (146), using (145). Doing this we put $s = \varepsilon - i\,\omega$, where ε is an infinitesimal positive quantity. We then have:

$$\ln \frac{1}{s\tau} = \ln \frac{1}{|\omega\tau|} + i\frac{\pi}{2}\mathrm{sign}(\omega) \qquad \textbf{(160)}$$

and $P_0(t)$ can be calculated by the real integral:

$$P_0(t) = \int_0^\infty \frac{(+\omega^2\tau^2)\left[\frac{\rho}{2}\cos\omega t + \frac{1}{\pi}\left(\rho \ln\frac{1}{\omega\tau} + 1 + \omega^2\tau^2\right)\sin\omega t\right]}{\omega\left[\left(\rho \ln\frac{1}{\omega\tau} + 1 + \omega^2\tau^2\right)^2 + \frac{\pi^2}{4}\rho^2\right]}\,d\omega \qquad \textbf{(161)}$$

This function decreases monotonically from $P_0(0) = 1$ to $P_0(\infty) = 0$. A numerical evaluation of (161) gives the following approcimate expressions:

$$P_0(x) \approx 1 - x^{1.75} \quad for\ x < 1 \qquad \textbf{(162)}$$

and

$$P_0(x) \approx \frac{1}{\ln x} \quad for\, x > 5 \tag{163}$$

where, again, $x = t/\tau$.
From (163) and (148) we find the *intensity* in the long-time-limit:

$$\lambda(t) \approx \frac{1}{t(\ln \frac{t}{\tau})^2} \tag{164}$$

The complete expression for $P_0(x)$ can be inverted by computer, and this makes possible a simulation of zero-point-noise. Such a simulation proceeds by the following algorithm:

1): We choose an $x = t/\tau$ and calculate $u = P_0(x)$.

2): We generate a random number v with a uniform distribution between 0 and 1.

3): If $v<u$ the number of events is zero, and simulation ends.

4): If $v>u$ we determine $x_1 = P_0^{-1}(v)$, and x_1 is then the time of the first event.

5): We then go back to 1) using x_1 as the new origin of time and $x-x_1$ as the new value of x. This procedure continues, summing the number of events, until it ends in point 3).

The following table gives results for 100 simulations with x=10 and ρ=1. The table shows the number, N, of simulations that result in n events:

n	0	1	2	3	4	5	6	7	8	9
N	28	16	13	11	10	8	5	6	2	1

The theoretical value of $\langle n \rangle$ is N(10)=2.5363, while the table above gives $\langle n \rangle$=2.52. The table also gives the standard-deviation $\sqrt{\langle (n-\langle n \rangle)^2 \rangle} = 2.41$. The fact that mean and standard-deviation are nearly equal indicates a geometric distribution for n, which it should be, according to (158).

In QS we shall assume, that the collapse of the wave-function takes place by the first noise-event from the detector. The probability of collapse before time t is then:

$$P_c(t) = 1 - P_0(t) = \frac{\rho\ g(\frac{t}{\tau})}{1 + \rho\ g(\frac{t}{\tau})} \qquad \textbf{(165)}$$

12. Collapse as fix-point-projection.

The last statement in chapter 11, that the collapse happens when the first event of quantum noise takes place may be true, but there are two reasons to modify it a bit or examine it somewhat closer:

First: The noise-events are clustered and most of them happen at time 0+ just after the quantum system has been connected to the detector. It may therefore be difficult to separate them and decide, which is the first.

Second: The collapse of the state-vector is a *projection* onto the ray or one-dimensional sub-space defined by the detector, and a projection may be repeated indefinitely and still remain the same projection.

These two considerations indicate that we may regard the collapse as caused by a whole cluster of noise-events — or a repeated projection onto the same ray.

This leads on to an idea originally proposed by the austrian second order cybeneticist Heinz von Foerster (1911-2002):

Objects are tokens of eigenbehaviour. This idea is symbolized by the fix-point-equation:

$$X = F(X) \qquad \textbf{(166)}$$

or by the *Oroborous* picture of a snake biting its own tail.

In an article by Louis H. Kauffman about von Foerster's idea it is said:*)

"Taking Heinz's suggestion to heart we find that an object in itself is a symbolic entity, participating in a network of interactions, taking on its apparent solidity and stability from these interactions. We ourselves are such objects, we as human beings are "signs for ourselves", a concept originally due to the american philosopher C. S. Peirce [ref. Ref. 3 to part I.2]. In many ways Heinz's eigenforms are mathematical companions to Peirce's work. We will not follow this comparison in the present essay, but the reader familiar with Peirce is encouraged to do so."

We may thus pursue the idea that *an objective property of a quantum system is a fix-point or an eigenform of the collapse-process.*

The number of noise-events sufficient to produce a collapse is therefore indefinite. This is not a serious drawback of the model, since it is inherent in the strong concept of fix-point, or eigenform. There is, however, another indefiniteness that needs to be repaired, namely the indefiniteness of the *time of measurement* hiding in the slowly increasing number-function $g(t/\tau)$. A real measurement takes place within a definite measuring time t_M, and if the collapse has not happened within this time, the measurement is concluded as a *null measurement.* We can define t_M, rather arbitrarily by the condition (comp. (165))

$$P_0(t_M) = \tfrac{1}{2} \;\; i.e. \; N(t_M) = \rho g(\frac{t_M}{\tau}) = 1 \qquad \textbf{(167)}$$

This condition can then be built in as a kind of "renormalization" of the detector-model of figure 20 by replacing the resistance R and the capacity C with the renormalized quantities R' and C' defined below:

$$R' = \frac{R}{g\,(\frac{t_M}{\tau})} \qquad C' = \frac{\tau}{R'} \qquad \textbf{(168)}$$

As in figure 11 and figure 15 we can then model the projection-operator by the expansion coefficient a in the form of a transformer, which leads to a further renomalization, shown in figure 21 (where we introduce the symbol $q = |a|^2$):

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*) **Louis H Kaufman** (2003), *Eigenforms – Objects as Tokens for Eigenbehaviors*, Cybernetics and Human Knowing, vol. 10, nos. 2-3.

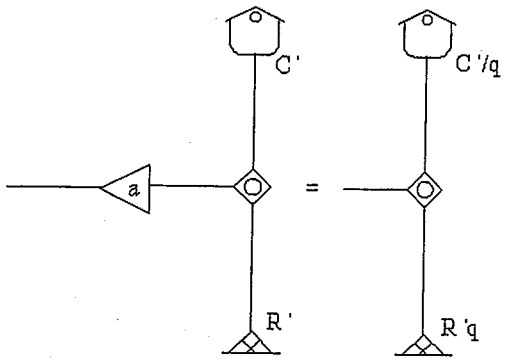

Figure 21. Projection-renormalization of detector model.

A repeated projection can now be represented by concatenation of a-transformers. The collapse as an eigenform of repeated projections is then derived from the model on figure 22 (where the model of figure 21 is represented by a box with a 'D' inside):

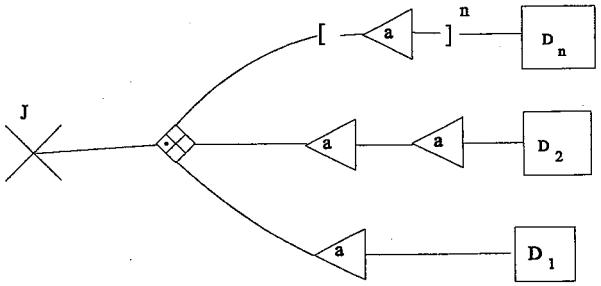

figure 22 Detection by repeated projection

The dissipative response-function for the combination of all these detectors is then (comp. (131)):

$$Z_1(\omega) = \frac{R''}{1+\omega^2\tau^2} \qquad \textbf{(169)}$$

where

$$R'' \sum_{n=1}^{\infty} R' q^n = R' \frac{q}{1-q} \qquad \textbf{(170)}$$

The dimensionless parameter ρ for the detector, reconstructed as figure 20 is then by (168) given as:

$$\rho g\left(\frac{t_M}{\tau}\right) = \frac{q}{1-q} \qquad \textbf{(171)}$$

So the probability of collapse (165) within time t_M becomes:

$$P_c(t_M) = \frac{\rho g\left(\frac{t_M}{\tau}\right)}{1+\rho g\left(\frac{t_M}{\tau}\right)} = \frac{\frac{q}{1-q}}{1+\frac{q}{1-q}} = q = |a|^2 \qquad \textbf{(172)}$$

which again (as in chapter 7) supports the probability-interpretation of the wave-function.

In the Copenhagen- (as well as the many-worlds-)interpretation of QM the probability-picture is used as an excuse for denying the objective validity of the collapse. We have now seen, that there is a way out of this anti-realism (or *nominalism*) The combination of the discretization of zero-point-noise and the fix-point-idea of von Foerster makes it possible to circumvent the main deficiency of the decoherence-picture, described in chapter 6 — the lack of a qualitative jump — and finally combine a realistic view of both the wave-function and its collapse. This is a crucial result of the QS-interpretation.

13. The conveyor model as an eigenform.

It has been stressed in chapter 8, that a measurement is an *irreversible* event, because it has to leave a permanent mark of registration. The description of the collapse as caused by dissipative noise emphasizes the dissipative, thermodynamic irreversibility as caused by a sink within the detector-model of

figure 20. While this is certainly true, there is another kind of irreversibility inherent in the measurement process, that we have hitherto neglected: As shown in figure 23 we may view a measuring apparatus A as a two-port-device with a 'Q-port' connected to the quantum system under inspection and an 'R-port' connected to the registering device. We want the apparatus to pass information from the Q-port to the R-port — but not the other way. The passage from the world of quantum phenomena to the classical world of registered information is irreversible in the sense that quantum uncertainties and superpositions are left out in the classical world. If the quantum system is in a superposition of state $|1\rangle$ and state $|2\rangle$ it is not the case that the pointer of the apparatus is left in a superposition between pointing to '1' and pointing to '2' although the Schrödinger equation for the combined Q and A-system would let it seem so. The classical world is an "either-or-world" while the quantum world is the "both-and-world" of complementarity.

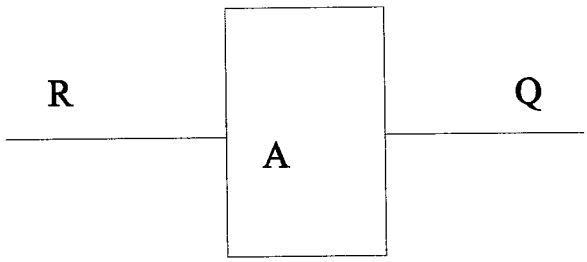

Figure 23. Measuring apparatus between Quantu world and classical Registration.

If the registration consists, e.g. in the punching of a hole in a card then we can't say that the state of the card "collapses" into a state of being punched or not, for the collapse-idea presupposes an initial superposition and there is no such thing in the classical world. Once we have passed from the Q- to the R-side of the A-box in figure 23 the system is in a definite thermodynamic state, and we may say that it evolves into one of two possible *attractors* — being punched or not, but we can't say that it "collapses" into one of these states. Surely the Q-system collapses when one of these attractors are realized, but this collapse is not due to a signal passing from the R-port to the Q-port, but to a noise-pulse from inside A that passes to both ports, as we shall see. This "second kind of irreversibility" may be described as an *activation* of the R — Q information bond, i.e. a process that blocks one of the two possible directions of signal-propagation inherent in the concept of an energy bond. So we may say, that a measuring device provides an *information bond* or an *activated energy bond* from Q to R. The blocking of signals from R to Q is not the same as dissipative irreversibility but rather a deviation from the property of *reciprocity*, described in Onsager's symmetry relations of chapter II.5.

In order to see how this activation may come about let us consider the generalized model of a measuring device, figure 24, that combines a *dissipative* element — a sink — with an *antireciprocal* element — a gyrator:

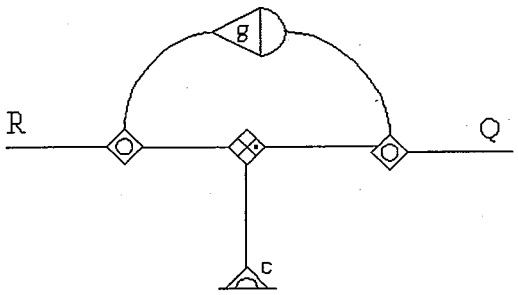

Figure 24. General non-reciprocal two-port.

Assuming now that we have effort-input in both ports — e_1 in the Q-port, e_2 in the R-port, the output-flows will be given by the following response-matrix:

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} c & g-c \\ -c-g & c \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \qquad \textbf{(173)}$$

We see, that the response is reciprocal, i.e. has a symmetric matrix, only for $g = 0$. We see also that the effort e_2 is blocked from influencing the flow f_1 in the case $g = c$. This activated two-port is called a *conveyor* and has a special icon, shown in figure 25.

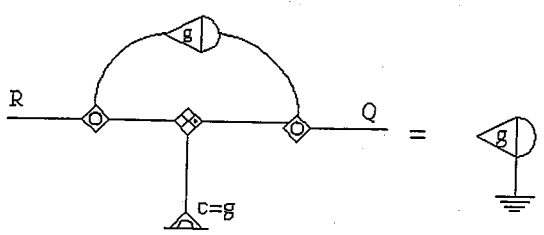

Figure 25. The conveyor.

It is seen that the conveyor-icon to the right in figure 25 contains the gyrator-icon. It also contains an "earth-connection" standing for the dissipative sink c. The vertical line in the middle of the conveyor-icon may be regarded as a picture of the "cut" separating the quantum world from the classical world. In chapter 6 it was mentioned, that Heisenberg saw the "cut" as connected with dissipative processes. Now we can see it as connected with activation — a special combination of dissipation and anti-reciprocity.

Now, it would seem that this special combination — the activation-condition g =c — would require a careful tuning of the parameters, which would make the pragmatic importance of the conveyor somewhat dubious, but we shall see, that this is not the case. We introduce the *transference-matrix* t of the two-port in figure 24 by the definition:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \qquad \textbf{(174)}$$

Solving the response-equation (173) we find:

$$t = \begin{pmatrix} \frac{c}{c-g} & \frac{1}{c-g} \\ \frac{g^2}{c-g} & \frac{c}{c-g} \end{pmatrix} \qquad \textbf{(175)}$$

Consider now the concatenation of two two-ports, A and B, shown in figure 26:

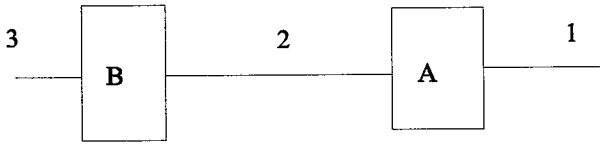

Figure 26. Concatenation of two-ports.

It is clear, then, that the transference-matrices, t_A and t_B for the systems A and B, and t_{AB} for the composite system satisfy the multiplication rule:

$$t_{AB} = t_B t_A \qquad \textbf{(176)}$$

The determinant of the transference-matrix (175) is:

$$\Delta = t_{11} t_{22} - t_{12} t_{21} = \frac{c + g}{c - g} \qquad \textbf{(177)}$$

We see, that $\Delta = 1$ for the reciprocal two-port, $g = 0$, $\Delta = \infty \infty$ for the conveyor, $g = c$, and $\Delta = 0$ for the case of reverse blocking, $g = -c$. For the realistic case $0 \leq g \leq c$ we have $\Delta > 1$. The determinants also satisfy the multiplication rule:

$$\Delta_{AB} = \Delta_A \Delta_B \qquad \textbf{(178)}$$

If we concatenate an infinity of non-reciprocal two-ports with $0 \leq g \leq c$ and $\Delta > 1$, the resultant matrix will have $\Delta = \infty$ and thus be a conveyor. In this way we see the conveyor — not just as a special, finely tuned non-reciprocal two-port, but as a *fix-point-eigenform* of the infinite concatenation of non-reciprocal two-ports. This is a good reason for distinguishing it with a special icon and consider it important for understanding the measuring process in QS.

Icon-reversal of the model figure 24 is equivalent to a sign-change of the gyrator g. So, a concatenation of two-ports with $-c \leq g \leq 0$ and $\Delta < 1$ will lead to a conveyor with $g = -c$ and $\Delta = 0$ — the icon-reversed conveyor.

The reciprocal two-port, $g = 0$, $\Delta = 1$, is, of course, also a fix-point for concatenation, but this is rather trivial, since it requires no special skills to prepare. The extreme kind of non-reciprocity — the activating property of the conveyor — is difficult to realize as a gyrator-effect — it requires a very strong magnetic field and an anisotropic material. Alternatively, it characterizes the amplification-processes associated with the measuring of microscopic effects, such as electronic operational amplifiers, or chemical reactions far from equilibrium, such as the development of photographic plates.

14. The noisy conveyor.

We shall now apply the FD-theorem to the general two-ports, considered in chapter 13 as models of a measuring apparatus.

We shall use the general model of figure 24 as a description of the apparatus A in figure 23, at first without specializing to the conveyor, g = c. Renaming the Q-port and the R-port in figure 23 as port 1 and port 2, respectively, and assuming the orientations to be towards A in both port 1 and port 2, the *admittance*-matrix is given by eq. (173):

$$\underline{\underline{Y}} = \begin{pmatrix} c & g-c \\ -(g+c) & c \end{pmatrix} \qquad \textbf{(179)}$$

Changing causality to flow-input in both ports with the same orientations we may determine the *impedance* matrix:

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \underline{\underline{Z}} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}; \underline{\underline{Z}} = \begin{pmatrix} \frac{c}{g^2} & \frac{c-g}{g^2} \\ \frac{c+g}{g^2} & \frac{c}{g^2} \end{pmatrix} \qquad \textbf{(180)}$$

The matrix-formulation of the FD-theorem in chapter II.5 gives the following expression for the Fourier-components of the flow-noise, δf_i and δf_j, in ports i and j:

$$< \delta f_i(\omega) \delta f_j^*(\omega) > = \frac{1}{\pi} \cdot \underline{\underline{Y_{ij}}}^D \cdot \theta(\omega,T) \qquad \textbf{(181)}$$

Where $Y^D(\omega)$ is the dissipative, i.e. hermitean, part of the admittance-matrix and $\theta(\omega,T)$ is the oscillator-energy, eq. (112).

Similarly, we have the effort-noises for flow-input:

$$<\delta e_i(\omega)\delta e_j^*(\omega)>=\frac{1}{\pi}\cdot\underline{\underline{Z}}_{ij}^{D}(\omega)\cdot\theta(\omega,T) \qquad \textbf{(182)}$$

It is now obvious, that the expressions (179) and (180) do not allow a calculation of the noise-width-function, because the dissipative admittance will cause an ultraviolet catastrophe. However, as we did before, we can "renormalize" the and impedance are frequency-independent, so the zero-point energy $\frac{1}{2}\hbar\omega$ noise by "dressing" the sink c with a storage element, as shown in figure 27:

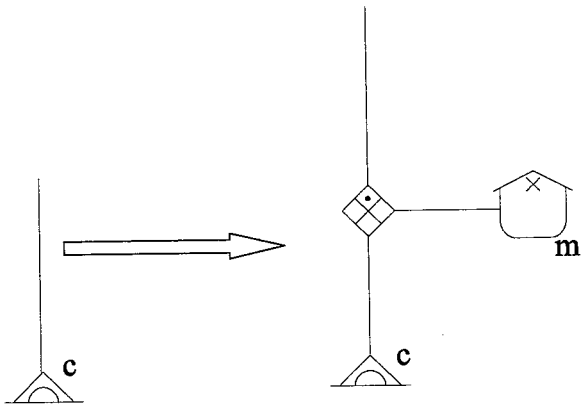

Figure 27. dressing a sink with a storage.

If c is a normal conductance the storage element takes the inertance of the charge-carriers into account. The quantity c in (179) and (180) then has to be replaced with a function c(s) of the Laplace-frequency s:

$$c(s)=\frac{c}{1+s\tau} \qquad \textbf{(183)}$$

where $\tau = c\,m$ is the relaxation time. Putting $s = -i\omega$ and taking the hermitean part of the matrices in (179) and (180) we find the dissipative response-matrices as function of the frequency ω:

$$\underline{\underline{Y}}^{D}(\omega)=\frac{c}{1+\omega^2\tau^2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} \qquad \textbf{(184)}$$

and:

$$\underline{\underline{Z}}^{D}(\omega)=\frac{c}{g^2(1+\omega^2\tau^2)}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \qquad \textbf{(185)}$$

For the conveyor with g = c = 1/R that all 8 noise-correlation-coefficients in (181) and (182) have the same ω-dependence as the first detector-model figure 20, eq. (131). They can therefore also each be associated with a finite number-function of discrete events, $N(t)=\rho g(t/\tau)$ as in (133) with $g(t/\tau)$ given by eq. (119). Considering discrete flow-pulses $\eta_i\delta(t-t_i)$ in port 1 and 2 we see from the negative non-diagonal elements of the matrix eq. (184) that the ηs in the two ports are *anticorrelated*, i.e. if η_i is +1 in port 1, then it is -1 in port 2 and vice versa. With the chosen orientations this means that a particle enters the apparatus in port 1 and leaves it in port 2.

Conversely, the non-diagonal impedance-elements in (185) are +1, which means that the effort-pulses h η_i $\delta(t-t_i)$ in the two ports are correlated, having the same value of η_i. This means that an effort-pulse can be interpreted as the simultaneous occurrence of *collapse* in port 1 and *detection* in port 2.

15. The anatomy of choice.

The general situation in QS, where the preparation of the state splits up into several rays, each of them connected with several detectors (figures 3 and 4) calls for the notion of two different kinds of choices: First, the quantum must choose between the different rays of the prepared superposition. This we call the *Q-choice.*

Second, when a ray is connected to several detectors, detection and collapse takes place in only one of them, and the choice between them we call the *M-choice.*

We have in chapters 12-14 described the M-choice as caused by the random influence of the zero-point-noise from the dissipative detector, considered as a sequence of discrete quantum-events. This is clearly an explanation that respects the principle of local realism, since it only involves the local expansion-coefficient of the state-vector to the local ray of the apparatus (figure 21) and its local dissipative parameter.

The Q-choice is more difficult to understand in a local-realistic way, since the exact specification of the ray of detection may be separated from the preparation-process both in space and time. In the third of Aspect's experiments (1982) a random switching takes place between two different directions of polarizer for each photon during their flight from source to detector, so a photon cannot "know" from birth, what it means to be detected as "x-polarized". It is as if a signal propagates from the actual detector to the newborn photon telling it what direction is 'x' and forcing it to choose, whether it shall be "x-polarized or not. This is difficult to accept, because such a signal would have to propagate *backwards in time.*

This suggestion was actually taken serious in one of the more fanciful interpretations of QM, *the transactional interpretation*, proposed by J. G. Cramer in 1986. Cramer referred to the "Wheeler-Feynmann-absorber-theory" that builds on the advanced solution to wave equations, that actually exist as mathematical solutions, e.g. to Maxwell's equations, but are normally considered to have no physical reality. In QS we shall prefer to maintain the normal conception of causality and thermodynamic irreversibility, as formulated, e.g. by Einstein, which is fully able to explain, why advanced solutions do not really exist. This means, however, that we have to develop an alternative to Cramer's "prescient" conception of the Q-choice.

In the transactional interpretation there is no need of an M-choice: All is decided in the future as a Q-choice of the ray of an actual detector ("actual" means that the detector exists in the present, but the Q-choice of it is decided in the future). In QS we shall need both a Q-coice and an M-choice, because we have to consider the possibility of a "null measurement" If a detector is chosen by the Q-choice, it still has the possibility of detecting nothing, and this must be decided by an M-choice of the apparatus in the proper time and place, if we do not accept the existence of advanced solutions. The reason why we don't is the following:

The theory of frequency-dependent linear response-functions is concerned with the response to time-homogeneous signals of the form e^{st}, where $s=\varepsilon-i\omega$. ω is the frequency and ε a small positive quantity. The reason why response-functions are only measurable in the right half-plane $\varepsilon>0$ is the causality condition, that we can only recognize something as a response, if the system has been undisturbed in the infinite past, i.e. if the stimulus e^{st} vanishes for $t=-\infty$. We can never claim that the stimulus vanished in the infinite future, because we do not know the future, and therefore an observed response is always retarded: it comes after the stimulus. However, if we calculate the response-function as a function of s, it will always be a real function of s. The observed response-function of s will be analytical — without poles and other singularities in the right half-plane, but the calculated

response-function can be defined in the whole complex s-plane and must have poles in the left half-plane $\varepsilon<0$. The response from such a pole will look like an advanced response, so the analytical continuation of the response-function to the left half-plane is the advanced response-function. It is thus correct to say that the advanced response-function *exists* as a solution to the fundamental equations, but it is not a measurable quantity, only an analytical continuation of the measurable response-function from the right half-plane. The advanced response-function in the left half-plane is separated from the measurable response-function in the right half-plane by a *branch-cut* along the imaginary axis, and there is a discontinuity over this cut, whose magnitude is the *dissipative* part of the response-function. The branch-cut prevents us from crossing the imaginary axis experimentally, and it is thus the dissipative properties of systems that makes advanced responses unobservable. This argument is essentially a modernized version of Einstein's argument from 1909, here summarized by D. Layzer:

"Einstein pointed out that the retarded and advanced descriptions of radiation processes occurring in any finite region of space-time are essentially equivalent, but the auxiliary conditions in the two descriptions differ in kind. In the retarded description all *macroscopic* radiation sources must be specified, while in the advanced description the *microscopic* absorption processes must be specified *in detail.* In practice one uses the retarded description because one does not have microscopic information about the absorbing matter. For the same reason, if one wishes to describe an irreversible process such as diffusion or heat conduction at the macroscopic level one must describe it as occurring in the 'forward' direction of time. In short, Einstein's argument demonstrates that the asymmetry of macroscopic radiation processes results from precisely those properties of matter in bulk that give rise to other macroscopically irreversible phenomena."

The weirdness of philosophies like the transactional interpretation (TI) and the many-worlds-interpretation (MWI — also called "the relative-state-interpretation") seems to arise from a peculiar form of orthodoxy — not uncommon among physicists — The belief, that only "first principles" as Schrödinger's, Hamilton's, and Maxwell's equations count as true physics, whereas "auxiliary conditions", such as our human inability to prepare an improbable initial state of a thermodynamic system, are considerations of an inferior kind. This is the attitude that make people believe, that irreversibility, chaos, flow of time, memory, and free will are illusory epiphenomena — that the real physical world is totally reversible and deterministic.

In QS we must abandon such reductionistic orthodoxy and try to understand the world from within our human preconditions, which include our ability to create and understand signs.

In the MWI the collapse of the wave-function is non-existent, because it does not follow from the Schrödinger equation, and in the TI it only exists in the future. In QS it occurs in the "now", from where it makes the future flow into the past.

Although both choices — Q and M — are decided in the present, they are not simultaneous. The Q-choice must precede the M-choice, and the latter must include the possibility of a denial — a null measurement. Somehow the Q-choice must be an opening question, or *proposal* to the detector, and this opening is a quantum-event, caused by an incoming particle, propagating forward in time. Using again the general two-port on figure 24 and its special case, the conveyor on figure 25 as our model of a detector, the choices as quantum events, in accordance with the QS-philosophy of such events, must be seen as causal shifts of the signal-structures at the junctions of the model. As the best way of surveying a causal structure we may choose the representation by *signal-rails*, introduced in chapter I.3.3.

In this way we find, that there are exactly four possible types of causality associated with the model on figure 24.

Figure 28 shows the signal-rail-structures of these four possibilities.

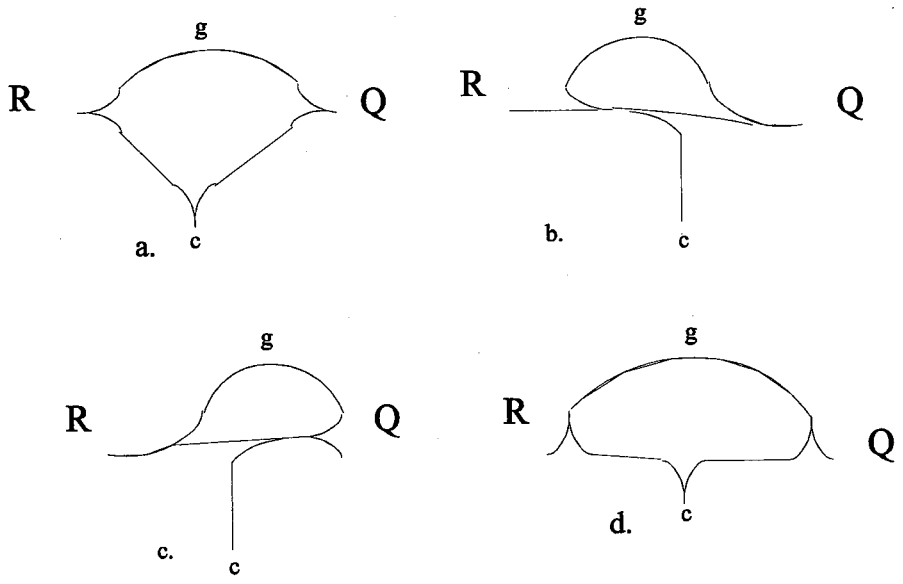

Figure 28. Causal structures of detector model.

The response-matrices for the cases a. and d. have already been given in (179) and (180). For the remaining two cases we find:

For case b.:

$$\begin{pmatrix} f_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} \frac{g^2}{c} & \frac{g-c}{c} \\ \frac{g+c}{c} & c \end{pmatrix} \begin{pmatrix} e_1 \\ f_2 \end{pmatrix} \qquad \textbf{(186)}$$

and for case c.:

$$\begin{pmatrix} e_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{c} & \frac{g-c}{c} \\ -\frac{c+g}{c} & \frac{c^2-g^2}{c} \end{pmatrix} \begin{pmatrix} f_1 \\ e_2 \end{pmatrix} \qquad \textbf{(187)}$$

We see that the conveyor, g=c, in all cases block for signals from port 2 to port 1.

In order to use the FD-theorem for these two cases we have to determine the *dissipative* response-matrices and then find the noise-correlations from (182) or eq. (110) in chapter II.5.

As in chapter II.5 we must aggregate the two bonds to a single two-dimensional vector-bond. This vector-bond will then have σ_z-metric, because the vector-components consist of one effort and one flow. Regarding the input-vector of e_1 and f_2 in (186) as a covariant effort and the output-vector of f_1 and e_2 as a contravariant flow we find that the response-matrix in (186) is $Y^{\cdot\cdot}$, the 3-variant of the admittance-tensor.

Replacing again the constant c with the frequency-dependent function $c/(1 + s\tau)$, putting $s = -i\omega$ and finding the hermitean part of $Y^{\cdot\cdot}(\omega)$ we get:

$$Y^{D\cdot\cdot} = \begin{pmatrix} \frac{g^2}{c} & \frac{g}{c} + i\omega\tau \\ \frac{g}{c} - i\omega\tau & \frac{1}{c} \end{pmatrix} \qquad \textbf{(188)}$$

Similarly, we get from (187) the 3-variant of the dissipative impedance-tensor:

$$Z^{D\cdot\cdot} = \begin{pmatrix} \frac{1}{c} & -1 - \frac{g}{c} i\omega\tau \\ -1 + \frac{g}{c} i\omega\tau & \frac{c^2 - g^2}{c} \end{pmatrix}$$

Unlike the dissipative response-functions (184) and (185) for the cases a. and d. we find in (188) and (189) an imaginary, antisymmetric part that increases proportional to ω. When this is combined with the zero-point oscillator-energy $\frac{1}{2}\hbar\omega$ it becomes impossible to renormalize the noise-correlations to finite width- or number-functions of time.

We have to accept, that the noise-correlations for the causal forms b. and c. are infinite, and this means, that these two forms are *unstable* and we only have the two pure causal forms a. and d. as anchor-points in the anatomy of choice.

Now, case a. with strong causality in both ports may be considered *closed*, because it requires a quantum event to assume dominance over an o-junction. Case d., on the other hand is *open* because it has weak causality in both ports, and weak bonds may be attached adiabatically without disrupting the causal structure.

We may assume that the detector is closed, until it is opened by a Q-choice. Such an event will then lead to one of the unstable forms b. and c. from where it must either return to a. or go on to d. by the M-choice.

We may thus identify the following two possible outcomes of a measurement- or detection-event:

a.) *a null-measurement*: a. → (b.c.) →a.
b.) *a positive detection*: a.→ (b.c.) →d.

After a positive detection ending in d. a second M-choice, possibly caused by classical noise from the passive recording device must lead back to a. concluding the cycle of measurement.

We have already seen the the opening Q-choice starting from a. is associated with a particle or a quantum entering the Q-port (port 1) and exiting the R-port. However, the final event, going from (b.c) or d.. to a. is associated with an effort-pulse in the Q-port and this is the sign of collapse of the wave-function.

Although we see that a null-measurement leads to collapse just as well as a positive detection the model does not clearly show that two different rays of Hilbert-space are involved: Indeed, the ray of null-measurement must be orthogonal to the ray of detection, but the model does not distinguish between these two rays, and this is, of course, a weakness.

In figure 29 the detector model of figure 24 is shown with an added bond of non-detection, -R, leading back to the Q-world to indicate the ray of null-measurement:

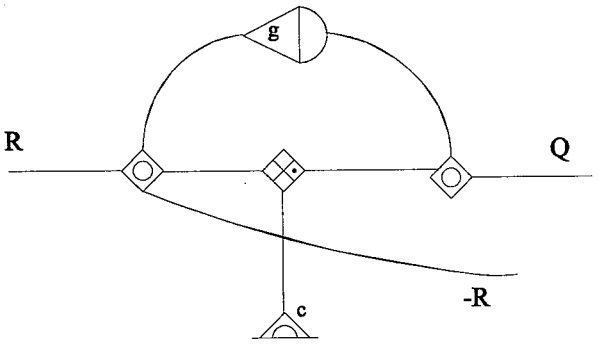

Figure 29. Detector with ray of non-detection.

In the Copenhagen-interpretation of QM the collapse is considered unreal — a mental inference rather than a physical event, and in the MWI it is a choice between different branches of "the multiverse" that takes place, presumably, in the synapses of the brain, if at all.

The above mentioned weakness in the QS-attempt to describe the collapse realistically could perhaps be resolved by a partial adoption of the Copenhagen view: The projection is real and caused by a dissipative noise-pulse, but the decision whether it is to the detection-ray or the orthogonal ray of null-measurement is left to mental inference, which is easy to perform by just looking at the detector.

Whether we accept this last adoption, or not, we still have the realistic view left, as is exposed by the decoherence-picture (chapter 6): All phase-coherence and quantum-interference is lost by the collapse.

16. Concluding remarks.

The heading of this part III of my thesis is "The energy-bond-graph approach to Quantum Semiotics". By this I mean, that the formalism of energy-bond-graphs should be able to build a new foundation of Quantum Mechanics, like I showed in part II, that it could be used for re-deriving the principles of special and general Relativity. This new way was first called Quantum Semeiotic in order to remind of C.S. Peirce's greek naming and spelling of his doctrine of signs (Σημειωτικ΄η), but later I preferred a more modern english spelling, like in Jesper Hoffmeyer's "Biosemiotics".

I had to admit, however, that I would not be able to define QS as encompassing all of QM (including Dirac's relativistic theory), but I still see QS as a possibility — a point in the horizon, that perhaps will be reached by a younger and more able researcher, and I hope that the EBG-formalism, as presented and applied in this thesis, may serve as a pointer to the right direction.

Nowadays one must distinguish between QM proper, as it is presented in many text-books, and the *interpretation* of QM, to which text-books most often pay only lip-service. Some of these interpretations are very strange and seem to violate well established principles of general philosophy and physics (e.g. falsifiability, or *fallibilism*, as Peirce called it). One could, of course, claim, that QM itself calls for some mysticism (or dizzyness, as Bohr would have it), but I regard the strangeness of these interpretations as a sign of crisis in physics.

The outline of QS, as here given, may be seen as an attempt to define a *non-paradoxical interpretation* of QM. Below I shall stress some points, where I think QS able to lift the veil of mysticism, that surround even the orthodox Copenhagen interpretation and seems to be the very fabric, that alternative interpretations are made of.

*

Anti-mysticism.

This attempt is *not* motivated by a dislike of mysticism. I highly respect the mystical traditions of religion and philosophy, like Taoism and Zen-buddhism and the Sufi-tradition of Islam, but I don't think that mysticism should be used as a lifeboat over waters of muddy strangeness in physics. The main virtue of

physics is that it is understandable, and we should not try to compromize that by appealing to "the Tao of Physics" in order to attract the interest of students of philosophy, psychology, or the fine arts.

I see QM as a paradigm of reasonableness, also in situations outside its usual domain of atomic physics. Classical logic is clearly not sufficient to cover even everyday situations of communication. It needs extension in some of the directions pointed out by Peirce, like the logic of vagueness, multivalued, temporal, and modal logic. All these extensions are covered by viewing *logic as semeiotic* and then we don't need a special "quantum logic".

Realism.

Realism is the belief, that the world exists independent of our preconceived notions of it.

In the middle ages the word "realism" often meant *concept-realism* — the belief that general concepts, *Universalia*, had an independent existence. For Platon they existed in their own *transcendent* "heaven", but for Aristotle they existed *immanently* within matter as *entelechia* — the principles of form. The contrary position to concept-realism was *Nominalism* — the belief that universalia only exist as names in our heads. This position, as formulated, e.g. by William of Occam and further by Francis Bacon was dominating up to recent times of positivism, but Gödel's discoveries in logic and meta-mathematics together with newcomings in pure Mathematics, such as B. Mandelbrot's concept of *fractals* — especially the *Mandelbrot-set* has led to a new wave of concept-realism in Mathematics — the conviction that mathematical concepts are *discovered* — not just *invented* like new words or rules of a game.

One of the main points of critique against the Copenhagen interpretation has been levelled against its *antirealism*. Some more or less apocryphical Bohr-quotations, like "there is no quantum-reality" or "the wave-function is just chalk on a blackboard" seem to justify this critique, But, as often stressed by the danish philosopher David Favrholdt, one may also find quotes that seem to exhibit Bohr as a realist. He certainly regarded atomic particles and measuring devices ("described in classical terms") as real things. His anti-realism — i.e. nominalism concerns the wave-function or "the very conditions for making predictions".

QS is realistic with regard to the wave-function and its collapse, and this is the main point, where it deviates from the Copenhagen-interpretation. The wave-function is of course just one of an infinity of solutions to the wave-equation, so it

is rather the equation as described by the underlying structure of an EBG-model as in figures 6, 7, and 8, that should be regarded as real. The EBG-icons are *real general objects* that are "classical terms" for many experimentalists, even though they are often not conceived in the abstract way here presented, but rather in more concrete electrical, rheological, or mechanical terms.

The relations represented by the icons are all linear and the storages contain both kinetic and potential energy, so the EBG-structure will be the medium for waves with a linear superposition principle.

When a wave is expanded into rays the complex expansion coefficients, a, are represented by transformers. The absolute squares $|a|^2$ multiply nearby sinks, where the passage of quanta are registered as delta-function-pulses (chapter 7). In this way $|a|^2$ gets the meaning of a probability. This interpretation can also be applied to the detection-end of the sign-relation, as shown in chapter 12. In this way QS supports a realistic view on the wave-function, probabilities, and quantum-events.

The FD-theorem.

The EBG-formalism and the FD-theorem (chapters 8-11) are the main keys to QS. We have accepted the usual QM-formalism as developed by Schrödinger, Heisenberg, and Dirac and shown that the FD-theorem follows rigorously from it, when it is combined with the EBG-theory of linear response.

This remarkable theorem is thus a gateway from QM to QS. However, in QS the dissipative noise is interpreted in a non-standard way — not just as concerning Fourier-components of continuous functions, but as "shot-noise" of quantum events. The restriction to zero-point-noise is renormalizeable for the width-or number-function for such events, and the FD-theorem gives exact expressions, both for waiting-time-distributions and probabilities for the number of events within a given time-interval. In the classical limit, $\beta=0$, we have gamma-distributions of time and Poisson-distribution of number, but in the quantum (zero-point) limit, $\beta=\infty$, we find a number-function $g(t/\tau)$ that increases rapidly in the beginning (producing the instantaneous Heisenberg-uncertainties) and very slowly (logarithmically) for later times. And the number-distribution in the long-time-limit was found to be geometrical, which is important for the realistic interpretation of the collapse as an eigenform, or fix-point behaviour.

Reality of the collapse.

In QS, unlike the Copenhagen-interpretation and the MWI, the collapse is seen as a real event in present time. This requires a theory of measurement, which has long been a missing link in QM. A full theory of measurement would require a detailed description of a real measurement-apparatus, but this has not been attempted here, because QS is seen as a theory of general signs — a metaphysical rather than a physical theory of measurement. The measurement-apparatus has therefore only been given an EBG-description of its most basic features — its dissipative and non-reciprocal response-properties. An important special case — the conveyor-model was introduced as a fix-point-eigenform of the general two-port-model, and we saw it as an icon of "the cut" separating between the quantum-world and the classical world. This makes it understandable, that systems governed by chemical reactions far from equilibrium, such as living organisms, belong to the classical world and thereby resolves such paradoxes as Schrödinger's cat.

Other interpretations also try to regard the collapse as real. R. Penrose has developed a theory of "objective reduction of the wave-function" based on the consideration of gravitational effects. The idea is, that QM requires a definite metric tensor, but the indecision of the Schrödinger-equation, leading to a superposition of pointer-positions of the measuring apparatus also leads to a superposition of metrics, that must be decided by a collapse. Small gravitational energy-differences associated with the different "pointer-positions" allows Penrose to use the energy-time-uncertainty-relation to calculate a waiting-time for the collapse. However, since the gravitational energy-differences tend to be very small, the calculated waiting times tend to be too long, and I think, that the present theory, where the waiting-time is scaled by the dissipative relaxation time of the apparatus gives a better estimate. This may be a point, where a possible falsification-experiment could be designed to decide between the two theories.

A third way of seeing the collapse as real is the decoherence-picture (chapter 6). If the decoherence-factor is described vaguely as "perturbations from the environment" it gives no definite expression for the waiting time, but if "the environment" is identified as the source of dissipative noise from the measurement apparatus, the decoherence-picture gives the same predictions as QS, because the width-function for the diffusion of the phase of the interference-terms (the non-diagonal elements of the density-matrix) is proportional to the number-function $g(t/\tau)$ here considered. It seems thus to be unnecessary or impossible to decide between decoherence and QS by measurement of waiting times. Philosophically, however, QS seems the more satisfactory, because it accounts for "the qualitative jump", which is rather explained away by decoherence as having happened FAPP (for all practical purposes). Penrose has seen decoherence as a main rival to his

own theory and has criticized it for the same reason.

Non-locality or entanglement.

The EPR-experiment, as realized by Aspect and others, has traditionally been seen as a proof of Quantum-Non-Locality (QNL), but this expression has in recent years been superseeded by the word "entanglement", perhaps because, as stressed, e.g., by Shimony, that there is no "action-at-a-distance" in other words: QNL is not real in a pragmatic sense: it does not allow for communication with superluminal speed. Instead, the word "entanglement" has been introduced as a more cautious way of expressing the fact, that a ray of superposition may refer to potential detections in different locations of space.

In QS this sort of entanglement is fully accepted and seen as a reflection of the distributive relations of junctions.

On the other hand: the EBG-formalism is *explicitly local* and therefore QS does not allow for real QNL.

I have earlier shown, that the observed violations of Bell's inequalities is not a sign of QNL, because it is possible to have them violated in purely local and classical scenarios of communication. In stead, one may see their violation as a sign of *contextuality*, which is basic in QM, where the properties of quantum systems only exist in a given experimental context.

In my first writings on QS twenty years ago I saw the coincidence-counters in Aspect's experiments as defining the common context for the two separated detections and proposed as a possible falsification of my idea a similar experiment wqithout coincidence-counters. This experiment was realized by G. Weihs and A. Zeilinger in Innsbruck in 1998 and I had to admit that my idea had been proven wrong.

Immediately after this I realized, that entanglement-phenomenon was incorporated in the EBG-formalism by the distributive relations of the junctions and gradually my present understanding of QS began to develop.

I have thus, painfully, had to realize the great benefits to science of fallibilism and falsification-experiments. Perhaps my ideas may be proven wrong once again by a clever experimentalist, and I shall rejoice for the sake of science.

FINIS, march 9th, 2004, Peder Voetmann Christiansen.

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