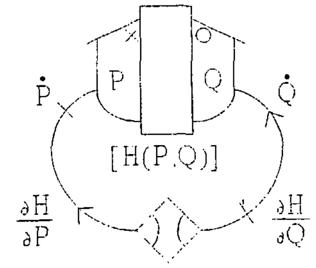
TEKST NR 419 2003

Energy Bond Graphs—a semiotic formalization of modern physics



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TEKSTER fra

IMFUFA ROSKILDE UNIVERSITETSCENTER
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES
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Energy Bond Graphs – A Semiotic Formalization of Modern Physics
Third International Gathering in Biosemiotics, Copenhagen 2003
IMFUFA tekst nr. 419-23 sider-ISBN 0106-6242

Abstract

This paper gives a brief introduction to the Energy Bond Graph (EBG) formalism – A modelling technique originally intended for Engineering and Ecological Energetics. The formalism is a shape-value notation built on C. S. Peirce's phenomenology and semiotics. Besides being a didactical tool for teaching physics to experienced students the formalism has proved valuable in connection with both the experimental and the theoretical physics research conducted at IMFUFA during the last 25 years.

The front page illustration shows the EBG formulation of Hamilton's equations.

Energy-bond-graphs — a semiotic formalization of modern physics.

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Biosemiotics, 3. International Gathering, Copenhagen, july 12th 2003

1. Scope of the formalism

An Energy Bond Graph (EBG) is a comprehensive diagrammatic representation of all the relations that constitute a dynamical system. As such, it is a mathematical model, but not just mathematical. The EBG formalism requires a total reduction to physical standard relations that exist both as hardware and as software. It thus resembles a program or a wiring diagram for an analog computer representing both a set of equations and a construction-recipe for a real system that acts out the equations. EBG modelling is a "glass- bead-game" — a "fundamentalist" way of doing physics (proceeding from real fundamentals), like Peirce's Existential Graphs are to logic. Peirce said in his second Lowell Lecture from 1903 that he had spent forty years of his life developing the existential graphs and he described its purpose in the following way that may just as well serve as an introduction to the EBG-formalism:

> "Before beginning, let us distinctly recognize the purpose which this system of expression is designed to fulfil. It is intended to enable us to separate reasoning into its smallest steps so that each one may be examined by itself. Observe, then, that it is *not* the purpose of this system of expression to facilitate reasoning and to enable one to reach his conclusion in the speediest manner. Were that our object, we should seek a system of expression which should reduce many steps to one; while our object is to subdivide one step into as many as possible. Our system is intended to facilitate the *study* of reasoning but not to facilitate reasoning itself. Its character is quite contrary to that purpose."

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The EBG-technique is not an easy way to mathematical models in physics, because it forces the user through an initial phase of semiotic reflections on categorization of variables, etc. This initial work, however, will lead to insights that are not easily gathered from more direct modelling techniques.

2. Classification of dynamic variables.

There are four kinds of dynamic variables in the EBG-formalism, and these are conveniently displayed in a Greimasian semiotic square. This method needs two pairs of binary opposites. The two members of a pair of opposites are placed diagonally against each other in the corners of a square, and then the four sides of the square identify the possibilities.

The two pairs of opposites are:
1: Level/rate, where the level-variables are accumulated, i.e. they can only change by time-integration of associated rates, whereas the rates may change abruptly.
2: directed (x)/undirected (o). Directed variables have direction both in space and time, i.e. they require a spatial orientation-convention and they change sign by time-reversal. Thus, we get the semiotic square of figure 1:

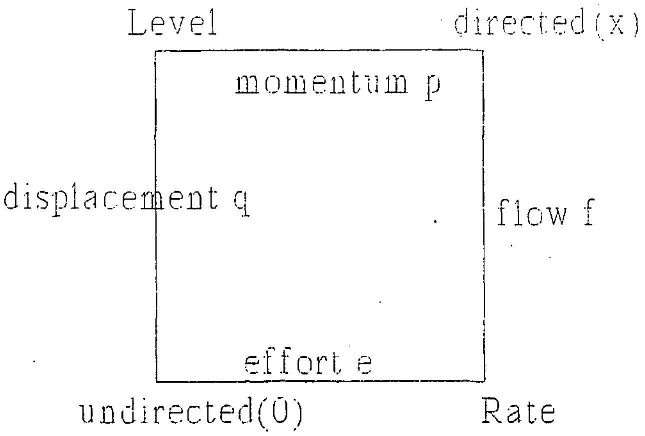


figure 1 Semiotic square of dynamic variables.

In EBG diagrams the levels are associated with "bird-house" storage icons (inspired by H.T. Odum)[1] marked x for kinetic energy and o for potential energy, while the rates are marked on the line-icon of an energy-bond with an arrow for the directed flow and a stroke for the undirected effort. The consideration that efforts (forces) are undirected reflects a deep law of classical mechanics, namely Newton's law of action and reaction.

Figure 2 below shows how the same system — a harmonic oscillator — is depicted by the present author (a) and with the more austere drawing style of the EBG-formalism's inventor H.M. Paynter (b).[2]

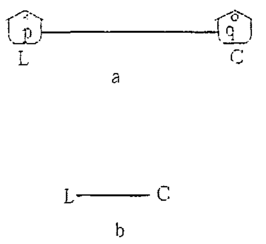


figure 2 Harmonic oscillator

3. Shape-value-notations

The difference between the two diagrams in figure 2 illustrates the concept of a *shape-value notation* (a) where the shape of the icons is chosen such as to convey an idea of their value or function and a *symbolic* notation where symbols like digits or letters are used to replace proper icons (b).

The present formulation was inspired by Odum's shape-value-notation for Ecological Energetics [1], but it was felt that the lack of physical precision in Odum's definitions of his icons could be remedied by using Paynter's bond-graphs [2] . However, Paynter did not attempt to create suitable shapes for his icons and reverted to a symbolic notation, which lead to some ambiguities. By using, e.g., the letter T as an icon for a transformer/transducer one misses the fact that a proper definition of the parameter of the object requires a destinction between its *primary* and its *secondary* port, and this distinction cannot be brought out by a symmetric shape like T, but only by an asymmetric icon (except in special cases), like the triangular shape in figure 6.

Some icons have associated parameters, denoted by symbols. The shape-value notation demands that the meaning of these symbols shall be uniquely defined by the shape of the icon. This demand means that it is sometimes necessary to have two different icons for the same physical function, as with the gyrators and the sinks shown in figure 6.

Attempts to create shape-value-notations for mathematics and logic have been studiously ignored by mathematicians, who for the last hundred years have preferred to continue in the trend laid out by Hilbert's shapeless axiomatization of Geometry. Thus was neglected Peirce's construction of a shape-value-notation for the sixteen binary logical cannectives[3] Peirce's notation was rediscovered by Shea Zellweger who has now developed it further to a complete "logic alphabet" thereby stressing the merits of shape-value notations for educational purposes[4] Peirce's early version of his existential graphs (the alpha graphs) was also a shape-value notation of logic and a precursor of Spencer-Brown's "Laws of Form" from 1969.[5]

By developing the EBG-formalism as a shape-value notation the present author hopes to have helped strengthening an important historical strain that may prove valuable to Physics and Biosemiotics.

4. Basic response properties.

In a response experiment we act on a system through an energy bond, which, as we have seen, contain four variables, q,f,e, and p. These four variables are not independent, because the levels q and p are time-integrals of the rates f and e. In the experiment we leave the system undisturbed from time -∞ to time 0. Then we choose one of the four variables as *stimulus s* and let it have a constant value from time 0 to ∞. As there are two causal groups of independent variables we can choose s from one group (e.g. (f,q)) and then observe the response r(t) as one of the variables from the other group (e,p). In the response-matrix shown in figure 2 there are thus two windows of possible response-properties, each containing four functions, but it can be shown that for *time-homogeneous* *systems* where the result does not depend on which instant we choose as time 0, two of the four functions in each window will be identical. Any physical system therefore has six different response-properties.

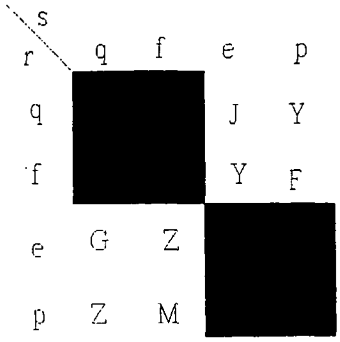


figure 3. the matrix of response-functions.

Response-semantics, i.e. the choice of proper nouns to denote the response-properties may then proceed from the *response-semiotics* of figure 3. In the following table we have combined usually used nouns with the six two-link Peircean sign classes: Read, e.g. (23) as "second of third".

(11) G: *Rigidity* or *elastic modulus*.
(12) Z: *Impedance* or *resistance*,
(22) M: *Inertance* or *Inductance*.
(13) F: *Lightness* or *Susceptibility*.
(23) Y: *Conductance* or *Mobility*.
(33) J: *Compliance* or *Capacitance*.

We have here defined the response-functions as functions of time from 0 to ∞, but by Laplace-Stieltjes-transformations they are defined as functions of a complex frequency $s=-i\omega$. These complex functions are usually just called *generalized susceptibilities*.
However, there are many advantages in distinguishing the different functions by different names because they are mathematically related by time-integration: G integrates to Z, Z integrates to M, and similarly for the set F,Y,J. In the frequency- (s-) domain time integration corresponds to division by s. Thus, by this semiotic/semantic approach it becomes

possible to identify and name a response-property after a quick glance at the oscilloscope. And a careful distinguishing by names serves to develop the somewhat mysterious ability called "physical intuition".

5. the energy bond, causality and orientation.

the most basic icon of the EBG-formalism — *the energy bond* — is drawn as a simple line. It denotes an energetic *interaction* between two system-components. The interaction is mediated by the two rate-variables that belong to the bond: the directed *flow*,f, indexed as an arrow and the undirected *effort*, e, indexed as a stroke. If system A acts on system B with an effort, then system B acts back on system A with a flow. This causality is then indicated by placing the effort-index closest to B and the flow-index closest to A. By using an arrow as the flow-index we are forced to choose an *orientation* of the bond. Every bond may have either orientation, but when we want to view the same physical situation with the opposite orientation, we have to change the sign of the symbolic expression for the flow (f changes to -f). The interaction between A and B described above can thus be depicted in the two ways shown in figure 4:

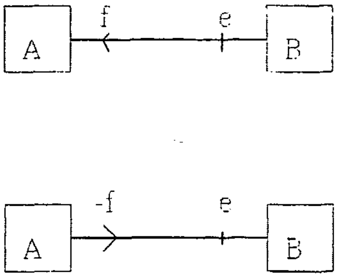


Figure 4 Interaction and orientation.

It must be stressed that the causality of an energy bond is not just a formal consideration made necessary by the algorithmic ordering. There exists a precise measurement prescription to decide the causal order of the rate-variables for each bond, based on the analysis of transients or noise.

Every energetic interaction is associated with a flow of energy. The energy flow in the direction of the orientation of the bond is the product of effort and flow.

In general effort and flow are defined as *vectors in a complex metric space of arbitrary dimensionality*. The energy flow is then defined as the *scalar* product of the effort- and the flow-vector. This generalization, however leads to the complication that the vectors must be represented as either *covariant* or *contravariant*. The flow-orientation rule shown in figure 4 then has to be modified such that the change of orientation is associated with a shift of variance of both the effort- and the flow-vector. By adopting the further rule that *the same physical situation may be described with either variance of the effort* (but not of the flow), the formalism becomes able to treat vectors of mixed kinds where some of the effort-components are flows and the corresponding flow-components are efforts. This feature allows for a very general treatment of response-experiments, where, e.g. the celebrated *Onsager symmetry relations* becomes a theorem of the EBG-formalism.

The general vector-formulation also demands that *each bond is associated with a metric tensor* that relates the covariant to the contravariant vectors. A unit metrical tensor is represented as a matrix with 1s in the diagonal and 0s outside. This is *Euclidean metric*. When coordinate vectors are defined by three spatial coordinates and one time-coordinate the flow-vector's first three components will be flows and the fourth will be an effort. This corresponds to a metric tensor with 1s in the three first places of the diagonal and -1 in the fourth place — *the Minkowsky-metric*. The group of metric-preserving transformations for this metric are *the Lorentz-transformations*, and the whole formalism of special relativity follows from this. Likewise, by adopting the general tensor-formulation of *Riemannian metric spaces* the theory of general relativity also follows from the EBG-formalism.

6. Definitions of basic icons

EBG-modelling always proceeds through three semiotic stages: from *icons* through *indices* to *symbols*. Each icon denotes a basic physical relation between the input and output variables of the associated bonds (the *ports*) When indices — flow-arrows and effort-strokes are marked on the bonds, symbolic expressions for these variables are written near the indices, whereby the meaning of the symbols and the function of the icon is defined. Note that *nearness* is itself an indexical element inherent in the meaning of the symbols. Symbols near the icons are also used for defining certain parameters of the icons (like storage-capacity and transformer-ratio).

The basic icons are clearly divided in three classes corresponding to Peirce's three phenomenological categories:

1: *Active* systems (sources of flow or effort) have an output-variable that is independent of the input-variable.

2: *Passive* or *reactive* systems have an output that is determined by the present or the previous values of the input.

3: *Dissipative* systems (the sinks) mediate between active and passive behaviour. Their response is mainly passive, like the voltage over a resistor, given by Ohm's law, but the passive response is superposed with an active component — the *noise* — that depends both on the sink-parameter and the temperature.

In figure 5 we show definitions of the active and the reactive systems.

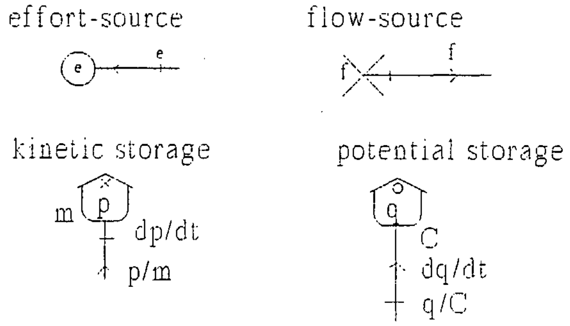
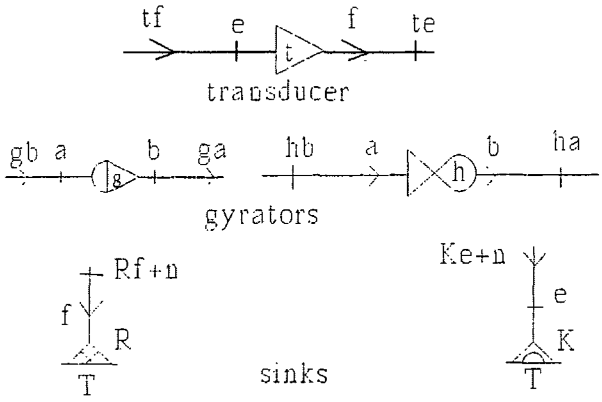


figure 5 active (sources) and reactive systems (storage)

Figure 6 shows the sinks and two passive 2-ports, transducer/transformer and gyrator. The definition does not distinguish between transducer and transformer, but the name "transducer is used when the parameter t has a physical dimension, whereas transformers are dimensionless. Transformers and gyrators are very different although they look similar. The parameters t and g may be given as level-variables elsewhere in the system, but then t must be an undirected level, whereas g must be directed. Such *parametric feedback* is useful for modelling nonlinearities.



fugure 6 Passive 2-ports and sinks (n denotes noise).

Finally, figure 7 shows the *junctions* as 3-ports, thus representing the only *triadic* relations of the formalism. The junctions are *topological* constraints corresponding to Kirchhoff's two laws of electrical networks. The o-junction corresponds to a node in the network (parallel connection) and the x-junction to a mesh (series connection). The similarities between o- and x-junction reflect the nearly dual symmetry between efforts and flows, but the dual symmetry is broken by the fact that flows are directed, while efforts are not, and this leads to subtle differences between the junctions, which, however, we shall not discuss here.

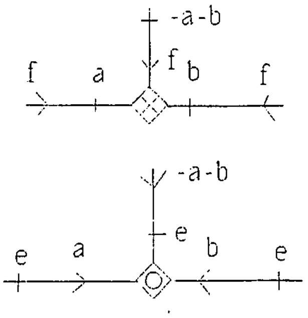


figure 7 The junctions.

7. Beyond laws of nature

A strange consequence of the EBG-game is that what we usually regard as laws of nature seem to vanish out of scope. Instead we have the rules of the game, which ensure that the laws of nature are obeyed. The vanishing of laws is thus a result of mathematization and semiotization. Similarly, in elementary physics the pupils learn the law of nature that forces are combined by the construction known as the parallelogram of forces, but in more advanced teaching they are just taught that forces are *vectors*, so the former law of nature becomes the rule for adding vectors.

We have seen that Newton's law of action and reaction hides within the rule that efforts are undirected or the flow-orientation rule of figure 4. Similarly, we can find Newton's second law of motion (force equals mass times acceleration) in the definition of the x-storage icon of figure 5, where m is the mass, p the momentum, e the force, and f the velocity (*p/m*). It was also mentioned that Onsager's symmetry- or reciprocity-relations is a theorem of the EBG-formalism. It is therefore not necessary for users of the formalism to know this as a law of nature, because it is automatically satisfied by all EBG-models that are made according to the rules.

Certain combinations of icons are not allowed, because they lead to *causal conflicts*. It is absolutely forbidden to connect two effort-sources directly or through a transformer. The sources are absolutely rigid in their causality, while other systems may be forced to yield. By connecting an effort source to a o-storage a mild conflict arises where the storage element is forced to give in and accept "differential causality". In this

> way we are allowed to ascribe a conductivity to an electric, capacitor C, but the conflict still shows itself in the fact that the frequency-dependent conductivity *Cs* goes to infinity for large frequencies, which means that the causality will break down if the effort of the source changes very rapidly. Causal conflicts may often be resolved by introducing extra sinks. This happens, e.g. when a car tries to accelerate too fast; the wheels will slide on the road — a sink has appeared to represent the friction between wheel and road. In the traditional way of describing physical systems — by equations — there is no formal treatment of causality, so the concept of causal conflict does not exist, but in reality it plays an important role, especially when systems break down.

Many laws of nature are formulated as partial differential equations, e.g. the diffusion- and wave-equations, Maxwell's electromagnetic equations, and the Schrödinger equation. Such equations are in EBG diagrams shown by combining icons, each representing an infinitesimal section of space in structures that are repeated in three dimensional space like the atoms in a crystal. As an example of this figure 8 shows the unit cell for representing Maxwell's equations in vacuum. Not shown in the diagram are storage elements connected to the junctions: x-storage to x-junctions representing the magnetic induction-vector $\bar{B}$, and o-storage to o-junctions representing the electric displacement-vector $\bar{D}$. The electric field $\bar{E}$ and the magnetic field $\underline{H}$ are represented, respectively, as the efforts and the flows in the bonds.

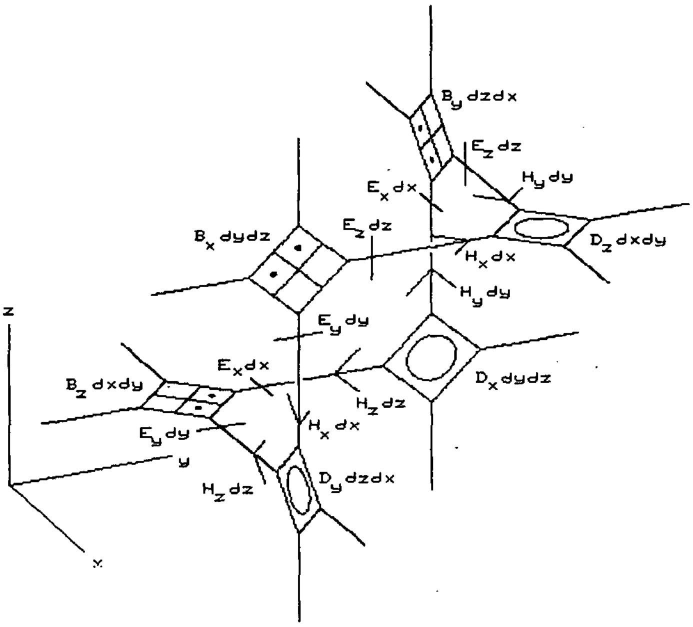
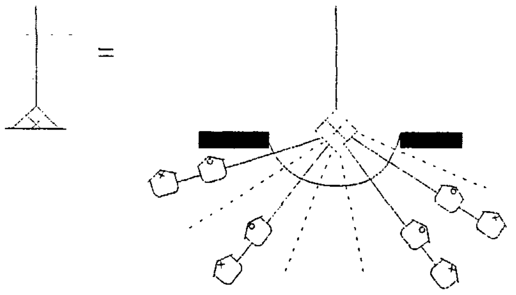


Figure 8 Unit cell for Maxwell's equations.

As mentioned, the sinks are sources of noise. This important insight follows from the *Fluctuation-Dissipation Theorem* by Callen and Welton (1951)[6]. As shown in figure 9 and the associated formula the complete complex impedance function $Z(\omega)$, $(\omega = is)$ may be spectrally resolved on the response-functions of harmonic oscillators. The real, or dissipative part of the impedance $R(\omega')$ is proportional to the density of oscillators on the real axis of their resonance frequencies ω'. The spectral resolution formula is then valid when ω has a positive imaginary part. This formula is translated to icons in the figure and gives justification for the choice of the sink-icon. The noise is simply the output from all these un-correlated oscillators in thermal equilibrium with the temperature T. For an ohmic resistance the oscillator-frequencies will be equally distributed over the real axis, and the noise-spectrum will be *white.*



$$Z(\omega) = \frac{i}{\pi}\int_0^\infty R(\omega')\left[\frac{1}{\omega-\omega'} + \frac{1}{\omega+\omega'}\right] d\omega'$$

figure 9 Spectral resolution of a sink

For a frequency-independent resistance R the prescription of the FD-theorem is simply that for each step dt of the numerical integration one has to add the effort-noise

$$n= N(\sqrt{(2RkT/dt)})$$

to the passive response of the sink. Here N(x) is a normally distributed random number with mean value 0 and standard deviation x. k is Boltzmann's constant, and T the absolute temperature. The formula is valid in *the classical limit*, when $dt >> \hbar/kT$.

It is noteworthy that the noise diverges to infinity when the steplength dt goes to zero. The noise is, as Peirce said,

"infinite in the here-and-nowness of immediate sensation, finite and relative in the recency of the past". (CP 6.135).

Every EBG-model containing dissipative elements thus becomes "animated with noise". It is easy to show that the level-variables of the system in this way get the exact fluctuation-moments that are prescribed by Statistical Mechanics, which in this way also becomes a result of the EBG-formalism.

The above expression for the effort-noise n demands that the sink-parameter R be positive. This is a further axiom of the formalism and it has the result that the energy flow to a sink, Rf^2 is always positive, thus ensuring that the laws of thermodynamics are always obeyed by EBG-models.

8. Completeness of the formalism

Within *Physics* and *Engineering* the EBG-formalism seems to be complete. Although new icons can be freely invented and added to the basic icons it seems always possible to make a full *reticulation*, i.e. to reduce these new functions to the basic icons. In practice it is easier to use a combination of basic and composite icons. The icons used by the author, shown in figure 10, are defined once and for all in an object-oriented drawing program (DrawPerfect 1.1)

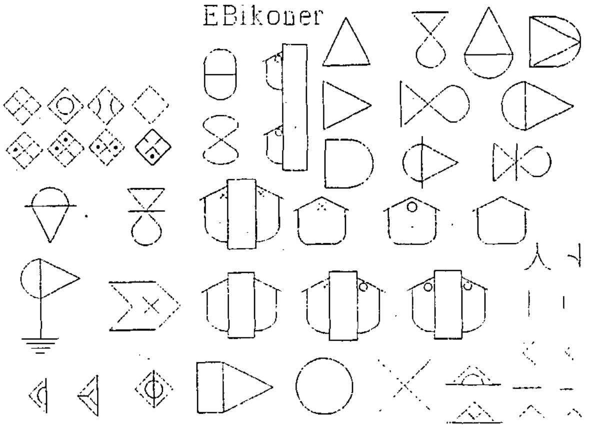


figure 10 Iconic objects of the EBG-formalism

A full proof of completeness is difficult to find (if not impossible), but many special cases have been considered[7] and algorithmic prescriptions for their translation to the EBG language have been found, of which I shall just mention a few:

1 *Chemical reactions* forwards and backwards were first described by a reaction-icon based on H.T. Odum's *work-gate-icon*, but it was later shown that the reaction-icon can be reticulated by sinks and gyrators. In this way it is possible to make EBG-descriptions of large biochemical reaction-networks, like photosynthesis.

2 Topology of electrical and rheological networks are easily translated to EBG diagrams of junctions. It turns out, however, that the x-junction is a series connection in electricity but a parallel connection in rheology (and vice versa for the o-junction).

3 Linear response can be reduced to electrical networks [8], and from there to EBG models. So in this case a full proof of completeness exists. On the other hand it is not always possible to translate EBG diagrams to electrical networks. (It may lead to short-circuits if one tries).

4 All systems that can be described with the Lagrangian and Hamiltonian methods of classical analytical mechanics can be translated to EBG-models by a relatively simple algorithm. So in this case the formalism is also proven complete, and it turns out that the EBG-models are generally more efficient for simulation than the Hamiltonian equations.

5 All relations exhibited by the basic icons are linear, but, as mentioned earlier, non-linearities may be introduced by parametric feedback from levels to transformers and gyrators. Such non-linearities will contain a time-delay, but many systems of classical mechanics require *simultaneous* non-linear relations. Such relations can also be given by an EBG diagram. In figure 11 it is shown how a simultaneous rendering of the relations s=sinθ and c=cosθ is reticulated:

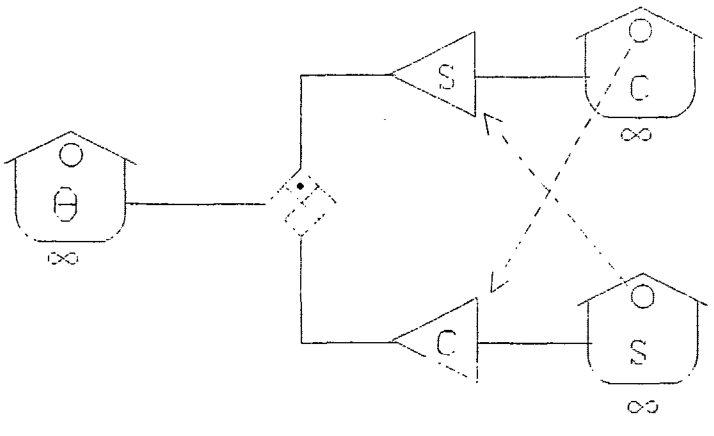


figure 11 Simultaneous reticulation of c=cosθ and s=sinθ.

Finally, figure 12 shows how a complicated mechanical system — a double-pendulum — is fully reticulated with all its cosines and square-roots:

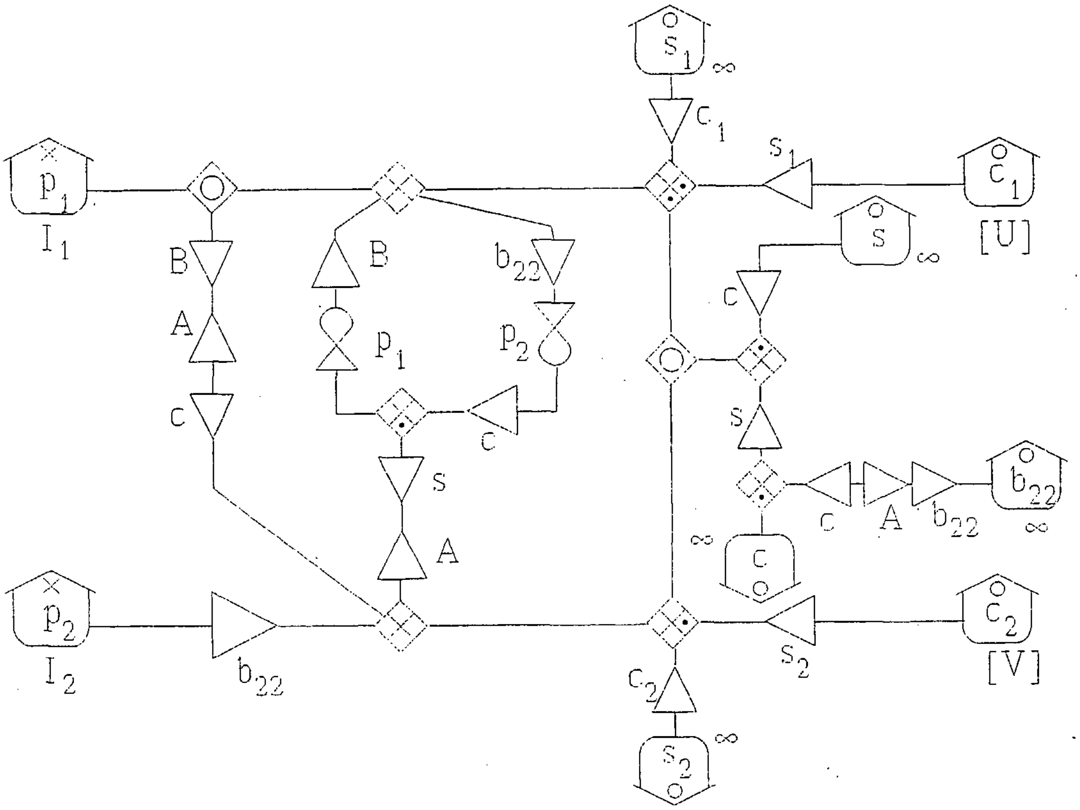


Figure 12 Full reticulation of double-pendulum.

9. Perspectives

Some biosemioticians are apprehensive about the dangers of physical reductionism. It must be admitted that EBG-formalism is an example of such reductionism, and even that it tries to carry physical reductionism to an extreme (by reducing basic physical concepts to even more basic semiotic concepts). Against such warnings I have several comments:

1): Kurt Gödel carried logical positivism to an extreme and found something on the other side of great importance to mathematics and philosophy.

2) An EBG-model is always open to expansion into the environment, except when the underlying theory demands a closure.

3) Regarding Biosemiotics: An EBG model is a sign of a general idea and thus, as Peirce said (CP 6.270) a living entity analogous to a person. The variables in an EBG-model are always superposed, both with thermal noise and quantum-fluctuations, that may be regarded as "living feeling" and spontaneity. If it is reductionistic to use such models in biology, it is, at least, not a simplistic kind of reductionism.

4) Regarding Quantum Mechanics: The EBG-formalism has a strong affinity to Quantum Mechanics, as it has to Relativity. If it becomes possible to construct an analog computer that can act out all the simultaneous feedbacks of EBG-models, it will be a quantum computer. The non-local constraints exhibited by the junctions give a simple explanation of Quantum-Non-Locality (the Einstein- Podolsky-Rosen experiments). Many other seemingly counter-intuitive quantum mechanical effects also appear as natural and understandable when viewed in the light of EBG-

reticulation of the physical sign-relations. A full investigation of this research program, called *Quantum Semiotic*, is in progress, but may take many years to complete.

Lejre, july 1. 2003

Peder Voetmann Christiansen.

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338/97 Kvantisering af nanolederes elektriske ledningsevne
Første modul fysikprojekt
af: Søren Dam, Esben Danielsen, Martin Niss,
Esben Friis Pedersen, Frederik Resen Steenstrup
Vejleder: Tage Christensen

339/97 Defining Discipline
by: Wolfgang Coy

340/97 Prime ends revisited - a geometric point of view -
by: Carsten Lunde Petersen

341/97 Two chapters on the teaching, learning and assessment of geometry
by: Mogens Niss

342/97 A global clean fossil scenario DISCUSSION PAPER prepared by Bernd Kuemmel
for the project LONG-TERM SCENARIOS FOR GLOBAL ENERGY DEMAND AND SUPPLY

343/97 IMPORT/EKSPORT-POLITIK SOM REDSKAB TIL OPTIMERET UDNYTTELSE AF EL PRODUCERET PÅ VE-ANLÆG
af: Peter Meibom, Torben Svendsen, Bent Sørensen

344/97 Puzzles and Siegel disks
by: Carsten Lunde-Petersen

345/98 Modeling the Arterial System with Reference to an Anesthesia Simulator
Ph.D. Thesis
by: Mette Sofie Olufsen

346/98 Klyngedannelse i en hulkatode-forstøvningsproces
af: Sebastian Horst
Vejledere: Jørn Borggren, NBI, Niels Boye Olsen

347/98 Verificering af Matematiske Modeller
- en analyse af Den Danske Eulerske Model
af: Jonas Blomqvist, Tom Pedersen, Karen Timmermann, Lisbet Øhlenschlæger
Vejleder: Bernhelm Booss-Bavnbek

348/98 Case study of the environmental permission procedure and the environmental impact assessment for power plants in Denmark
by: Stefan Krüger Nielsen
project leader: Bent Sørensen

349/98 Tre rapporter fra FAGMAT - et projekt om tal og faglig matematik i arbejdsmarkedsuddannelserne
af: Lena Lindenskov og Tine Wedege

350/98 OPGAVESAMLING - Bredde-Kursus i Fysik 1976 - 1998
Erstatter teksterne 3/78, 261/93 og 322/96

351/98 Aspects of the Nature and State of Research in Mathematics Education
by: Mogens Niss

352/98 The Herman-Swiatec Theorem with applications
by: Carsten Lunde Petersen

353/98 Problemløsning og modellering i en almendannende matematikundervisning
Specialerapport af: Per Gregersen og Tomas Højgaard Jensen

354/98 A Global Renewable Energy Scenario
by: Bent Sørensen and Peter Meibom

355/98 Convergence of rational rays in parameter spaces
by: Carsten Lunde Petersen and Gustav Ryd

356/98 Terrænmodellering
Analyse af en matematisk model til konstruktion af digitale terrænmodeller
Modelprojekt af: Thomas Frommelt, Hans Ravnkjær Larsen og Arnold Skimminge
Vejleder: Johnny Ottesen

357/98 Cayleys Problem
En historisk analyse af arbejdet med Cayleys problem fra 1870 til 1918
Et matematisk videnskabsfagsprojekt af: Rikke Degn, Bo Jakobsen, Bjarke K.W. Hansen, Jesper S. Hansen, Jesper Udesen, Peter C. Wulff
Vejleder: Jesper Larsen

358/98 Modeling of Feedback Mechanisms which Control the Heart Function in a View to an Implementation in Cardiovascular Models
Ph.D. Thesis by: Michael Danielsen

359/99 Long-Term Scenarios for Global Energy Demand and Supply
Four Global Greenhouse Mitigation Scenarios
by: Bent Sørensen (with contribution from Bernd Kuemmel and Peter Meibom)

360/99 SYMMETRI I FYSIK
En Meta-projektrapport af: Martin Niss, Bo Jakobsen & Tune Bjarke Bonné
Vejleder: Peder Voetmann Christiansen

361/99 Symplectic Functional Analysis and Spectral Invariants
by: Bernhelm Booß-Bavnbek, Kenro Furutani

362/99 Er matematik en naturvidenskab? - en udspænding af diskussionen
En videnskabsfagsprojekt-rapport af: Martin Niss
Vejleder: Mogens Nørgaard Olesen

363/99 EMERGENCE AND DOWNWARD CAUSATION
by: Donald T. Campell, Mark H. Bickhard, and Peder V. Christiansen

364/99 Illustrationens kraft - Visuel formidling af fysik
Integreret speciale i fysik og kommunikation
af Sebastian Horst
Vejledere: Karin Beyer, Søren Kjørup

365/99 To know - or not to know - mathematics, that is a question of context
by: Tine Wedege

366/99 LATEX FOR FORFATTERE - En introduktion til LATEX og IMFUFA-LATEX
af: Jørgen Larsen

367/99 Boundary Reduction of Spectral Invariants and Unique Continuation Property
by: Bernhelm Booss-Bavnbek

368/99 Kvartvejsraport for projektet SCENARIER FOR SAMLET UDNYTTELSE AF BRINT SOM ENERGIBÆRER I DANMARKS FREMTIDIGE ENERGISYSTEM
Projektleder: Bent Sørensen

369/99 Dynamics of Complex Quadratic Correspondences
by: Jacob S. Jalving
Supervisor: Carsten Lunde Petersen

370/99 OPGAVESAMLING - Bredde-Kursus i Fysik 1976 - 1999
Eksamensopgaver fra perioden 1976 - 1999. Denne tekst erstatter tekst nr. 350/98

371/99 Bevisets stilling - beviser og bevisførelse i en gymnasial matematik undervisning
Et matematikspeciale af: Maria Hermannsson
Vejleder: Mogens Niss

372/99 En kontekstualiseret matematikhistorisk analyse af ikke-lineær programmering:
Udviklingshistorie og multipel opdagelse
Ph.d.-afhandling af Tinne Hoff Kjeldsen

373/99 Criss-Cross Reduction of the Maslov Index and a Proof of the Yoshida-Nicolaescu Theorem
by: Bernhelm Booss-Bavnbek, Kenro Furutani and Nobukazu Otsuki

374/99 Det hydrauliske spring - Et eksperimentelt studie af polygoner og hastighedsprofiler
Specialeafhandling af: Anders Marcussen
Vejledere: Tomas Bohr, Clive Ellegaard, Bent C. Jørgensen

375/99 Begrundelser for Matematikundervisningen i den lærde skole hhv. gymnasiet 1884-1914
Historiespeciale af Henrik Andreassen, cand.mag. i Historie og Matematik

376/99 Universality of AC conduction in disordered solids
by: Jeppe C. Dyre, Thomas B. Schrøder

377/99 The Kuhn-Tucker Theorem in Nonlinear Programming: A Multiple Discovery?
by: Tinne Hoff Kjeldsen

378/00 Solar energy preprints:
1. Renewable energy sources and termal energy storage
2. Integration of photovoltaic cells into the global energy system
by: Bent Sørensen

379/00 EULERS DIFFERENTIALREGNING
Eulers indførelse af differentialregningen stillet over for den moderne
En tredjesemesters projektrapport på den naturvidenskabelige basisuddannelse
af: Uffe Thomas Volmer Jankvist, Rie Rose Møller Pedersen, Maja Bagge Pedersen
Vejleder: Jørgen Larsen

380/00 MATEMATISK MODELLERING AF HJERTEFUNKTIONEN
Isovolumetrisk ventrikulær kontraktion og udpumpning til det cardiovaskulære system
af: Gitte Andersen (3.moduls-rapport), Jakob Hilmer og Stine Weisbjerg (speciale)
Vejleder: Johnny Ottesen

381/00 Matematikviden og *teknologiske kompetencer* hos kortuddannede voksne
- Rekognosceringer og konstruktioner i grænselandet mellem matematikkens didaktik og forskning i voksenuddannelse
Ph. d.-afhandling af Tine Wedege

382/00 Den selvundvigende vandring
Et matematisk professionsprojekt
af: Martin Niss, Arnold Skimminge
Vejledere: Viggo Andreasen, John Villumsen

383/00 Beviser i matematik
af: Anne K.S.Jensen, Gitte M. Jensen, Jesper Thrane, Karen L.Å.W. Wille, Peter Wulff
Vejleder: *Mogens Niss*

384/00 Hopping in Disordered Media: A Model Glass Former and A Hopping Model
Ph.D. thesis by: Thomas B. Schrøder
Supervisor: Jeppe C. Dyre

385/00 The Geometry of Cauchy Data Spaces
This report is dedicated to the memory of Jean Leray (1906-1998)
by: B. Booss-Bavnbek, K. Furutani, K. P. Wojciechowski

386/00 Neutrale mandatfordelingsmetoder – en illusion?
af: Hans Henrik Brok-Kristensen, Knud Dyrberg, Tove Oxager, Jens Sveistrup
Vejleder: Bernhelm Booss-Bavnbek

387/00 A History of the Minimax Theorem: von Neumann's Conception of the Minimax Theorem - - a Journey Through Different Mathematical Contexts
by: Tinne Hoff Kjeldsen

388/00 Behandling af impuls ved kilder og dræn i C. S. Peskins 2D-hjertemodel
et 2. moduls matematik modelprojekt
af: Bo Jakobsen, Kristine Niss
Vejleder: Jesper Larsen

389/00 University mathematics based on *problemoriented student projects: 25 years of* experience with the Roskilde model
By: Mogens Niss
Do not ask what mathematics can do for modelling. Ask what modelling can do for mathematics!
by: Johnny Ottesen

390/01 SCENARIER FOR SAMLET UDNYTTELSE AF BRINT SOM ENERGIBÆRER I DANMARKS FREMTIDIGE ENERGISYSTEM Slutrapport, april 2001
Projektleder: Bent Sørensen
Projektdeltagere: DONG: Aksel Hauge Petersen, Celia Juhl, Elkraft System[#]: Thomas Engberg Pedersen[##], Hans Ravn, Charlotte Søndergren, Energi 2[#]: Peter Simonsen, RISØ Systemanalyseafd.: Kaj Jørgensen[*], Lars Henrik Nielsen, Helge V. Larsen, Poul Erik Morthorst, Lotte Schleisner, RUC: Finn Sørensen[**], Bent Sørensen
[#]Indtil 1/1-2000 Elkraft, [##] fra 1/5-2000 Cowi Consult
[*] *Indtil 15/6-1999 DTU* Bygninger & Energi, [**] fra *1/1-2001 Polypeptide Labs.*
Projekt 1763/99-0001 under Energistyrelsens Brintprogram

391/01 Matematisk modelleringskompetence – et undervisningsforløb i gymnasiet
3. *semesters* Nat.Bas. projekt af: Jess Tolstrup Boye, Morten Bjørn-Mortensen, Sofie Inari Castella, Jan Lauridsen, Maria Gøtzsche, Ditte Mandøe Andreasen
Vejleder: Johnny Ottesen

392/01 "PHYSICS REVEALED" THE METHODS AND SUBJECT MATTER OF PHYSICS
an introduction to pedestrians (but not excluding cyclists)
PART III: PHYSICS IN PHILOSOPHICAL CONTEXT
by: Bent Sørensen.

393/01 Hilberts matematikfilosofi
Specialerapport af: Jesper Hasmark Andersen
Vejleder: Stig Andur Pedersen

394/01 "PHYSICS REVEALED" THE METHODS AND SUBJECT MATTER OF PHYSICS
an introduction to pedestrians (but not excluding cyclists)
PART II: PHYSICS PROPER
by: Bent Sørensen.

395/01 Menneskers forhold til matematik. Det har sine årsager!
Specialeafhandling af: Anita Stark, Agnete K. Ravnborg
Vejleder: Tine Wedege

396/01 2 bilag til tekst nr. 395: Menneskers forhold til matematik. Det har sine årsager!
Specialeafhandling af: Anita Stark, Agnete K. Ravnborg
Vejleder: Tine Wedege

397/01 En undersøgelse af solvents og kædelængdes betydning for anomal swelling i phospholipiddobbeltlag
2. modul fysikrapport af: Kristine Niss, Arnold Skimminge, Esben Thormann, Stine Timmermann
Vejleder: Dorthe Posselt

398/01 Kursusmateriale til "Lineære strukturer fra algebra og analyse" (E1)
Af: Mogens Brun Heefelt

399/01 Undergraduate Learning Difficulties and Mathematical Reasoning
Ph.D Thesis by: Johan Lithner
Supervisor: Mogens Niss

400/01 On Holomorphic Critical quasi circle maps
By: Carsten Lunde Petersen

401/01 Finite Type Arithmetic
Computable Existence Analysed by Modified Realisability and Functional Interpretation
Master's Thesis by: Klaus Frovin Jørgensen
Supervisors: Ulrich Kohlenbach, Stig Andur Pedersen and Anders Madsen

402/01 Matematisk modellering ved den naturvidenskabelige basisuddannelse
- udvikling af et kursus
Af: Morten Blomhøj, Tomas Højgaard Jensen, Tinne Hoff Kjeldsen og Johnny Ottesen

403/01 Generaliseringer i integralteorien
- En undersøgelse af Lebesgue-integralet, Radon-integralet og Perron-integralet
Et 2. modul matematikprojekt udarbejdet af: Stine Timmermann og Eva Uhre
Vejledere: Bernhelm Booss-Bavnbek og Tinne Hoff Kjeldsen

404/01 "Mere spredt fægtning"
Af: Jens Højgaard Jensen

405/01 Real life routing
- en strategi for et virkeligt vrp
Et matematisk modelprojekt af: David Heiberg Backchi, Rasmus Brauner Godiksen, Uffe Thomas Volmer Jankvist, Jogvan Martin Poulsen og Neslihan Saglanmak
Vejleder: Jørgen Larsen

406/01 Opgavesamling til dybdekursus i fysik
Eksamensopgaver stillet i perioden juni 1976 til juni 2001
Denne tekst erstatter tekst nr. 25/1980 + efterfølgende tillæg

407/01 Unbounded Fredholm Operators and Spectral Flow
By: Bernhelm Booss-Bavnbek, Matthias Lesch, John Phillips

408/02 Weak UCP and Perturbed Monopole Equations
By: Bernhelm Booss-Bavnbek, Matilde Marcolli, Bai-Ling Wang

409/02 Algebraisk ligningsløsning fra Cardano til Cauchy
- et studie af kombinationers, permutationers samt invariansbegrebets betydning for den algebraiske ligningsløsning *før* Gauss, Abel og Galois
Videnskabsfagsprojekt af: David Heiberg Backchi, Uffe thomas Volmer Jankvist, Neslihan Saglanmak
Vejleder: Bernhelm Booss-Bavnbek

410/02 2 projekter om modellering af influenzaepidemier
Influenzaepidemier- et matematisk modelleringsprojekt
Af: Claus Jørgensen, Christina Lohfert, Martin Mikkelsen, Anne-Louise H. Nielsen
Vejleder: Morten Blomhøj
Influenza A: Den tilbagevendende plage – et modelleringsprojekt
Af: Beth Paludan Carlsen, Christian Dahmcke, Lena Petersen, Michael Wagner
Vejleder: Morten Blomhøj

411/02 Polygonformede hydrauliske spring
Et modelleringsprojekt af: Kåre Stokvad Hansen, Ditte Jørgensen, Johan Rønby Pedersen, Bjørn Toldbod
Vejleder: Jesper Larsen

412/02 Hopfbifurkation og topologi i væskestrømning – en generel analyse samt en behandling af strømningen bag en cylinder
Et matematisk modul III professionsprojekt af. Kristine Niss, Bo Jakobsen
Vejledere: Morten Brøns, Johnny Ottesen

413/03 "Elevernes stemmer" Fysikfaget, undervisningen og lærerroller, som eleverne opfatter det i det almene gymnasium i Danmark
Af: Carl Angell, Albert Chr. Paulsen

414/03 Feltliniediagrammer En vej til forståelse?
Et 1. modul fysikprojekt af: Ditte Gundermann, Kåre Stokvad Hansen, Ulf Rørbæk Pedersen
Vejleder: Tage Emil Christensen

415/03 FYSIKFAGET I FORANDRING Læring og undervisning i fysik i gymnasiet med fokus på dialogiske processer, autenticitet og kompetenceudvikling
Ph.d.-afhandling i fysikdidaktik af: Jens Dolin

416/03 Fourier og Funktionsbegrebet
- Overgangen fra Eulers til Dirichlets funktionsbegreb
Projektrapport af: Rasmus Brauner Godiksen, Claus Jørgensen, Tony Moyer Hanberg, Bjørn Toldbod
Vejleder: Erik von Essen

417/03	The Semiotic Flora of Elementary particles By: Peder Voetmann Christiansen
418/03	Militærmatematik set med kompetencebriller 3. modul projektrapport af: Gitte Jensen og specialerapport af: Jesper Thrane Vejleder: Tine Wedege