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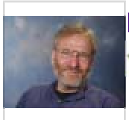
Review



Bernhelm Booss-Bavnbek evaluated the article as:

Brief summary:	Another jewel of a paper
Doi:	10.14293/S2199-1006.1.SOR-MATH.A6G464.v1.RTHMVX
Level of importance:	
Level of validity:	
Level of completeness:	
Level of comprehensibility:	

Comments:

From history we know that the philosopher Immanuel Kant was quite categorical both with his own life style and his way of thinking: Schedules should be kept. However, Kant cancelled his classes whenever he received a new book or paper by J.-J. Rousseau in his remote Prussian university town of Koenigsberg. Einstein required more to cancel a class. He wrote in his diary on 9 November, 1918: "Vorlesung ausgefallen wegen Revolution." I know of not so few mathematicians who feel like Kant: Every time a new SIAM-News column or paper by PJD reaches them, they leave everything to study PJD's most recent comments to the practice and meaning of the work and life of mathematicians. I do.

Why is that? Because PJD's writing is deep, well-informed, erudite; it is structured; and it is inspiring. Provocative too, but only to a level of pain of a massage refreshing body and mind and demanding some changes of behavior without ever getting dogmatic.

1. Another jewel of a paper
The present paper's depth lies in PJD's concept of reasonable practice, both in theoretical mathematics and mathematics applications, contrary to the proclaimed but not always obtainable full reliability of mathematical calculations. Based on his life-long and highly successful work with the numerical analysis of special functions (see also his influential [3] with Philip Rabinowitz), he advocates a Peircean view upon the quality criteria of mathematical reasoning with approximations.

In their textbook Concrete Mathematics [4, Chapter 9, pp. 425] for beginners in computer science, Donald E. Knuth and his co-authors explain: "Exact answers are great when we can find them; there's something very satisfying about complete knowledge." But there's also a time when approximations are in order. If we run into a sum or a recurrence whose solution doesn't have a closed form (as far as we can tell), we still would like to know something about the answer; we don't have to insist on all or nothing. And even if we do have a closed form, our knowledge might be imperfect, since we might not know how to compare it with other closed forms." So far, so good. But what is a good approximation? When is it reasonable to stop an iteration? Which conclusions can we draw after a necessarily limited, finite number of repetitive calculation steps? Is there out there a correct answer? Are we getting closer; moving in circles; or led astray by our algorithm and series of intermediate results? That is a common uncertainty in mathematics, sciences, and segments of everyday life when neither common sense nor established logical inferences can be applied. To that Charles Sanders Peirce coined the term abduction. It describes the "pragmaticist" way of forming beliefs when neither deduction nor induction can do.

While not a survey paper for the math community, PJD does get a lot around on his 4 1/2 pages. There are no angles of the related numerical analysis he is not aware of. As he explains, there are hundreds of stop rules for numerical iterative schemes, there are monotonous procedures and interval arithmetics and many other ideas, but none of general validity. His examples throw a spotlight on the gap between the constructivist and "existentialist" views of the mathematical practice.

In spite of the blurred topic, the paper is quite convincingly structured, with a wide range of numerical examples of sequences of numbers and calculations, followed up by epistemological considerations, and concluded by PJD's personal advices. The paper is provocative by wiping out the customary sharp frontiers between pure and applied mathematics and between absolute truth and pragmatic beliefs. It is inspiring in many ways. To me, mostly because the paper forced me to

- recall Kronecker's criticism of the concept of irrational numbers on similar grounds like PJD's headaches. Though it is reported that Kronecker showed the same gentleman like cultivated scholar behavior we know from PJD, when Kronecker, e.g., acknowledged the beauty and strength of Lindemann's proof of the transcendency of the number π, "under the assumption that the meaningless concept of a transcendent number had some meaning";

- check the stop rules I learned from Lothar Collatz [2, p. 123] and the frightening Strutt's chart of instability regions for the seemingly simple Mathieu's differential equation $y''(x) + (\lambda + \gamma \cos x)y(x) = 0$ in the $\lambda \times \gamma$-plane;

- consult the classic textbook by Rice and Rice (see also the review [1]) which has a whole chapter on The nature of errors and uncertainty (pp. 309-330) with subsections on Uncertainty in the mathematical model, Uncertainty in algorithm construction, Errors in computation, and Safeguards against errors and uncertainty - and advocates the same wisdom like PJD;

- confirm in Google Translate that there is no difference between the existentialist exist and the constructivist can be found in Scandinavian languages: English (and continental European) "exist" = Danish "findes".

2. Minor criticism
Besides the dispensable typo "N" instead of "n_0" on p. 3, column 2, line 20, I felt a bit offended by the slow beginning of the paper on page one, left column. That may be necessary for non-mathematicians in the readership. For mathematicians that beginning is confusing because we hardly ever teach checking the convergence of a sequence of numbers without specifying a rule of generation. First in the second column, in the dialogue with the investigator, named AZ, it becomes clear what PJD's goal is in this paper.
Neither can I recognize "the currently promulgated definition that a function is an association of every element of a set A with a unique element of a set B". Contemporary textbooks in Calculus – single variable (like the one of the Calculus Consortium, initially funded by a National Science Foundation Grant under leadership of Deborah Hughes-Hallett, Andrew McGleason, and William McCallum) begin with the rule of four, namely that functions can be represented by tables, graphs, formulas, and descriptions in words.
Moreover, I would have preferred to refer to Peirce and his concept of abduction explicitly instead of the reference to the also clever but in relation to Peirce epigonic Polya.

3. Readability
As with all writings of PJD, he devoted much care to making his examples, his claims, and his ideas comprehensible to a wide readership also of non-mathematicians, demanding only their concentration and a modest mathematical capability.

References
[1] A. Borodin, Reviews: Introduction to Computer Science. Problems, Algorithms, Languages, Information and Computers by J.K. Rice and J.R. Rice'. Amer. Math. Monthly 77/10 (1970), 1124-1125.
[2] L. Collatz, Differentialgleichungen, seventh ed., Leitfaden der Angewandten Mathematik und Mechanik [Guides to Applied Mathematics and Mechanics], vol. 1. B. G. Teubner, Stuttgart, 1990, Eine Einführung unter besonderer Berücksichtigung der Anwendungen. [An introduction with special consideration of applications].
[3] P. J. Davis and P. Rabinowitz, Methods of numerical integration. Dover Publications Inc., Mineola, NY, 2007, Corrected reprint of the second (1984) edition.
[4] R. L. Graham, D. E. Knuth and O. Patashnik, Concrete mathematics. Addison-Wesley Publishing Company Advanced Book Program, Reading, MA, 1989, A foundation for computer science.

Competing interests: None