Learning and teaching math: easy and hard. How ?

Extended version

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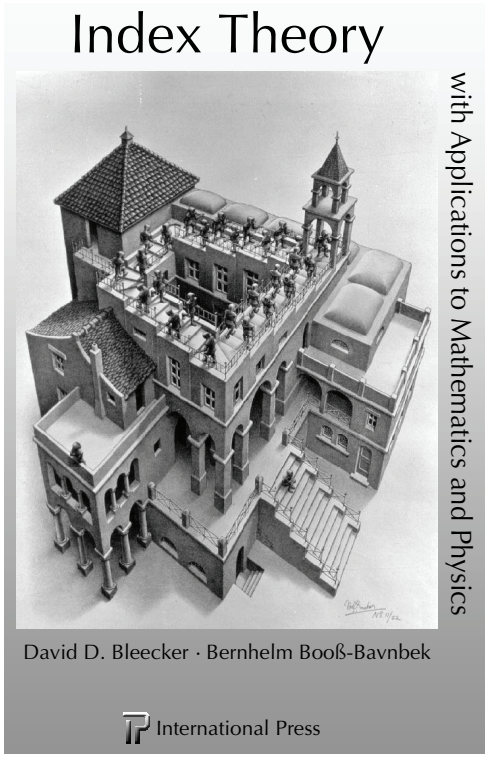
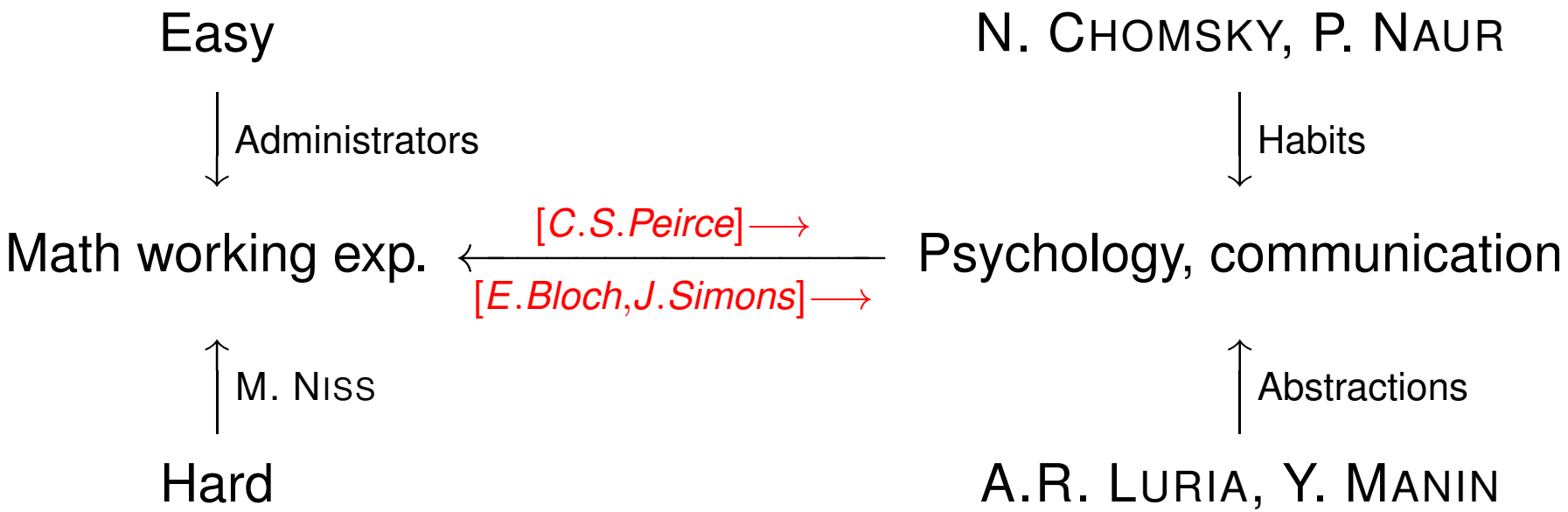


Outline

1. Introduction
   - The meaning of math working experience
2. Looking back - who/what has changed?
   - Not the students: mind stability v. cataclysmic changes
   - Way of talking about our subject
     - Today less metaphysical exaggerations
     - Seminal changes of math content widely ignored
     - Administrative frame: Continuing dismantling of the public sector
3. Math is hard — How and why?
   - Students
   - Lagrange, Gauss, Peirce, Hirzebruch, Singer, Manin, Arnol'd, Hörmander
4. Math is easy — Really?!
   - Schopenhauer vs. Gauss
   - Chomsky, Atiyah, Lagrange, Peirce, Gramsci, Freire, Naur, Kierkegaard

Goal of this talk and basic assumptions



1. Math is damn hard (until one understands).
2. Math is invisible in context (until one looks beneath the surface).
3. C.S. PEIRCE's desperate hope: Use the working experiences of mathematicians and physicists to substantiate a philosophy of sustainable behaviour.
4. PEIRCE's claim to philosophy: Investigate the reason and the conditions for the possibility of inter-subjective communication between humans (K. OEHLER).

Possible math working experience lessons for handling ambiguity in human communication — three cases

ERNST BLOCH (1885–1977), German marxist philosopher on forms of political communication: While the right wing are experts in lies, they have proved able to address humans, contrary to the left wing: they are experts in reliable analysis but continuously in danger to address things.

JAMES SIMONS, US-American mathematician. In 1966 he proved a breaking new result about multidimensional varieties. Then he left mathematics. In 2014 he became no. 87 in wealth in the US and invited speaker in the Einstein Memorial Address of the American Mathematical Society. His recipe for getting rich: “Mathematical analysis + Common sense + Luck".

JOHANNES GALTUNG, Norwegian peace researcher’s pray: O my Lord, give me the strength to fight where I can change, the patience to abstain from hopeless fights, and the wisdom to distinguish between the two situations.

Looking back — student mind set changed?

The mind of the students - has it changed?

- General “wisdom": The browse-generation, the me-generation? Hardly reliable verdicts.

No! Strong evolutionary evidence for mind stability over time:

1. Case dog breeding: still 80% lupine after 10^4 selections.
2. Case Cromagnon aesthetics: Curiosity and imagination undestroyable
   - La grotte de Lascaux,
   - Le tombe de Tarquinia,
   - Tiziano Vecellio,
   - Paula Modersohn–Becker,
   - Jackson Pollock.
3. Counterarguments: MARX’s *Das Sein bestimmt das Bewusstsein*; PEIRCE’s 800-years cataclysms; JULIAN JAYNES’ modern bicameral mind.

$\Longrightarrow$ No evidence for short term changes. Look elsewhere!

Meaning — Content — Frame

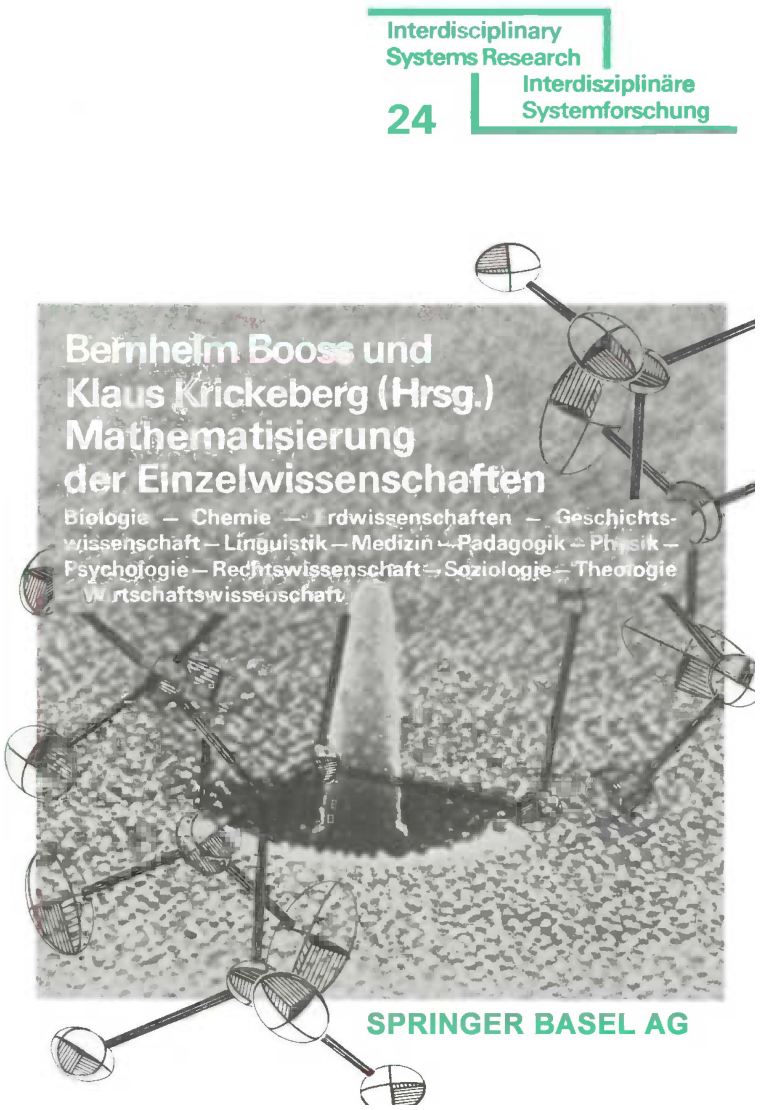
Mind set stability

Long-term mind stability

$\Longrightarrow$

Students' short-term mind changes highly improbable

Looking back — vane promises? I



1970s' metaphysical exaggerations in our way of talking about *mathematization* and *structure*:

- Biology, Medicine
- Chemistry
- Physics, Geophysics
- Linguistics
- Educational Studies, Psychology
- Law, Theology
- History, Sociology
- Economy

$\bigoplus$ Science optimism $\Leftarrow$ Military invincibility perception $\Leftarrow$ *Triade* of WWII
$\bigoplus$ Pernicious structuralism proliferation

Looking back — vane promises? II

- P. GÖRNER, 1976: Naive “Modelling cycle", unfortunately widely proliferated since then.
- N. LUHMANN, 1976: “Lieber Herr Booß ... Ich stelle mir jetzt für die Zusammenarbeit zwischen Mathematikern und Soziologen folgendes Modell vor: Der Soziologe berichtet dem Mathematiker über die weichen Stellen in seiner Theorie, von denen er erkennen kann, dass grössere Begriffsschärfe, grössere Kontrollierbarkeit der Konsistenz und vor allem grössere Eliminierungseffekte einen analytischen Gewinn erbringen würden. Der Mathematiker müsste dann prüfen, ob er zumindest Suchhinweise, wenn nicht gar Modelle oder Erfahrungen mit Modellkonstruktionen zur Verfügung stellen kann. Meine Befürchtung ist, dass genuin soziologische Theorieüberlegungen, die nicht vorweg im Blick auf mathematische Modelle oder statistische Methoden der Datenanalyse konzipiert sind, ihre

Looking back — vane promises? III

Abstraktionsgewinne mit einer Unschärfe bezahlen müssen, die für den Mathematiker nichts mehr besagt."

- Dansk matematisk landsmøde, 1971, Working paper: “Skulle man ... kort sige, hvad der er karakteristisk for den ‘matematiske tænkemåde’, kunne man gøre det ved at påstå, at den består i at generalisere. Både på et meget primitivt niveau — begrebet variabel er vel det første eksempel, man møder på et generaliseringstræk — og på overordnede niveauer, hvor generalisering træder i værk over for komplekser og relationer, samt som sidste fase, hvor generaliseringen virker på totale strukturer..."

Looking back — changes of math content ignored? I

Some Turning Points in the History of Mathematics That Have Had Consequences in the Philosophy of Mathematics (PHILIP J. DAVIS)

1. Pythagorean Theorem; $\sqrt{2}$ (Existence)
2. Euclid's Elements (Axiomatics; Idealization)
3. Algebraization of arithmetic circa 15th C (Formalization)
4. Discovery of the complex numbers (Semantics)
5. Algebraization of geometry; Descartes (Downgrading the visual)
6. Invention of Calculus; Newton, Leibniz (Existence of infinitesimals)
7. Algebra goes abstract; Galois, Hamilton (Formalization)
8. Mathematical logic; Boole, Frege, Russell, Whitehead (Logicism)
9. Non-Euclidean geometry (Conflict between empiricism and axiomatics)
10. Axiomatization of the real numbers and of analysis; Cauchy, Weierstrass, et al. (Formalization)
11. Cantorian set theory (Existence)
12. Space goes abstract; Riemann, Klein, Peano, Hilbert (Formalism, Degradation of the visual)
13. Gödel v. Hilbert's Program (Destruction of Logicism)
14. Electronic digital computing machines (Preeminance of the discrete over the continuous)
15. Increasing relevance of stochasticism (Ontology)

2/15 came up in my time!

Looking back — changes of math content ignored? II

⊕ The Seven Great Math Riddles

1. Solving algebraic equations,
   CARDANO ET AL., D'ALEMBERT, ABEL, GALOIS;
2. Transcendency of π, LINDEMANN;
3. Continuum Hypothesis, COHEN;
4. Four Colours Suffice, APPEL, HAKEN;
5. Fermat's Last Theorem, WILES;
6. Poincaré Conjecture, PERELMAN;
7. Riemann Hypothesis, ?

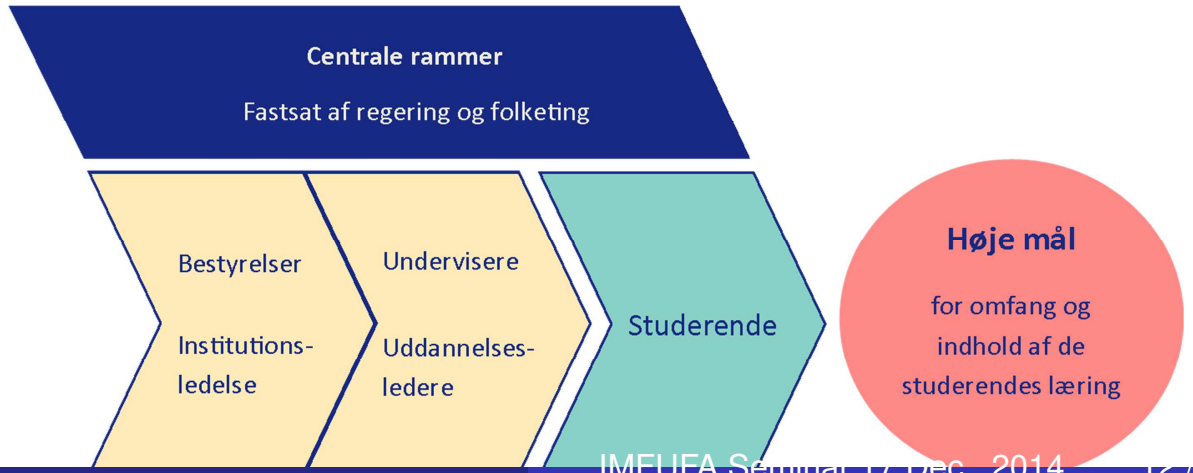
4/7 solved in my time!

Looking back — Continuing dismantling of the public sector, loss of social capital, and over-administration

1961-63: IBM 7090 = 709–T transistorized mainframe computer, rented for $ 63,500 a month (equal to $ 520,000 in 2014),
at TH Darmstadt and Uni Bonn:
$2 \times (1.8\,10^{10}\text{DM} + 340\text{m}^2)$,
long time before industry.

1966-76: Top salary for young mathematicians (plus personal secretary and student help) in public research institutions.

2014: Expert Committee on *Quality in Higher Education in Denmark: New Ways - a higher education system for the future*: More leadership instead of professional peer and student debate



Math is hard — How and why? I

- NatBatch project [CHRISTEL, CHRISTINA, AND MIKE]: *So confused* about no congruence between
  1. complexity of problem formulation;
  2. complexity of solution.
- LAGRANGE (1813): *So sorry* about
  1. having contributed to continuously growing complications; and
  2. students from non-wealthy families who have to study math.
- I.M. SINGER: *So proud* - addressing only problems that *require the hardest math*.
- V. ARNOL´D: *So sad* — In Moscow, professors now are more clever than students — what decline!
- F. HIRZEBRUCH: *So demanding* — MPIs without teaching are meaningless.

Math is hard — How and why? II

- Y. MANIN, based on A. LURIA: Abstractions, however powerful and appealing, are *non-natural* for human mind (contrary to logicism's claims).
- C.S. PEIRCE: a) Handling doubts and fixation of belief in levels:
  1. The method of tenacity — contrary to social impulse
  2. The method of authority — contrary to widening one's view
  3. (The method of doubt and discussion — makes inquiry a matter of fashion and taste)
  4. The method of science — public truth on reality, pragmatic maxim.

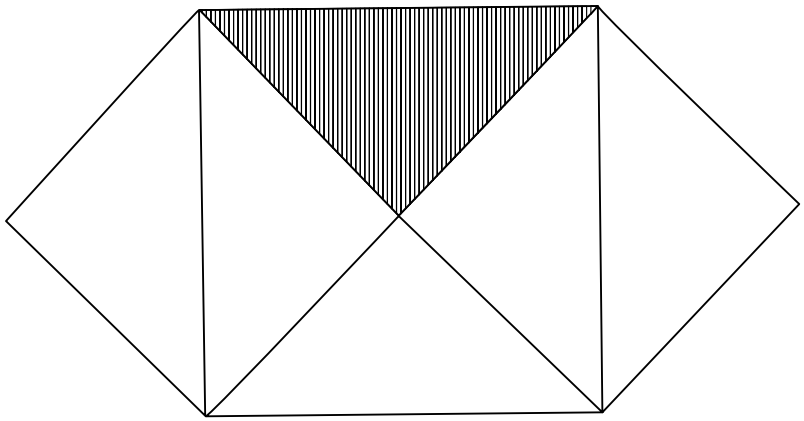
  b) *Anthropological* message - necessary to transgress common sense in artificial environment
- C.F. GAUSS, L. HÖRMANDER, typical referee reports (proving that reading math papers is always hard, also for the greatest math geniuses): *Incomprehensible — wrong — I did it long time ago*
- Highest compliment: *It's clear*

Contractible? Yes or No? How and Why?

Math is easy — Really? I

- SCHOPENHAUER, 1818: EUCLID's complications are monstrous and dispensable

book, but why it is so remains in doubt. Similarly, Pythagoras' theorem tells us about an occult quality of the right-angled triangle: Euclid's stilted, indeed underhand, proof leaves us without an explanation of why, while the following simple and well-known figure yields more insight into the matter in one glance than that proof, and also gives us a strong inner conviction of the necessity of this property and of its dependence on the right angle:



- GAUSS' answer, unfortunately: On the contrary, hide the genesis of arguments in the service of thorough simplification!

Math is easy — Really? II

- CHOMSKY: Generative grammar, most challenging math tasks are solved at the age of 2
- ATIYAH: Evolving unity and transparency (myth, refuted by BBB&PJD)
- LAGRANGE: Nature favours planetary approximations by grouping heavenly bodies according to very small and enormous eccentricity (myth?)
- PEIRCE, GRAMSCI, FREIRE, NAUR:
  a)Trace the habits of nature,
  b) relate to our form(s) of life, adolescence, clash of cultures
  c) translational power (and 2 contradictions) by
  1. coding math experiences and
  2. make them transferrable for adaption in new contexts
- Two contradictions all math learning and teaching has to live with:
  1. Result v. Process;
  2. Abstraction v. Context
- KIERKEGAARD: Seduction and passion