

Mathematics and War: an Invitation to Revisit

Physicists, chemists, and biologists have a tradition of discussing meta-aspects of their subject – among which the military use and misuse of the knowledge they produce. Similar concerns are rare among mathematicians.

No rule without exceptions. During the Vietnam war, a number of appeals were circulated among US mathematicians (with reverberations in particular in France and Japan and at the ICM in Moscow in 1966 and Nice in 1970) not to engage in war-related work. One such appeal was published in the *Notices of the AMS* in January 1968. Grothendieck's resigning from mathematics was a consequence of this debate. [Godement 1978], no longer debate but politico-economical analysis, was written from a mathematician's perspective even though it did not deal with mathematical research in particular. [Gross 1978], also written by a mathematician, was shorter but concentrated on mathematics.

In the new context of the euro-missile controversy of the early 1980s, military research came into the focus in debate of universities of Western Germany. [Booß & Høyrup 1984] was an offspring of this new discussion concentrated on mathematics; the broad discussion is reflected in [Tschirner & Göbel (eds) 1990]. The “Forum on Military Funding of Mathematics” published in the *Mathematical Intelligencer* 1987 no. 4 reflects problems arising for the US mathematical community from the “Strategic Defense Initiative” in the same phase.¹ Some more publications followed, mainly with historical emphasis.

As warfare is now again becoming an all-too-obvious aspect of our world and a no less obvious part of “Western” policies, time seemed ripe for taking up the issue anew. August 29–31, 2002, 42 mathematicians, historians of mathematics, military historians and analysts, and philosophers gathered in the historic military port of Karlskrona, to discuss four questions:²

- To what extent has the military played an active part throughout history, and in particular since World War II, in shaping modern mathematics and the careers of mathematicians?

¹ Cf. also [Davis 1989].

² We use the opportunity to thank Maurice and Charlyne de Gosson and the Blekinge Institute of Technology and its Mathematics Department for organizing splendidly this conference, supported by Stig Andur Pedersen of The Danish Network for History and Philosophy of Mathematics (MATHNET) and Reiner Braun of The International Network of Engineers and Scientists for Global Responsibility (INES). From the conference, a kind of enlarged proceedings will appear as Bernhelm Booß-Bavnbek & Jens Høyrup (eds), *Mathematics and War*. Basel & Boston: Birkhäuser, 2003. Much of what is said in the following draws on this volume. To the theme in general, see also [Booß & Høyrup 1984], [Epple & Remmert 2000], [Godement 1994 and 2001], [Meigs 2002], and [*The AMRC Papers* 1973].

- Are mathematical thinking, mathematical methods, and mathematically supported technology³ about to change the character and performance of modern warfare, and if so, how does this influence the public and the military?
- Which were, in times of war, the ethical choices of outstanding individuals like the physicist Niels Bohr and the mathematician Alan Turing? To what extent can general ethical discussions provide guidance for working mathematicians?
- What was the role of mathematical thinking in shaping the modern international law of war and peace? Can mathematical arguments support actual conflict solution?

Perspectives from mathematics

All mathematicians know the tales, reliable or not, about Archimedes and his defence of Syracuse. They may also have heard about early Modern ballistics and fortification mathematics and the importance of trigonometry for navigation. All these cases of mathematics being implied in conquest, warfare or preparation of war have one thing in common: that which was combined with technical and military knack was almost exclusively already existing mathematics. In this respect such examples do not differ from the use of simple accounting mathematics in logistics – which after all is likely to have been much more important from the military point of view. Mathematics served as a toolbox, and military officers may have been the largest group that received some general mathematical training; but the involvement of mathematics as a general endeavour with the military institution was not very intimate, and specifically military applications had no independent role as a shaping force for mathematics. Tartaglia's composition of straight lines and circles in ballistics was clearly inspired from gunnery and the war against the Turks. When Galileo introduced the parabolic law this origin was already left behind, and the theory was linked instead to the philosophical discussion of local movement – and was largely irrelevant for the firing of guns because of the influence of air resistance, as pointed out explicitly by Galileo.

Even to this rule there is an exception. That part of the Sumero-Babylonian legacy which is most spoken of in general histories of mathematics – namely the invention and implementation of the place value system – may be a child of war. In c. 2074 BCE, king Shulgi organized a military reform in the Sumerian Empire, and the next year an administrative reform which, under the pretext of a state of emergency that became permanent, seems to have enrolled the larger part of the working population in quasi-servile labour crews and made overseer scribes accountable for the performance of their crews calculated in abstract units worth $\frac{1}{60}$ of a working day (12 minutes) and according to fixed norms. In the ensuing bookkeeping, all work therefore had to be calculated precisely and converted, which asked for multiplications and

³ This “broad concept” of mathematics is the one that serves in the following; it also embraces computers and computer science.

division in huge numbers. Therefore, a place value system with base 60 was introduced for intermediate calculations.⁴ Its functioning presupposed the use of tables of multiplication, reciprocals and technical constants and the training of these in school; the implementation of a system whose basic idea had been “in the air” for some centuries therefore asked for decisions made at the level of the state and set through with great force. Then as in many later situations, only war provided the opportunity for such social power of will.

Apart from that the conclusion stands that “the involvement of mathematics as a general endeavour with the military institution was not very intimate, and specifically military applications had no independent role as a shaping force for mathematics” until a century ago. Since around 1500 BC, as already mentioned, the employment of fortification mathematicians and the teaching of naval and artillery officers certainly played a social role for mathematics by providing job opportunities and a market for mathematics text books (copiously decorated with military symbols).

This we may regard as the past. The contemporary situation can be said to start around the First World War, and to reach full development during the Second World War.

During World War I, two important new military technologies depended on *mathematics in the making*: sonar, and aerodynamics. They were so impressive that Picard, in spite of his own patriotism (which non-French cannot help seeing as pure chauvinism), regretted the perspective that young mathematicians might opt in future for applied mathematics only [*Proc. ... 1920*: xxviii]. In general, however, the immediate role of the pure sciences, mathematical and otherwise, was that of providing manpower that could be converted into first-class creative engineers – not restricted to applying a set of standard rules but able to implement theoretical knowledge and make it function in practice; this was also the role of most of the mathematicians that were actually involved in the war effort (if they were not, as was the use in France, sent into the trenches). Nobody will claim that mathematics was in any way decisive for the outcome of the war, nor that WW-I applications of mathematics left important traces in the post-war world (civil aviation still belonged to the future).

Picard's worries proved unfounded. Main-stream mathematics soon reverted to the pre-War model, even more swiftly than the precariously erected organization of planned science was dismantled. Aerodynamics of course survived, but only as a current among others.

All of this was different in World War II, either quantitatively or qualitatively: the organization of science intended to support the war effort was a major concern of both Axis and Allied powers; mathematical technologies (radar, sonar, the decipher computer, *the* bomb) can

⁴ Since it was a floating-point system with no indication of absolute place, it could only be used for intermediate calculations – just as the slide rule of engineers in quite recent times. Since intermediate calculations have not survived, the exact dating of the implementation can only be determined from indirect arguments. See, e.g., [Høyrup 2002: 314].

be argued to have been war-decisive; computers, nuclear energy, jet propulsion – all mathematically constructed and computed for the war – have changed our world beyond recognition after 1945. All of these build on pre-War theoretical insights;⁵ some of them (computers, jet motors) were not only “in the air” before the war but functioned as prototypes; but in all cases the war, by making available huge means without counting costs and benefits, made it possible to boost a development which otherwise might have taken decades⁶ – and perhaps, in cases like the proliferation of DDT and atomic reactors, might have been stopped at an early moment when the problems they create became visible.

During the War, mathematicians in large numbers were recruited, many of them to teach sailors and air-crew members basic trigonometry (etc.), but many also to serve as best-level creative engineers. Afterwards, the latter have often tended to regard what they did dismissively (“I did not write one line that was publishable”), perhaps because puzzle-solving with no further theoretical impact did not look important in the mathematician's hindsight; this assessment notwithstanding, what was done depended critically on mathematical ingenuity and training. A striking example is O. R. Frisch and R. Peierls' mathematical formulation of the essential questions surrounding the construction of a uranium bomb in March 1940 and their “back-of-an-envelope” discovery that its critical mass was so small that military use was feasible.⁷

In some cases, of course, the solving of problems defined by the war *did* have important theoretical impact – we all know about the emergence of computer science, information theory, Monte Carlo simulation, operations research, and statistical quality control.

This time, nothing was dismantled after the War (many mathematicians, of course, hurried away from military research) – the Cold War was already on. In the slightly longer run (a decade or so), civil re-application of the new mathematical war techniques caused profound transformation of these and violent acceleration of their development: only the war effort had allowed the creation of the first costly computers, but only commercial use allowed mass

⁵ At times fully detached from every technical application; N. Wiener and E. Hopf had calculated the radiation equilibrium at stellar surfaces, but their theory could be applied to the expanding surface of the exploding bomb [Wiener 1964: 142f]; A. A. Markov had investigated his eponymous processes as pure mathematics and illustrated the applicability of the concept on linguistic material [A. A. Youschkevitch 1974: 129]; in the Manhattan Project they turned out to be relevant for solving diffusion equations and for describing nuclear branching processes.

⁶ The parallel to the invention of the place value system in Sumer is striking. In that case, parallel processes not furthered by a military government indeed asked for much longer time: in China the unfolding took more than a millennium, in India it never really took place before the “Indian” system was brought back from abroad.

⁷ See [Gowing 1964: 40–43, 389–393] and [Dalitz & Peierls 1997: 277–282]. This latter volume presents Peierls as a physicist, but his actual chair was in “applied mathematics”; even a “broad concept” of mathematics does not free us from delimitation problems.

production, open competition, intensive development efforts and reduction of costs. We may add that only the freeing from the pressure of immediate applicability (“better a fairly satisfactory answer now than the really good answer two years after defeat”) gave space for fruitful interaction between theoretical understanding and applications in for instance computer science.

When discussing mathematical research for military purposes, both during World War II and in recent decades, we should differentiate several situations and problems.

- Firstly, we must distinguish the application (sometimes creative, sometimes repetitive) of existing tools within the military institution itself (ballistic computation, modelling, ...) from creative mathematical research outside this institution but directed toward military goals.
- Secondly, we should remember that mathematical research consists in more than the production of theorems of presumed military use. Several institutions (Süss's original planning of the German Oberwolfach Institute in 1944, the American Mathematics Research Center in Wisconsin) exemplify an efficient model, *a two-way chain*, which *grosso modo* works as follows:⁸ A core group of highly skilled mathematicians familiar with the direct problems of the military employer (efficiency of bombing, controlled spread of bacteriological agents, better radar detection and avoidance of enemy detection, or whatever it may be) find out which of these can be approached mathematically, undertake an initial translation, and direct the translated problems to other experienced mathematicians who are well-informed about and centrally located within the whole mathematical milieu; these parcel out the questions into problems which colleagues may take up as mathematically interesting, perhaps even without knowing that they enter into a network of military relevance; once such questions have been answered, the same chain functions backwards, reassembling the answers and channelling the global solution to the employer (only the availability of large amounts of money distinguishes this from how mathematics of civilian relevance can be created).

This is only one among several models. We know that it was planned to function in World War II Germany but was implemented too late to become efficient; we know that it has functioned in the US. We know less about the organization of military mathematical research in the late Soviet Union, but it appears that here, as in production and research in general, the civil and military domains were more efficiently separated than in the West.

Rounding off what can be said in “the perspective from mathematics” we may make some

⁸ Concerning the Oberwolfach Institute, this structure follows from analysis of the material presented by Gericke [1984], cf. [Høyrup 1985]; on the same institution, see also [Remmert 1999]. For the Wisconsin Institute, see [*The AMRC Papers*, 1973].

general observations.

- Mathematical war research has resulted in certain fundamental theoretical innovations. It is striking, however, that all of these appear to depend on an exceptional mathematician. The names of Turing, von Neumann, Shannon, Wald, and Pontryagin must suffice to make the point.
- However, the utility of mathematics *for the treatment of military problems* does not depend critically on the presence of an exceptional mathematician. Mathematicians in large numbers have proved themselves unexpectedly able to function as creative mathematical engineers, in the sense explained above.
- This ability has largely depended on their ability to become familiar with methods and approaches of various mathematical disciplines and to synthesize these. The still persistent unity of mathematics is thus demonstrated *ad oculos*, if not in the mathematical journals then in the uses.
- It should not be forgotten that the traditional application of the toolbox of already existing mathematics goes on, now at the level created by recent mathematical research.

Military perspectives

At the conference, the point was strongly made by Colonel Svend Bergstein that actual war cannot be calculated, no more today than in Clausewitz's times; not only too many unpredictable external factors are involved, also the aspects of human behaviour that are most atavistic and contrary to reason.

Nevertheless, and as a matter of fact, mathematics – that is, mathematical thinking, mathematical methods, and mathematics-based technology – has become an integral and even essential part of modern warfare. (This does not mean that mathematics has become the major expense of the military apparatus – mathematics and what goes with it is a cheap way to use expensive resources more efficiently.)

Once more, we may list various aspects of this role and utility of mathematics as discussed at the conference and in other contexts.⁹

- Mathematics serves in managing the institution. Purchases of weapons systems are planned, war-games and logistics are calculated.
- Weapons and weapons systems are optimized and their efficiency during action enhanced. This regards munitions (including missiles and bombs provided with guidance systems); delivery systems (including for instance aeroplanes provided with electronic countermeasure circuitry); the reconnaissance, control and communication interface (“to

⁹ Evidently it is difficult to find *any* technology which has been created during the last decades which is not somehow driven by mathematics. The list discusses such facets of the matter as go beyond what holds for any practice that involves computers or microelectronics.

ensure that the right forces are at the right spot at the right moment, and with the right information about the enemy” – Svend Bergstein); and, across all of these, high-speed cryptography. The improvement of data transmission technologies is of general importance for many of these questions, but the creation of data is not only a presupposition for their transmission but also field which nowadays makes use of even more sophisticated mathematics.¹⁰

- Similarly, the strategic planning of the possible use of the weapons systems depends on mathematical calculation; even the dismantlement of weapons systems without the risk of destabilizing disequilibrium in the SALT negotiations was analyzed mathematically.
- Perhaps unexpected by civilians but emphasized by military analysts, simple accounting mathematics *performed by mathematically trained independent personnel and not by the active warriors* is mandatory if strategic gains and losses are to be assessed realistically – leading officers, like all of us, are easy victims of self-deceiving optimism and pessimism according to circumstances.
- At the opposite end of the scale, mathematics may also be an indispensable tool. Thus, when the effect of fragmentation bombs on human bodies was to be predicted but humanitarian concerns prohibited testing on pigs, mathematical simulation was put into play.
- Ideologically, the waging of war is made more acceptable to the public by the presentation of warfare as a question of mathematical calculation, and thus of war as “more rational and clean”. Whereas Hitler preached German invincibility by presenting the Wehrmacht as “Fast as German greyhounds, tough as German *Lederhosen*, hard as Krupp steel”, mathematics presents modern warfare as “fast by avionics, precise by GPS, safe by mathematically optimized operations planning”.
- Similarly, certain uses of mathematical representations of the task to be performed may serve to make the agent see it as a normal manipulation of symbols and thus to eliminate the need for appeals to atavistic instincts – say, seeing a village to be bombed as triangles similar to those of a computer game may facilitate the killing. (Evidently, being at a height of 5 kilometres already has much the same effect).

Utility is one thing, possible backfiring that must be taken into account is another. Firstly, seeing war as “more rational and clean” may affect (and often appears to affect) not only the public but also the political planners. This is pernicious, not only for the victims but already for

¹⁰ Interestingly, the analysis of the damages of the intestine of a wounded soldier by magnetic resonance imaging (MRI) and the localization of enemy ground forces by synthetic aperture radar (SAR) build on the same mathematics – both, indeed, squeeze out of a “short antenna” as much information by systematic repeated use as could be gained from an extended antenna without advanced mathematics [Schempp 1998: 44 and *passim*].

the planners themselves who may engage recklessly their armed forces in operations and wars that are less easily won than predicted by the machine-rational perception of the character of war.

Less dangerous for planners but just as much for victims is the relative inexpensiveness of present-day mathematically supported asymmetric warfare for the attackers – if the subjugation of Serbia in the Kosovo war cost only 7 billion US \$, that is, 700 \$ per Yugoslav capita, the temptation is great to solve all similar problems in a similar way. (In the moment such a war turns out as things develop to involve the use of ground forces, costs of course explode, and we are brought back to the situation discussed in the previous paragraph).

Another feature of the mathematization of warfare, also contributing to the ongoing militarization of our world but not restricted to the field of easy asymmetric wars and punitive operations, is the transformation of the “Krupp model” into an “infinite Krupp model”. War and prepared war is always between two (possibly more) parts – Clausewitz would speak of a *Zweikampf*, a duel, which has now become a “duel of systems”. In the nineteenth century, Friedrich Krupp would first develop nickel-steel armour that could resist existing shells, then chrome-steel shells that could pierce this armour, then high-carbon armour plate that resisted these, then cap-shot shells that could break this plate – and that was the end of it. In the duel between soil-air missiles and aeroplanes, no physical limit prevents an ongoing sophistication and ensuing arms race. Cap-shots shells were and remained extremely expensive; so are stealth bombers and fighters, but such measures as depend solely on sophistication of soft- and hardware have neither budgetary nor intellectual definitive bounds. Processes depending on physics and chemistry may have definitive natural boundaries. Those depending solely on mathematics seem to have none. The ensuing virtual absence of limits enhances the stress on both sides, and thus the speed and instability of such a race.

Ethics

Mathematics, according to a familiar view, is a neutral tool. As once formulated by Jerry Neyman, “I prove theorems, they are published, and after that I don't know what happens to them”.

This is certainly an important feature of the mathematical endeavour, and does not only hold for theorems and theorem production. Also the teaching of mathematics, the production of high-level general mathematical competence in the population, is a precondition not only for the waging of modern war but also for the functioning of our whole technological society (quite apart from its cultural value).

But the title “mathematics and war” implies ethical dilemmas. In order to avoid having the ethical discussion end up in “I feel...”/“but I feel”, we may start by looking at the actual ethical choices of some well-known figures.

- Laurent Schwartz used his high academic prestige to make his resistance to the French and American wars in Algeria and Vietnam more efficacious; he saw no connection between his work in mathematics and his political commitment (and as far as his own theoretical

production is concerned it may be difficult to find any immediate and direct link).

- Niels Bohr, when becoming aware of the German nuclear bomb project, supported the competing Anglo-American project; when discovering the dangers that were to arise from the success of the latter, he issued warnings to responsible politicians (Churchill, Roosevelt) and to the public (the “Open letter”) – using his prestige as an originator of the underlying theory and as a collaborator (and arguably overrating the impact his interventions might have).
- Alan Turing, quite sceptical of British society (for political as well as personal reasons), put his outstanding abilities in the service of war with total loyalty when he felt it was needed; unlike Bohr, he did so without ever putting himself into focus.
- Kinnosuke Ogura had been a strong promoter of (Marxist-inspired) democratic modernization of Japan, and had opposed Japanese policies as being parallel to German and Italian Fascism. After the beginning of the aggression against China in 1937, however, patriotism and the project to use war as a way to modernization urged him to play a central role in the organization of Japanese mathematics in the service of the military state. After the war he regretted, without specifying too directly what he had done.
- John von Neumann, like Turing, applied his outstanding abilities in war research. Neumann did so both during World War II and in the early Cold War; whereas Turing had been a loyal participant about whose personal attitudes in the matter we know nothing, Neumann made the creation of the H-bomb a personal project which (well served by Stanislaw Ulam) he did all he could to promote – his aim being to make possible a preemptive first strike.
- Lev S. Pontryagin gave up an extremely fruitful research line in algebraic topology and created control theory. In hindsight this appears to have been caused by a will to serve his socialist country by solving the problems of guiding intercontinental ballistic missiles – thus making *impossible* the first strike.
- Decades before, G. S. Hardy had tried to avoid that usefulness of his science which consists in “accentuat[ing] the existing inequalities in the distribution of wealth, or more directly promot[ing] the destruction of human life” by concentrating on supposedly useless number theory. Ironically, he repeated this phrase in 1940, when number theory was soon to become a cryptographic resource.
- The radical pacifist Lewis Fry Richardson published his path-breaking *Weather Prediction by Numerical Process* in 1922 after having made sure that 64000 “computers” (human beings furnished with desk calculators) would need more than one day to predict the weather one day ahead. This he saw as a guarantee that numerical weather prediction could not be put to military use.

To what extent can these serve as exemplars and role models? Firstly, they show that two fundamentally different situations must be distinguished. One is that of Laurent Schwartz and Hardy: deep scepticism towards their own society, or aspects of that society as a warring power.

The other is that of the remaining examples: they accepted their own society and its warfare or armament policies, either in general or under actual circumstances – certainly with different degrees of identification.

In the second situation, the ethical dilemmas are few. Obviously, one will see no objections to doing his best. Certainly, dilemmas are not totally absent: one may still, like von Neumann, give an extra push; one may, like Turing, be fully loyal but leave the political decisions to those who are officially entitled to take them (whether politicians, citizens in general, or military men); or one may, like Bohr, use one's particular standing and insight and moderate, warn, or point to alternative options.

The situation of the sceptic is less clear-cut. Very few of us are in a situation (the situation, say, of von Neumann and Pontryagin) where nobody else could do what we are doing; these few may influence matters directly by deciding to cooperate or not to cooperate.

Most mathematicians, if they chose not to cooperate in mathematics research and teaching, will have little effect, and little of what most mathematicians do in research as well as teaching is directed toward a specific application. Deciding to abstain from working with a particular discipline because it seems “corrupt” is mostly futile. Giving up mathematics is not only giving up military applications but anything mathematics can be used for – and whatever cultural value we may ascribe to mathematics.

However, the practice of the mathematician consists in more than the abstract production and dissemination of theorems. Any mathematician is in a *particular* situation, and in any particular situation there are specific conditions and a specific room for decisions. One may, for instance, widen one's own insight and global understanding of the role of mathematics, and try to share it with students, colleagues and the public – or one may chose to remain and leave as blind as comfortable. One may be a teacher in one or the other position, teaching within a highly stratified or a more egalitarian education system; one may organize the research of an institution, one may be a prestigious researcher, or one may be the newly appointed young colleague. One may be in the top of the “AMRC chain”, one may be in its periphery knowing or not knowing to belong there, or perhaps be wholly outside it. In each situation, the scope of ethical choices is different, and no general ethical rules or advice can be issued (a somewhat less abstract discussion of the matter can be found, however, in [Booß & Høyrup 1984]). What can be said in general is that the neutrality of mathematics *per se* does not entail the neutrality of these ethical choices.

An enlightenment perspective???

The Enlightenment believed that reason *might* serve general progress; Rousseau and Swift pointed out that too often reason is used in the service of purely technical rationality and for purposes of sub-optimization, with morally and physically disfiguring effects. According to Defoe's Robinson Crusoe, “Reason is the Substance and Original of the Mathematicks”. Where does that leave mathematics with regard to disfigurement and progress today?

Much of what was said above concerning the utility of mathematics for the military points rather to disfigurement. Most alarming of all are probably not the actual uses but the ideological veil of rationality, cleanness and surgical accuracy which is derived from the mathematization of warfare. By generalization one might claim that this applies not only to the military aspects of our modern technical society but to the technically rational society as a whole.

However, one of the ways in which mathematics serves the military points in the opposite direction: that sober-minded elimination of self-deceiving optimism and pessimism which can be provided by mathematical reasoning and calculation. Mathematics-based reason at its best should allow us also in larger scale to unlearn conventional wisdom, to undermine facile indoctrination, to distinguish the possible from glib promises. It *might* help us, if not to find any absolutely best way – this is too much to expect from rational analysis – then at least to evade the worst. If reason is “the Substance and Original of the Mathematics”, mathematics *might* serve to make clear to us that war is fundamentally irrational and unreasonable not only in commonplace ideological generality but in specific detail. Admittedly, technical rationality prevails over reason for the moment, both concerning the general political situation and the uses to which mathematics is put.

Just as mathematical theories, mathematics *as a general undertaking* is ethically neutral – or, better, ethically ambiguous: responsibility remains with its practitioners, disseminators, and users.

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Suggested illustrations:

1. Oberwolfach today

Caption: When listening to a music CD we enjoy and do not think of the military origin of the coding involved. Similarly, mathematicians going to the wonderful Mathematical Research Institute *Oberwolfach* enjoy the ambience and do not think about the fact that the institute originated as a military research institution in 1944 - apparently well planned for the purpose though too late to become efficient.

2. Rejewski ceremony

Caption: Leading Polish army officers were present at a ceremony in 2001 when a memorial plaque was unveiled at the tomb of the cryptologist Marian Rejewski (1905-1980), here some generals with Rejewski's daughter and the president of the Polish Mathematical Society. Not many mathematicians have experienced similar honours in life or posthumously. Already as a mathematics student Rejewski had been recruited by the Cipher Bureau of the General Staff of the Polish Army in 1929. Rejewski then created a mathematical method for breaking the German *Enigma* code of that time. It does great credit to the Polish Cipher Bureau officers that they realized so soon the potential of mathematics in cryptological research.

3. The first page of Ogura's 1944 book on *Mathematics in Wartime*

Caption: Kinnosuke Ogura (1885-1962) was an excellent Japanese mathematician, struggling for the modernization and democratization of his home country. However, during the *Greater East Asia War* he was trapped in the mobilization of the Tennoist aggression against China. The picture shows the first page of Ogura's 1944 book on *Mathematics in Wartime*. He is writing a bellicose appeal to mobilize mathematics for the Tennoist victory. After 1945 lending of extant library copies was banned and the book was silently excluded from Ogura's *Collected Works* in 8 volumes.

4. busgroup.jpg

Caption: There is no known picture of Turing during the wartime period, but this photograph shows Alan Turing (at left) with his athletic club in 1946. At this point he was engaged in designing a first digital computer at the National Physical Laboratory, London. This used his wartime knowledge of electronic technology to put his 1936 theory of the universal machine into a practical form. The codebreaking machinery at Bletchley Park, although very advanced, had never actually used Turing's fundamental idea of the universal machine and the stored program, but as soon as the war ended Turing set to work to bring it to reality.

5. Kolmogorov's *Theory of Firing*, four front pages 1942, 1945, 1948, 2002

Caption: The 1945 work by Kolmogorov and collaborators on *Firing Theory* was interesting enough to be translated by the RAND Corporation in 1948 and is still regarded as fundamental in quite recent US military education. It is one of the apparent paradoxes of the relation between mathematics and warfare that the preceding paper by Kolmogorov was published in 1942 for

everybody to read (including the enemy). Could it be that the 1942 paper was too mathematical and too general to inform military practice directly? And on the other hand: was the 1945 work after all too closely linked to a specific problem to inspire further mathematical development?

6. Vostok launch rocket of the first manned spacecraft (launched on April 12, 1961)

Caption: According to information received from Samara Aerospace University, the launchings of Sputnik 1 and Sputnik 2 in 1957 were made without recourse to the Pontryagin Maximum Principle. Only bringing down cosmonauts safely and guaranteeing that intercontinental missiles surviving a first strike would hit New York presupposed the Maximum Principle, which was in the public domain well before that. For Sputnik 1 and 2, however, Pontryagin assisted to find the correct weight of the space craft (same source).

7. Circular error probable, table

Caption: The average precision of bombing and firing is commonly characterized by the *Circular Error Probable (CEP)*, that is, the radius of a disc around the goal point within which (on average) 50% of the shots hit while 50% fall outside. (Kolmogorov's approach was more sophisticated). The table shows how CEP has decreased dramatically in aerial bombing over the last 60 years and how the efficiency of a bomber increased correspondingly. The table gives the calculated number of bombs required for destroying (hitting once) a 20m x 30m object.

8. Target selection and order of battle, scheme of modern air raid build-up

Caption: In World War II, the destruction of a major composite target might ask for the deployment of 1000 bombers. Nowadays a similar task may be effectuated by, say, 29 heavy bombers (lower level of the above schematized front view of the attack - heights are indicated in kilometres). But these have to be supported by another set of 275 ground attack fighter and fighters ("SEAD package") to suppress enemy air defence (bottom). Higher up, 24 "Intelligence-Surveillance-Reconnaissance" (ISR) aircrafts guide the action of the lower levels – and on top, dozens of ISR spacecrafts participate. Not depicted in the scheme are the about 200 tankers for air-to-air refuelling; the about 60 transporters for flying in supply, ordnance, maintenance provisions; the hundreds of sea-launched Tomahawk Land Attack Cruise Missiles; and the swarms of unmanned aerial vehicles. Much more than informatics is thus involved in the support of the mission itself, and the dramatic decrease of CEP has to be paid for by dramatic increase of support crafts. However, air raids are still the cheapest way to wage a punitive war (forbidden by international law, but practised), inflicting huge economic losses on the enemy at extremely low operational costs.

9. Demolished Varadin Danube River Bridge in Novi Sad, Yugoslavia

Caption: The destruction of the bridges across the important international waterway Danube, some of them in the North of Yugoslavia and hence far away from Kosovo where the Yugoslav

military operational capability should be hindered, was unlawful by The Geneva Protocol I. The counter-argument given is that this kind of warfare is, after all, cost-efficient in human lives, even for the target population – as illustrated by the undamaged blocks of flats standing near to the crushed bridge. The as yet mysterious health problems of Nato soldiers who participated in the Gulf and Kosovo wars tell us that other damages that do not show up on photographs may turn up in medical statistics. An even greater cost of this high-precision warfare supported by mathematics is its very introduction of the concept of justified risk-less punitive wars without bloodshed. This creates invincibility illusions, lowers the barrier against war and talks people into accepting war.

10. Frontispiece Grotius' book, 1646

Title-page of the second edition of Grotius' *De Jure Belli Ac Pacis*. As indicated by the armillary sphere, the publisher was also engaged in mathematical publishing. Historians of international law credit Hugo Grotius with the creation of modern international law, as in particular established in the Peace Treaty of Westphalia of 1648 and the Charter of the United Nations of 1945, and trace the origins of it back to patterns of mathematical thinking of striking public appeal in Grotius' time. Military analysts of our time blame the striking public appeal of mathematics supported modern warfare for undermining international law.

11. The mosaic from <http://www.barca.fsnet.co.uk/archimedes.htm> (Städelsches Kunstinstitut, Frankfurt/M., Germany)

Caption: "Noli turbare circulos meos": When Archimedes's city was conquered in spite of his astounding mathematical engineering, he pretended (according to the famous anecdote) that he had only made pure mathematics.

Box to accompany Figure 8 (Scheme of modern air raid build-up)

Precision Bombing Acronyms

AWACS	Airborne warning and control system for air and combat control
B-◇	Long-range bomber with weapon payload of more than 10 tons
COM	Military communication / signals intelligence spacecraft
E-8	Joint surveillance and targeting attack radar system JSTARS
EA-6B	"Prowler" carrier-borne radar jammer
EC-130	"Compass Call" communication jammer
F-◇	Fighter and fighter ground attack aircraft
GPS	Global positioning system navigation satellite
IR-NRO	Infra-red (US) National Reconnaissance Office spacecraft
ISR	Intelligence, surveillance, reconnaissance package
LM-NRO	Imaging radar (US) National Reconnaissance Office spacecraft
MET	Weather satellite

RC-135	"Rivet Joint" signals intelligence gathering aircraft
SAM-◇	Surface-air missile
SEAD	Suppressing enemy air defence package
U-2	Optical spy plane

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