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**COMPLEX STRUCTURES
IN THE NASH-MOSER CATEGORY**

by

Jens Gravesen

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IMFUFA

ROSKILDE UNIVERSITETSCENTER
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES
FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, Roskilde University Center, Postbox 260, DK-4000 Roskilde,
DENMARK

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Abstract. Working in the Nash-Moser category, it is shown that the harmonic and holomorphic differentials and the Weierstrass points on a closed Riemann surface depend smoothly on the complex structure. It is also shown that the space of complex structures on any compact surface forms a principal bundle over the Teichmüller space and hence that the uniformization maps of the closed disk and the sphere depend smoothly on the complex structure.

1. Introduction

In my thesis [5], I needed to show that the harmonic and holomorphic differentials, the Weierstrass points and the uniformization maps of a Riemann surface depend continuously on the complex structure. It was done by explicit calculations, but later I realized that these results are a consequence of the Nash-Moser inverse function theorem, and the purpose of this paper is to demonstrate this. The result concerning the uniformization maps is also contained in the work of Earle et al. [1], [2].

In section 2 the basic definitions and notation of the Nash-Moser category are introduced. In section 3 the harmonic and holomorphic differentials and the Weierstrass points are considered and finally, in section 4 Teichmüller theory is studied.

2. The Nash-Moser Category

A *Fréchet space* is a complete Hausdorff metrizable locally convex topological vector space.

Let F and G be Fréchet spaces, U an open subset of F , and $P: U \subseteq F \rightarrow G$ a continuous nonlinear map. The *derivative* of P at the point $f \in U$ in the direction $h \in F$ is defined by

$$DP(f)h = \lim_{t \rightarrow 0} \frac{P(f + th) - P(f)}{t}.$$

If the limit exists, then we say that P is differentiable at f in the direction h . We say that P is continuously differentiable (or C^1) on U if the limit exists for all $f \in U$ and all $h \in F$ and if $DP: (U \subseteq F) \times F \rightarrow G$ is continuous. Note that the continuity requirement is weaker than the usual definition for a Banach space. The n th derivative is inductively defined by

$$D^n P(f)\{h_1, \dots, h_n\} = \lim_{t \rightarrow 0} \frac{D^{n-1} P(f + th_n)\{h_1, \dots, h_{n-1}\} - D^{n-1} P(f)\{h_1, \dots, h_{n-1}\}}{t}$$

and will be regarded as a map

$$D^n P: (U \subseteq F) \times F \times \dots \times F \longrightarrow G.$$

We say P is of class C^n , if $D^n P$ exists and is continuous, and we say P is smooth or of class C^∞ , if P is of class C^n for all n .

The topology of a Fréchet space can always be defined by some countable collection of seminorms $\|\cdot\|_n$, such that

- (1) If $\|f\|_n = 0$ for all n , then $f = 0$.
- (2) A sequence which is Cauchy in each seminorm is convergent.

A *grading* on a Fréchet space is a collection of seminorms $\{\|\cdot\|_n \mid n = 0, 1, 2, \dots\}$ defining the topology, such that

$$\|f\|_0 \leq \|f\|_1 \leq \|f\|_2 \leq \dots$$

A *graded Fréchet space* is a Fréchet space with a choice of grading.

A linear map $L: F \rightarrow G$ between graded spaces is called *tame*, if it satisfies a *tame estimate* of degree r and base b , i.e., if

$$\|Lf\|_n \leq C\|f\|_{n+r} \quad \text{for all } n \geq b,$$

where the constant C may depend on n . If L is bijective and both L and L^{-1} are tame, then L is called a *tame isomorphism*.

Two different gradings on the same Fréchet space is called *tamely equivalent*, if the identity map is a tame isomorphism.

A map $P: U \subseteq F \rightarrow G$ satisfies a *tame estimate* of degree r and base b if

$$\|P(f)\|_n \leq C(1 + \|f\|_{n+r}) \quad \text{for all } n \geq b,$$

where the constant C may depend on n . If U is open and P is continuous, then P is called *tame* if it satisfies a tame estimate in a neighbourhood of each point. If P happens to be linear, then the two definitions of tame coincide.

A map $P: U \subseteq F \rightarrow G$ is called *smooth tame* if it is smooth and all its derivatives are tame. A composition of smooth tame maps is smooth tame.

Let B be a Banach space with norm $\|\cdot\|_B$ and let $\Sigma(B)$ denote the space of all sequences $\{f_k\}$ of elements in B such that

$$\|\{f_k\}\|_n = \sum_{k=0}^{\infty} e^{nk} \|f_k\|_B \leq \infty \quad \text{for all } n \geq 0.$$

Then $\Sigma(B)$, equipped with the norms above is a graded space.

Let F and G be graded spaces. We say that F is a *tame direct summand* of G if there exist tame linear maps $L: F \rightarrow G$ and $M: G \rightarrow F$ such that the composition $ML: F \rightarrow F$ is the identity.

A graded space is called *tame* if it is the direct summand of $\Sigma(B)$ for some Banach space B .

A direct summand of a tame space is tame and the cartesian product of two tame spaces is tame.

Let M be a compact manifold possibly with boundary and corners, and let $C^\infty(M)$ denote the space of smooth functions $M \rightarrow \mathbb{R}$ equipped with the C^∞ -topology. The space $C^\infty(M)$ has gradings defined by the supremum norms $\|\cdot\|_n = \|\cdot\|_{C^n(M)}$, the Hölder norms $\|\cdot\|_n = \|\cdot\|_{C^{n+\alpha}(M)}$ with $0 < \alpha < 1$ or the Sobolev norms $\|\cdot\|_n = \|\cdot\|_{L_p^n(M)}$ with $1 < p < \infty$. By the Sobolev inequalities, the gradings are all equivalent, and in [Ha] it is shown that $C^\infty(M)$ is tame. It is also shown that if $C_0^\infty(M)$ is the closed subspace of functions in $C^\infty(M)$ which vanish on the boundary together with all their derivatives, then $C_0^\infty(M)$ is tame. But we have to consider a space between $C_0^\infty(M)$ and $C^\infty(M)$.

(2.1) LEMMA. If M is a compact manifold with boundary ∂M and $C^\infty(M, \partial M)$ is the closed subspace of functions in $C^\infty(M)$, which vanish on the boundary, then $C^\infty(M, \partial M)$ is tame.

PROOF: Let B be a neighbourhood of ∂M diffeomorphic to $\partial M \times [0, 1]$.

Choose a smooth function $\sigma: [0, 1] \rightarrow [0, 1]$ which is 1 in a neighbourhood of 0 and 0 in a neighbourhood of 1.

Let $i: C^\infty(M, \partial M) \hookrightarrow C^\infty(M)$ be the inclusion and define $\rho: C^\infty(M) \rightarrow C^\infty(M, \partial M)$ by

$$\rho(f)(x) = \begin{cases} f(x) & \text{if } x \notin B \\ f(y, t) - \sigma(t)f(y, 0) & \text{if } x = (y, t) \in \partial M \times [0, 1] = B. \end{cases}$$

The maps i and ρ are tame and $i \circ \rho$ is the identity, so $C^\infty(M, \partial M)$ is a tame direct summand of $C^\infty(M)$ and hence tame. \square

Let M be a compact manifold possibly with boundary and corners. If ξ is a vector bundle over M , then the space $C^\infty(M, \xi)$ of smooth sections in ξ is a tame space. If η is another vector bundle, then $L(\xi, \eta)$ denotes the bundle of vector bundle morphisms $\xi \rightarrow \eta$.

(2.2) LEMMA. If ξ, η and ζ are vector bundles over M , then the composition map

$$C^\infty(M, L(\eta, \zeta)) \times C^\infty(M, L(\xi, \eta)) \longrightarrow C^\infty(M, L(\xi, \zeta))$$

is smooth and tame.

The space $\text{Diff}_k(\xi, \eta)$ of differential operators of degree k from ξ to η can be identified with the space of smooth sections in the bundle $L(J^k \xi, \eta)$, where $J^k \xi$ is the k 'th jet bundle, so we have

(2.3) COROLLARY. If ξ, η and ζ are vector bundles over M , then the composition map

$$\text{Diff}_l(\eta, \zeta) \times \text{Diff}_k(\xi, \eta) \longrightarrow \text{Diff}_{k+l}(\xi, \zeta)$$

is smooth and tame.

A *tame manifold* is a manifold with charts in tame spaces and whose transition functions are smooth tame maps.

If ξ is a smooth bundle over M , then the space $C^\infty(M, \xi)$ of smooth sections in ξ is a tame manifold.

A *tame Lie group* is a tame manifold G , which has a group structure such that the multiplication map $G \times G \rightarrow G$ and the inverse map $G \rightarrow G$ are smooth tame maps.

If M is a compact manifold possibly with boundary and corners, then the diffeomorphism group is a tame Lie group. If $\partial M = \emptyset$, then it is shown in [Ha], and the general result follows from lemma 2.1

The *Nash-Moser category* is the category whose objects are tame spaces and whose maps are smooth tame maps. The importance of this category is that we have the Nash-Moser inverse function theorem :

(2.4) THEOREM. Let F, G be tame spaces and $P: U \subseteq F \rightarrow G$ a smooth tame map. Suppose that the equation for the derivative $DP(f)h = k$ has a unique solution $h = VP(f)k$ for all $f \in U$ and all $k \in G$, and that the family of inverses $VP: U \times G \rightarrow F$ is a smooth tame map. Then P is locally invertible, and each local inverse P^{-1} is a smooth tame map.

3. Complex Structures in Dimension Two

Let M be a compact two-dimensional manifold possibly with boundary and corners. Choose a volume form Ω and let F be the subbundle of $\text{End}(TM)$ consisting of endomorphisms J with $J^2 = -1$ and $\Omega(v, Jv) \geq 0$ all $v \in TM$. As the dimension of M is two, any almost complex structure is a complex structure, so we define the space $\mathcal{C}(M)$ of complex structures on M as the space of smooth sections J in F , equipped with the C^∞ -topology. By general theory, $\mathcal{C}(M)$ is a smooth tame submanifold of $C^\infty(M, \text{End}(TM))$.

The volume form Ω together with a complex structure J determine a metric on M by

$$(v, w)_J = \frac{1}{2}(\Omega(v, Jw) + \Omega(w, Jv)).$$

By duality we can consider J as acting on one-forms, and then $-J$ is the Hodge star-operator for $(\cdot, \cdot)_J$. We also let $-J$ denote the Hodge star operator acting on zero- and two-forms, i.e., $Jf = -f\Omega$ and $Jf\Omega = -f$.

The metric $(\cdot, \cdot)_J$ on M induces a Hermitian metric on the bundle $\wedge^i M_{\mathbb{C}}$ of complex valued i -forms on M . We also denote this metric by $(\cdot, \cdot)_J$, and in terms of J it can be expressed as $(\varphi, \psi)_J \Omega = \varphi \wedge -\overline{J\psi}$. The space of metrics on a vector bundle consists of smooth sections in the bundle of symmetric two-tensors, so it is a smooth tame manifold. We clearly have

(3.1) LEMMA. The map $J \rightarrow (\cdot, \cdot)_J$ is a smooth tame map from the space of complex structures to the space of metrics.

On $C^\infty(\wedge^i M_{\mathbb{C}})$ we have an inner product defined by

$$(3.2) \quad (\varphi, \psi)_J = \int_M (\varphi, \psi)_J \Omega = \int_M \varphi \wedge -\overline{J\psi}.$$

The complex structure J induces a splitting $\wedge^1 M_{\mathbb{C}} = \wedge_J^{1,0} M \oplus \wedge_J^{0,1} M$ of the complex one-forms into $(1, 0)$ -forms and $(0, 1)$ -forms, and a corresponding splitting of the exterior differential $d = \partial_J + \bar{\partial}_J$. The projection $\pi_J^{k,l}$ onto $\wedge_J^{k,l} M$ is given by $\pi_J^{1,0} = \frac{1}{2}(1 - iJ)$ and $\pi_J^{0,1} = \frac{1}{2}(1 + iJ)$, so the operators ∂_J and $\bar{\partial}_J$ have the expressions

$$\partial_J = \begin{cases} \pi_J^{1,0} \circ d = \frac{1}{2}(1 - iJ) \circ d, & \text{on } \Omega^0 M_{\mathbb{C}} \\ d \circ \pi_J^{0,1} = d \circ \frac{1}{2}(1 + iJ), & \text{on } \Omega^1 M_{\mathbb{C}}, \end{cases}$$

$$\bar{\partial}_J = \begin{cases} \pi_J^{0,1} \circ d = \frac{1}{2}(1 + iJ) \circ d, & \text{on } \Omega^0 M_{\mathbb{C}} \\ d \circ \pi_J^{1,0} = d \circ \frac{1}{2}(1 - iJ), & \text{on } \Omega^1 M_{\mathbb{C}}, \end{cases}$$

and the Laplacian is given by

$$\Delta_J = d_J^* d + d_J d_J^* = JdJd + dJdJ.$$

So corollary 2.3 implies

(3.3) LEMMA. *The map $J \mapsto \Delta_J$ is a smooth tame map $\mathcal{C}(M) \rightarrow \text{Diff}_2(\wedge M)$ and the maps $J \mapsto \partial_J$ and $J \mapsto \bar{\partial}_J$ are smooth tame maps $\mathcal{C}(M) \rightarrow \text{Diff}_1(\wedge M_{\mathbb{C}})$*

Now we are going to consider harmonic and holomorphic differentials and Weierstrass points, so in the rest of the section we will assume that M is closed, i.e., that $\partial M = \emptyset$. We fix a basis $(\alpha_1(J), \dots, \alpha_{2g}(J))$ for the J -harmonic one-forms by demanding that $\int_{c_i} \alpha_j(J) = \delta_{i,j}$ for $i, j = 1, 2, \dots, 2g$, where $(c_1, c_2, \dots, c_{2g})$ is a fixed canonical homology basis. The J -harmonic one-forms are solutions to $\Delta_J \alpha = 0$, so by [6, II theorem 3.3.3] we have

(3.4) LEMMA. *The map $J \mapsto \alpha_j(J)$ is a smooth tame map $\mathcal{C}(M) \rightarrow C^\infty(\wedge^1 M)$.*

As we get a basis $(\omega_1(J), \omega_2(J), \dots, \omega_g(J))$ for the J -holomorphic differentials by putting $\omega_j(J) = \alpha_j(J) - iJ\alpha_j(J)$ for $j = 1, 2, \dots, g$, see [4, Proposition III.2.7.], we have

(3.5) LEMMA. *The map $J \mapsto \omega_j(J)$ is a smooth tame map $\mathcal{C}(M) \rightarrow C^\infty(\wedge^1 M)$.*

If we consider $\wedge^1 M_{\mathbb{C}} \times \mathcal{C}(M)$ as a vector bundle over $M \times \mathcal{C}(M)$, and we put

$$\wedge^{k,l}(M \times \mathcal{C}(M)) = \{(\alpha, J) \in \wedge^1 M_{\mathbb{C}} \times \mathcal{C}(M) \mid \alpha \in \wedge_J^{k,l} M\},$$

then we have

(3.6) LEMMA. *Let α_j be as in lemma 3.4. Then α_j is a smooth tame section in $\wedge^1 M_{\mathbb{C}} \times \mathcal{C}(M)$. The spaces $\wedge^{1,0}(M \times \mathcal{C}(M))$ and $\wedge^{0,1}(M \times \mathcal{C}(M))$ are vector bundles over $M \times \mathcal{C}(M)$ and if ω_j is as in lemma 3.5, then ω_j is a smooth tame section in $\wedge^{1,0}(M \times \mathcal{C}(M))$ and $\bar{\omega}_j$ is a smooth tame section in $\wedge^{0,1}(M \times \mathcal{C}(M))$.*

PROOF: We only have to observe that the fibers of $\wedge^{1,0}(M \times \mathcal{C}(M))$ is one-dimensional, and if $(p, J) \in M \times \mathcal{C}(M)$, then one of the forms $\omega_j(J)(p)$ are non-zero. \square

Likewise we will consider $\Omega^1 = C^\infty(M, \wedge^1 M_{\mathbb{C}}) \times \mathcal{C}(M)$ as a vector bundle over $\mathcal{C}(M)$, and we put

$$\begin{aligned} \Omega^{k,l} &= \{(\alpha, J) \in \Omega^1 \mid \alpha \in C^\infty(M, \wedge_J^{k,l} M)\}, \\ \text{Harm} &= \{(\alpha, J) \in \Omega^1 \mid \alpha \text{ is } J\text{-harmonic}\}, \\ \text{Hol} &= \{(\alpha, J) \in \Omega^{1,0} \mid \alpha \text{ is } J\text{-holomorphic}\}, \\ \overline{\text{Hol}} &= \{(\alpha, J) \in \Omega^{0,1} \mid \alpha \text{ is anti-}J\text{-holomorphic}\}. \end{aligned}$$

(3.7) LEMMA. The spaces $\Omega^{1,0}$ and $\Omega^{0,1}$ are vector subbundles of Ω^1 , and $\Omega^1 = \Omega^{1,0} \oplus \Omega^{0,1}$. The space Harm is a trivial vector subbundle of Ω^1 , Hol is a trivial vector subbundle of $\Omega^{1,0}$ and $\overline{\text{Hol}}$ is a trivial vector subbundle of $\Omega^{0,1}$.

PROOF: First we show that $\Omega^{k,l}$ is a vector bundle. If $J_1 \in \mathcal{C}(M)$, then the map $C^\infty(M, \wedge_{J_1}^{k,l}) \times \mathcal{C}(M) \rightarrow \Omega^{k,l}$ defined by $(\alpha, J) \mapsto (\pi_{J_1}^{k,l} \circ \alpha, J)$ is bijective if restricted to fibers close to the fiber over J_1 . We claim it defines a local trivialization. Let J_2 be another complex structure. The transition function

$$C^\infty(M, \wedge_{J_1}^{k,l} M) \times \mathcal{C}(M) \longrightarrow C^\infty(M, \wedge_{J_2}^{k,l} M) \times \mathcal{C}(M)$$

is given by

$$(\alpha, J) \longmapsto \left(\left(\pi_J^{k,l} \Big|_{\wedge_{J_2}^{k,l}} \right)^{-1} \circ \pi_J^{k,l} \circ \alpha, J \right).$$

As the map is induced by bundle maps, it is smooth and tame, and we can conclude that $\Omega^{k,l}$ is a vector bundle over $\mathcal{C}(M)$.

Clearly, the inclusion $\Omega^{k,l} \hookrightarrow \Omega^1$ and the projection $\Omega^1 \rightarrow \Omega^{k,l}$ are smooth tame maps, so $\Omega^{k,l}$ is a subbundle of Ω^1 . It is trivial that $\Omega^1 = \Omega^{1,0} \oplus \Omega^{0,1}$, and that Harm, Hol and $\overline{\text{Hol}}$ are trivial vector bundles follows from lemma 3.4 and lemma 3.5. Finally, if $\alpha_1, \dots, \alpha_{2g}$ are as in lemma 3.4 and $\langle \cdot, \cdot \rangle$ is as in (3.2), then we can define a smooth tame projection $P: \Omega^1 \rightarrow \text{Harm}$ by

$$P(\alpha, J) = \left(\sum_{j=1}^{2g} \langle \alpha, \alpha_j(J) \rangle_J \cdot \alpha_j(J), J \right),$$

and we see that Harm is a subbundle of Ω^1 . Similar it is seen that Hol and $\overline{\text{Hol}}$ are subbundles of respectively $\Omega^{1,0}$ and $\Omega^{0,1}$. \square

If $\omega_1, \dots, \omega_n$ are holomorphic sections in $\wedge_J^{1,0} M$, with local expressions $\omega_j = f_j dz$, then the Wronski determinant

$$\det[\omega_1, \dots, \omega_n] = \begin{vmatrix} f_1 & f_2 & \dots & f_n \\ f_1' & f_2' & \dots & f_n' \\ \vdots & \vdots & & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & \dots & f_n^{(n-1)} \end{vmatrix} dz^{\frac{n(n+1)}{2}}$$

is a well-defined holomorphic section in $\left(\wedge_J^{1,0} M \right)^{\frac{n(n+1)}{2}}$. Let $(\omega_1(J), \dots, \omega_g(J))$ be a basis for the J -holomorphic differentials as in lemma 3.5. By corollary 4.5 we can choose a local parameter z_J , which depends smoothly and tamely on J so we see that $\det[\omega_1, \dots, \omega_g]$ is a smooth tame section of $\left(\wedge^{1,0}(M \times \mathcal{C}(M)) \right)^{\frac{g(g+1)}{2}}$. As the J -Weierstrass points are the zeros of $\det[\omega_1(J), \dots, \omega_g(J)]$, we get

PROPOSITION (3.8). *The Weierstrass points depend continuously on the complex structure.*

With the same notation as above, assume that $z_J = 0$ defines the same point $p \in M$ for all $J \in \mathcal{C}(M)$ and that p is a non-Weierstrass point in the complex structure J_0 . For J in a neighbourhood of J_0 , $\det[\omega_1(J), \dots, \omega_g(J)] \neq 0$. So the inverse matrix $[\omega_1(J), \dots, \omega_g(J)](0)^{-1}$ exists, and it depends continuously on J . If

$$(\xi_1(J), \dots, \xi_g(J)) = (\omega_1(J), \dots, \omega_g(J))[\omega_1(J), \dots, \omega_g(J)](0)^{-1},$$

then $(\xi_1(J), \dots, \xi_g(J))$ is a basis for the J -holomorphic differentials adapted to the point p , and we have shown

LEMMA (3.9). *If p is a non J_0 -Weierstrass point, then for J in a neighbourhood of J_0 , we can find a basis, continuously depending on J , for the J -holomorphic differentials adapted to the point p .*

4. Teichmüller theory

The identity component $\text{Diff}_0(M)$ of the diffeomorphism group acts on $\mathcal{C}(M)$ by

$$\mathcal{C}(M) \times \text{Diff}_0(M) \longrightarrow \mathcal{C}(M): (J, \varphi) \longmapsto J \cdot \varphi = \varphi_*^{-1} \circ J \circ \varphi_*,$$

and the Teichmüller space is $\mathcal{T}(M) = \mathcal{C}(M) / \text{Diff}_0(M)$.

We can find a subgroup $\mathcal{D}(M)$ of $\text{Diff}_0(M)$, such that $\mathcal{D}(M)$ acts freely on $\mathcal{C}(M)$ and $\text{Diff}_0(M) = \mathcal{D}(M) \times \text{Aut}(M)$, where $\text{Aut}(M)$ is the holomorphic automorphism group of M (it depends only on the topology of M). If M is the sphere, the torus, the unit disk or an annulus, then $\mathcal{D}(M)$ is the set of $\varphi \in \text{Diff}_0(M)$ which fixes a certain number of points. In all other cases $\mathcal{D}(M) = \text{Diff}_0(M)$, see [1] and [2].

We want to show that the action of $\text{Diff}_0(M)$ on $\mathcal{C}(M)$ admits slices, and we will do it by using the analogous result for the space of metrics.

The space $\mathcal{M}(M)$ of Riemannian metrics on M is $C^\infty(M, E)$, where E is the open subbundle of S^2T^*M consisting of positive definite symmetric two-tensors, and so $\mathcal{M}(M)$ is a smooth tame manifold.

The diffeomorphism group acts on $\mathcal{M}(M)$ by pullback and it is a result of Ebin [3] that there exist slices for this action. In [7] it is shown that Ebin's result holds in the Nash-Moser category.

A complex structure can also be considered as a conformal class of metrics, and if g is a metric, then the corresponding endomorphism $J_g \in \mathcal{C}(M)$ is rotation through $\frac{\pi}{2}$.

(4.1) LEMMA. *The map $\mathcal{M}(M) \rightarrow \mathcal{C}(M): g \mapsto J_g$ is smooth tame and equivariant.*

PROOF: The map is clearly equivariant, so we only have to show that it is smooth and tame. We have that $\mathcal{M}(M) = C^\infty(M, E)$, $\mathcal{C}(M) = C^\infty(M, F)$ and that the map $g \mapsto J_g$ is induced by a smooth bundle map $E \rightarrow F$ and hence is smooth tame. \square

If J is a complex structure, then we let g_J denote the unique constant curvature metric such that $J_{g_J} = J$ and M_{g_J} has volume one.

(4.2) LEMMA. *The section $J \mapsto g_J$ is smooth tame and equivariant.*

PROOF: The map is clearly equivariant, so we only have to show that it is smooth and tame. By lemma 3.1 there exists a smooth tame section, so the result follows from the following lemma. \square

Let for a metric $g \in \mathcal{M}(M)$, \tilde{g} be the unique constant curvature metric such that g and \tilde{g} are conformal equivalent and $M_{\tilde{g}}$ has volume one.

(4.3) LEMMA. *The map $\mathcal{M}(M) \rightarrow \mathcal{M}(M): g \mapsto \tilde{g}$ is smooth tame.*

PROOF: We can write $\tilde{g} = e^{2f}g$ and it is enough to show that the map $g \mapsto f$ is smooth tame.

The curvature K_g of g depends on g and its first two derivatives and the map $g \mapsto K_g$ is smooth and tame. Similarly, if Δ_g is the Laplacian for g and $\text{vol}(M_g)$ is the volume of M in the metric g , then the maps $g \mapsto \Delta_g$ and $g \mapsto \text{vol}(M_g)$ are smooth tame.

If $\tilde{g} = e^{2f}g$, then $K_{\tilde{g}} = e^{-2f}(K_g - \Delta_g f)$. We want $\varepsilon = K_{\tilde{g}}$ to be constant and the volume of M to be one. We can achieve this by first demanding ε to be -1 , 0 or 1 , depending on the topology, and then adjust the volume. So we shall show that the solution to

$$(*) \quad \Delta_g f + \varepsilon e^{2f} = K_g,$$

with $\varepsilon = 0, \pm 1$, is a smooth tame function of g . If $\varepsilon = 0$, then the solution is not unique, but we can use the volume to fix it, and in the following we will argue as though the solution is unique. We define $P: C^\infty(M) \times \mathcal{M}(M) \rightarrow C^\infty(M) \times \mathcal{M}(M)$ by

$$P(f, g) = (\Delta_g f + \varepsilon e^{2f}, g).$$

If we let $V(f, g)k = D_g \Delta_g(f)k$, then

$$DP(f, g)(h, k) = ((\Delta_g + 2\varepsilon e^{2f})h + V(f, g)k, k).$$

As $(\Delta_g + 2\varepsilon e^{2f})$ is a smooth tame family of differential operators, the same is true of the inverses, so by the Nash-Moser inverse function theorem, the map P has a smooth tame inverse and hence the solution of $(*)$ depends smoothly and tamely on g . \square

As a corollary we get

(4.4) THEOREM. In the Nash-Moser category $\mathcal{C}(M) \rightarrow \mathcal{T}(M)$ is a principal $\mathcal{D}(M)$ -bundle.

PROOF: The equivariant section $\sigma: \mathcal{C}(M) \rightarrow \mathcal{M}(M)$ of lemma 4.2 induces a section $\bar{\sigma}: \mathcal{T}(M) \rightarrow \mathcal{M}(M)/\mathcal{D}(M)$, and we have a commutative diagram

$$\begin{array}{ccc} \mathcal{M}(M) & \xrightarrow{\sigma} & \mathcal{C}(M) \\ \downarrow & & \downarrow \\ \mathcal{M}(M)/\mathcal{D}(M) & \xrightarrow{\bar{\sigma}} & \mathcal{T}(M) = \mathcal{C}(M)/\mathcal{D}(M). \end{array}$$

As the projection $\mathcal{M}(M) \rightarrow \mathcal{M}(M)/\mathcal{D}(M)$ admits local sections, we see that the same is true for $\mathcal{C}(M) \rightarrow \mathcal{T}(M)$. \square

Instead of using the result for $\mathcal{M}(M)$ we could show that the action of $\text{Diff}(M)$ on $\mathcal{C}(M)$ satisfies the requirements in the slice theorem of [7]. But the easiest way to do that is to use lemma 4.2 and imbed $\mathcal{C}(M)$ as an $\text{Diff}(M)$ invariant submanifold of $\mathcal{M}(M)$, because then clearly the action on $\mathcal{C}(M)$ satisfies the same requirements as the action on $\mathcal{M}(M)$.

As $\mathcal{T}(M)$ is contractible, the bundle $\mathcal{C}(M) \rightarrow \mathcal{T}(M)$ is trivial, so we have $\mathcal{C}(M) = \mathcal{T}(M) \times \mathcal{D}(M)$. In particular, if M is the sphere or the closed disk, then $\mathcal{T}(M)$ is a point, so $\mathcal{C}(S^2) = \mathcal{D}(S^2)$ and $\mathcal{C}(\bar{D}) = \mathcal{D}(\bar{D})$. If we let M_J denote M equipped with the complex structure J , then we can formulate it as

(4.5) COROLLARY. If M is the sphere or the closed unit disk and J_0 is the standard complex structure, then there exists a smooth tame map $J \mapsto \varphi_J$ from $\mathcal{C}(M)$ to $\text{Diff}(M)$, such that $\varphi_{J_0} = \text{id}$ and $\varphi_J: M_J \rightarrow M_{J_0}$ is holomorphic.

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