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Li, Ping
The rigidity of Dolbeault-type operators and symplectic circle actions
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MSC Classification: 37B05 58J20 32Q60 37J10

Keywords: rigidity; dolbeault-type operator; symplectic circle action; Hamiltonian circle action

Review text:
Besides providing a highly readable review of some corner stones in the field of circular actions on symplectic manifolds, this short paper proves interesting new results on the relations between symplectic structures and geometric analysis. Already in the visionary paper [*S. Smale*, J. Math. Mech. (now: Indiana Univ. Math. J.) 14, 1049–1055 (1965; Zbl 0166.36102)] an intimate relation between dynamical systems on manifolds and elliptic operators were worked out. The present paper addresses the following problem: Given a closed connected symplectic manifold (M, ω) of dimension $2n$ and a circular action on it that is symplectic, i.e., assume the closedness of the 1-form $\omega(X, \cdot)$, where X is the generating vector field of this circle action. Under what conditions can we conclude that the action is Hamiltonian, i.e., the exactness of the mentioned 1-form yielding a smooth moment map f on M such that $\omega(X, \cdot) = df$?
The following facts have been well known:
(1) An obvious necessary condition for a symplectic circle action to be Hamiltonian is to have nonempty fixed points.
(2) For Kähler manifolds and for 4-dimensional symplectic manifolds, that condition is also sufficient.
(3) However, [*D. McDuff*, J. Geom. Phys. 5, No.2, 149–160 (1988; Zbl 0696.53023)] gave an example of a 6-dimensional symplectic manifold with symplectic circle action of non-empty (but connected) fixed point set which is not Hamiltonian.
Since then, many authors chased the possible conjecture that a symplectic circular action with isolated fixed points must be Hamiltonian, until now without a finite clarification. The present paper is inspired by various results concerning the weights of the fixed points and by a link to the index theory of elliptic operators due to [K.È. Fel'dman, Russ. Math. Surv. 56, No. 5, 978–979 (2001); translation from Usp. Mat. Nauk 56, No. 5, 187-188 (2001; Zbl 1051.57040)]. That link is not so surprising taking in regard the close interrelation between fixed point theorems and index theory. More precisely, Feld'man had shown that the Todd genus of a manifold admitting a symplectic circle action with isolated fixed points is either 0, in which case the action is non-Hamiltonian, or 1, in which case the action is Hamiltonian.
The author of the present paper considers some Dolbeault-type elliptic operators on a compact, almost-complex manifolds admitting a circle action compatible with the almost-complex structure. He defines the equivariant indices of these operators under this circle action and, closely following [*F. Hirzebruch*, *T. Berger*, and *R. Jung*, 'Manifolds and modular forms', Wiesbaden (1992; Zbl 0767.57014)], he proves their invariance under *all* circle actions having isolated fixed points. This is the meaning of the word rigidity in his title, taken from [*R. Bott* and *C. Taubes*, J. Am. Math. Soc. 2, No.1, 137–186 (1989; Zbl 0667.57009)].
When an almost-complex manifold admits a compatible circle action with isolated fixed points, the author's rigidity result produces many identities concerning the weights on the fixed points. In particular, he obtains an operational re-formulation of Fel'dman's result for that class of manifolds.
Reviewer: Bernhelm Booß-Bavnbek (Roskilde)

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