

[Feferman, Solomon](#)

Lieber Herr Bernays! Lieber Herr Gödel! Gödel on finitism, constructivity, and Hilbert's program. (*English*) [Zbl 06220601](#)

Baaz, Matthias (ed.) et al., Kurt Gödel and the foundations of mathematics. Horizons of truth. Cambridge: Cambridge University Press (ISBN 978-0-521-76144-4/hbk). 111-133 (2011).

This is a substantial contribution to mathematical logic and to a special trait of its history to be explained below. It is written by one of the most prominent experts and certainly the most outspoken debater in the field, the doyen of the philosophers of mathematics, Prof. em. Solomon Feferman, Fellow of the American Mathematical Society among a multitude of other honours, and chief-editor of [*K. Gödel*, 'Collected Works. I-V'. New York: Oxford Univ. Press, hardbacks and pbk/reprints (1995-2013); (1986; [Zbl 0592.01035](#)); (2001; [Zbl 0973.01104](#)); (1990; [Zbl 0698.01023](#)); (2001; [Zbl 1074.01014](#)); (2001; [Zbl 1074.01015](#)); (1995; [Zbl 0826.01038](#)); (2003; [Zbl 1026.01019](#)); (2013; [Zbl 1276.01015](#)); (2003; [Zbl 1026.01020](#)); (2013; [Zbl 1276.01014](#))].

Usually, the topic of mathematical logic invokes three different feelings among mathematicians: We may become (i) curious whether such investigations can tell us something new about our way of working and the reliability of our results; we may (ii) recognize the high level of abstraction and ingenious constructions in this (very marginal) subfield of mathematics no matter how irrelevant and remote it may be conceived from our own work; or we may, as most mathematicians do, (iii) discard any occupation with it as a waste of time and something which has been on the agenda of mathematicians several generations ago but lost its relevance totally. The treatise under review invites to positions (i) and (ii), even Prof. Feferman would be the first to explain his understanding for the common choice of position (iii).

Adapting the first view, we are in good company of giants of mathematics like Leopold Kronecker, Hermann Weyl and Aleksandr Danilovich Aleksandrov: Most famous is Kronecker's dismissal of the irrational numbers, 'wenn er z. B. eine Definition nur dann für zulässig erklärt, wenn sie in jedem Falle durch eine endliche Anzahl von Schlüssen erprobt werden kann' according to the obituary [*H. Weber*, *Math. Ann.* 43, 1–25 (1893; [JFM 25.0033.04](#))]. At once, Kronecker recognized the beauty of Hermite-Lindemann's proof of the transcendency of π and the usefulness of non-finite concepts in large fields of algebra, analysis, mechanics and astronomy, concepts which he considered indispensable for his time and also for ontological reasons. However, he kept his belief in [*L. Kronecker*, *J. Reine Angew. Math.* 101, 337–355 (1887; [JFM 19.0063.03](#)), p. 338], 'dass es dereinst gelingen wird, den gesammten Inhalt aller dieser mathematischen Disciplinen zu "arithmetisiren", d. h. einzig und allein auf den im engsten Sinne genommenen Zahlbegriff zu gründen'. His prophecy became true for many highly esteemed mathematical problems like the Riemann Hypothesis (RH). While RH in its common form would fall under Kronecker's verdict, there are several purely arithmetic assertions (in the sense of Kronecker) that are demonstrably equivalent to the Riemann hypothesis, see [*Y. Manin*, *A course in mathematical logic for mathematicians*. 2nd ed., Graduate Texts in Mathematics 53. Berlin: Springer (2010; [Zbl 1180.03002](#)), pp. 14 and 355]. Moreover, as predicted by Kronecker, for most purposes of numerical analysis we need no longer consider $2, e$ or π as numbers but it suffices to consider them as algorithms that can be nicely described in finite algebraic terms. Yet, for many numerical algorithms justifying the commonly used termination criteria remains an unsolved problem as emphasized in [*P. J. Davis*, "The relevance of the irrelevant beginning", *ScienceOpen Research* (2014; doi: 10.14293/A2199-1006.01.SOR-MATH.6G464.v1)].

Weyl's dismay with the continuum (or, equivalently, his doubt of the meaning of the *power set of the set of integers*) and Alexandrov's concerns regarding the concept of compactness (his doubt of the meaning of the set of all subsequences of a sequence) were nourished by the same reservations as Kronecker's but without having found a pragmatic solution yet, see e.g. *P. Cohen's* proof in [Proc. Natl. Acad. Sci. USA 50, 1143–1148 (1964; [Zbl 0192.04401](#))] that the Continuum Hypothesis neither can be proved nor disproved by arguments obeying Zermelo-Fraenkel's axioms (ZF). If we agree that $2^{\aleph_0} = \aleph_1$? is a reasonable question and if we grant that the axioms of set theory and the logical means of expression and deduction in ZF actually exhaust the apparatus for constructing proofs in modern mathematics, then in the words of [[Zbl 1180.03002](#), p. 107] “we can say that the continuum problem is the first known example of an absolutely undecidable problem. Although Gödel's incompleteness theorem provides concrete examples of undecidable propositions in any formal system having reasonable properties, these examples can be decided in an ‘obvious’ way in some higher system. The situation with the continuum problem seems much more difficult.”

That should provide sufficient reason for studying the present treatise carefully. The reader shouldn't feel annoyed by the many repetitions from the author's introduction to the chapter on the correspondence with Bernays in [[Zbl 1026.01019](#); [Zbl 1276.01015](#), pp. 40–79]. There, Feferman was disciplined enough to follow his own editorial principles, namely ‘to provide historical context to the correspondence, explain the contents to a greater or lesser extent, and, where relevant, discuss later developments or provide a critical analysis’ [[Zbl 1026.01019](#); [Zbl 1276.01015](#), p. vi]. Clearly, as editor, he had to restrain his own views and his own feelings invoked by his search for Gödel's motivations behind his development of formal apparatus. In the present treatise, he enjoyed a much greater freedom and he used it marvelously. Out of that substantial correspondence between Bernays and Gödel, ranging from 1930 to 1975, and through his own meticulous analysis of finest nuances in the formal concepts and interpretations these two men exchanged with each other over time, Feferman distilled a moving picture of a man, Kurt Gödel, who – clearly among other aspects of his life and work – fought all his life for Hilbert's recognition, even after Hilbert's death, about his work, both to the extent it supported Hilbert's ideas and it confined them.

Then, what can we learn from these letters and their interpretation by the author of the present treatise? Paraphrasing a paradox stated in [*M. Otte*, *Analytische Philosophie – Anspruch und Wirklichkeit eines Programms*. Hamburg: Felix Meiner Verlag (2014), p. 7] we may conclude: ‘1. Die Wege mathematischen Denkens sind viel zu komplex, als dass wir auf Versuche ihrer Formalisierung und Mathematisierung verzichten könnten! Formalisierung und Mathematisierung sind unsere Fenster zur Welt und zum mathematischen Denken. 2. Die Wege mathematischen Denkens sind viel zu komplex, als dass Formalisierung und Mathematisierung allein unsere Probleme in ihnen lösen könnten.’ In admiration both for Gödel and the present author's explanations we have to agree with [[Zbl 1180.03002](#), p. 243] regarding the mathematization of our thinking as mathematicians: ‘It is amazing that within formal mathematics it is possible to say something about such informal things.’

For the entire collection see [[Zbl 1253.00009](#)].

Reviewer: [Bernhelm Booß-Bavnbeck \(Roskilde\)](#)

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