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Introduction

As conjectured in [1], some spectral invariants of a closed partitioned manifold $M = M_0 \cup_\Sigma M_1$ with $M_0 \cap M_1 = \partial M_0 = \partial M_1 = \Sigma$ can be coded by the intersection geometry of the Cauchy data spaces along the partitioning hypersurface Σ of associated differential operators. The Yoshida–Nicolaescu Formula belongs to this program. It was proved in [**16**] in dimension 3 and subsequently generalized in [**12**] and modified by several authors (see e.g. [**2**], [**3**], [**7**], [**8**]). It expresses the spectral flow of a family of Dirac operators with the same principal symbol but continuously varying connections by the Maslov intersection index of the Cauchy data spaces.

A closer look at the presently available spectral flow formulas shows that they differ quite a bit in the kind and strength of the underlying assumptions and the substance of the claims made. It seems to us that our framework (based on standard functional analysis involving only elementary distribution theory) permits a wider applicability than rather deep means (like the symbolic calculus and approximation theory).

First of all, the Cauchy data spaces are treated in slightly different ways. On one side, the Cauchy data spaces are established as L^2–closures of smooth

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sections over the partitioning hypersurface Σ, coming from the restriction to the boundary of all smooth solutions over one of the parts M_j of the partitioned manifold M. In the case of the Dirac operator, this Cauchy data space can be represented as the range of the L^2–extension of the pseudo–differential Calderón projection and established as a Lagrangian subspace of the symplectic Hilbert space $L^2(-\Sigma)\dot{+}L^2(\Sigma)$ (for the somewhat delicate details see e.g. [4], pp. 75–104).

On the other side, the Cauchy data spaces can be established as subspaces of the symplectic Hilbert space $\boldsymbol{\beta} := D_{\max}/D_{\min}$ of natural boundary values, i.e. the boundary values of sections belonging to the maximal domain $D_{\max}$ of the operator (so in [**2**], [**3**], see also [**11**]). One can embed $\boldsymbol{\beta}$ as a non–closed subspace into the distribution space $H^{-1/2}(-\Sigma)\dot{+}H^{-1/2}(\Sigma)$. This treatment of the Cauchy data spaces is independent of pseudo–differential analysis and has several conceptual and technical advantages: the arguments work for any symmetric elliptic differential operator of first order which satisfies a weak unique continuation property and admits an extension defining a self–adjoint Fredholm operator. Moreover, no product structures are required near the boundary. Finally, the space $\boldsymbol{\beta}$ is basic and most suitable for a short and general proof of the closedness and the Lagrangian property of the Cauchy data spaces and the continuity of the corresponding transformation from symmetric operators to Lagrangian subspaces. As observed already in [**14**], establishing this continuity is the crucial step in proving spectral flow formulas.

In [**2**], this $\boldsymbol{\beta}$–program was carried out by standard functional analysis arguments and without assuming regularity at the endpoints or differentiability of the curve of operators, or other supplementary technical assumptions. The result was a quite general spectral flow formula, and the present paper is a continuation of [**2**]. We show, also by standard arguments, that an additional continuous transformation can be obtained, at least in the case when all metric structures are product near the separating (or non–separating) hypersurface, namely from the Cauchy data spaces of the $\boldsymbol{\beta}$–theory into the 'conventional' L^2–theory. This gives a slight generalization and a new proof of the Yoshida–Nicolaescu Formula.

In Section 1 we address a fairly general situation in symplectic functional analysis and prove a 'criss–cross' reduction theorem for the Maslov index. In Section 2 we give a condensed version of the General Spectral Flow Formula of [**2**]. In Section 3 we connect the two preceding sections and give a new proof of the Yoshida–Nicolaescu Formula. In the Appendix we correct an erroneous description in [**2**], §§ 1.1-1.2, and add a lemma to be inserted at the end of § 1.2 in [**2**].

1. Criss-Cross Reduction of the Maslov Index

Let $\boldsymbol{\beta}$ and L be symplectic Hilbert spaces with symplectic forms ω_β and ω_L, respectively. Let

$$\boldsymbol{\beta} = \boldsymbol{\beta}_- \dot{+} \boldsymbol{\beta}_+ \quad \text{and} \quad L = L_- \dot{+} L_+ \tag{1.1}$$

be direct sum decompositions by transversal (not necessarily orthogonal) pairs of Lagrangian subspaces. We assume that there exist continuous, injective mappings

$$i_- : \boldsymbol{\beta}_- \longrightarrow L_- \quad \text{and} \quad i_+ : L_+ \longrightarrow \boldsymbol{\beta}_+ \tag{1.2}$$

with dense images and which are compatible with the symplectic structures, i.e.

$$\omega_L(i_-(x), a) = \omega_\beta(x, i_+(a)) \quad \text{for all } a \in L_+ \text{ and } x \in \boldsymbol{\beta}_- . \tag{1.3}$$

Let λ_0 be a fixed Lagrangian subspace of $\boldsymbol{\beta}$. We denote the Fredholm Lagrangian Grassmannian of λ_0 by $\mathcal{FL}_{\lambda_0}(\boldsymbol{\beta})$. It is the space of all Lagrangian subspaces of $\boldsymbol{\beta}$ which make a Fredholm pair with λ_0. Its topology is defined by the operator norm of the projections. Its fundamental group is $\mathbf{Z}$, and the mapping of the loops in $\mathcal{FL}_{\lambda_0}(\boldsymbol{\beta})$ onto $\mathbf{Z}$ is given by the Maslov index. It is an intersection index of the loop with the Maslov cycle

$$\mathcal{M}_{\lambda_0}(\boldsymbol{\beta}) := \{\lambda \in \mathcal{FL}_{\lambda_0}(\boldsymbol{\beta}) \mid \lambda \cap \lambda_0 \neq \{0\}\}.$$

By functional analytical means, the definition of the Maslov index can be extended unambiguously to all continuous curves in $\mathcal{FL}_{\lambda_0}(\boldsymbol{\beta})$ (see **[2]** and **[3]**, inspired by **[15]**; we refer to **[6]** for various aspects of the Maslov index and also to **[9]**, **[10]** for a cohomological treatment).

We are going to prove the following

THEOREM 1.1. *Under the assumptions* (1.1), (1.2), *and* (1.3),
(a) *we have a continuous mapping*

$$\tau : \mathcal{FL}_{\boldsymbol{\beta}_-}(\boldsymbol{\beta}) \longrightarrow \mathcal{FL}_{L_-}(L)$$

(b) *which maps the Maslov cycle of $\boldsymbol{\beta}_-$ into the Maslov cycle of L_- and*
(c) *preserves the Maslov index*

$$\mathbf{mas}\,(\{\lambda_t\}_{t\in[0,1]}, \boldsymbol{\beta}_-) = \mathbf{mas}\,(\{\tau(\lambda_t)\}_{t\in[0,1]}, L_-)$$

for any continuous curve $[0,1] \ni t \mapsto \lambda_t \in \mathcal{FL}_{\boldsymbol{\beta}_-}(\boldsymbol{\beta})$.

We prove Theorem 1.1 in a series of small lemmata.

1.1. Definition of the mapping τ. We consider the direct sum

$$\mathcal{D} := \boldsymbol{\beta}_+ \oplus L_- ,$$

where $\boldsymbol{\beta}$ and L are identified with subspaces of $\mathcal{D}$. Then we define the mapping τ simply by

$$\tau(\lambda) := \lambda \cap L \quad \text{for } \lambda \in \mathcal{FL}_{\boldsymbol{\beta}_-}(\boldsymbol{\beta}). \tag{1.4}$$

Clearly

$$\tau(\lambda)=\{i_-(a)+x \mid a\in\boldsymbol{\beta}_-,\ x\in L_+ \text{ such that } i_+(x)+a\in\lambda\}. \tag{1.5}$$

To prove that range$(\tau)\subset\mathcal{FL}_{L_-}(L)$ and that τ is continuous, we introduce an alternative description of $\tau(\lambda)$ in terms of bounded operators associated to λ. For a given $\lambda\in\mathcal{FL}_{\boldsymbol{\beta}_-}(\boldsymbol{\beta})$, we fix a direct sum decomposition

$$\lambda=(\lambda\cap\boldsymbol{\beta}_-)\dot{+}\mu$$

with a suitable closed μ. Let $\pi_+:\boldsymbol{\beta}\to\boldsymbol{\beta}_+$ denote the projection along $\boldsymbol{\beta}_-$. Then to claim that $(\lambda,\boldsymbol{\beta}_-)$ is a Fredholm pair is equivalent to claiming that the projection $\pi_\lambda:=\pi_+|_\lambda:\lambda\to\boldsymbol{\beta}_+$ is a Fredholm operator. So we deduce that $F_\lambda:=\pi_+(\lambda)=\pi_\lambda(\mu)$ is closed. For later use we notice that

$$\dim\boldsymbol{\beta}_+/F_\lambda\ =\ \dim(\boldsymbol{\beta}_+\dot{+}\boldsymbol{\beta}_-)\,/\,(\lambda+\boldsymbol{\beta}_-)\ =\ \dim\lambda\cap\boldsymbol{\beta}_-\ <+\infty. \tag{1.6}$$

By the injectivity of $\pi_+|_\mu$ we can write μ as the graph of a uniquely determined bounded operator

$$f_\mu:F_\lambda\longrightarrow\boldsymbol{\beta}_-\,.$$

Then we rewrite

$$\tau(\lambda)=i_-(\lambda\cap\boldsymbol{\beta}_-)+\mathrm{graph}(\varphi_\lambda)\,, \tag{1.7}$$

where

$$\varphi_\lambda:\ \begin{array}{ccc} i_+^{-1}(F_\lambda) & \longrightarrow & L_- \\ x & \mapsto & i_-\circ f_\mu\circ i_+(x) \end{array}.$$

Since φ_λ is bounded and its domain is closed in L, its graph is also closed in L, and so is $\tau(\lambda)$ which differs from graph(φ_λ) only by a space of finite dimension. But moreover:

LEMMA 1.2. *For each $\lambda\in\mathcal{FL}_{\boldsymbol{\beta}_-}(\boldsymbol{\beta})$, we have $\tau(\lambda)\in\mathcal{FL}_{L_-}(L)$.*

PROOF. In the exact sequence

$$\begin{array}{ccccccccc} 0 & \longrightarrow & F_\lambda & \hookrightarrow & \boldsymbol{\beta}_+ & \xrightarrow{p} & \boldsymbol{\beta}_+/F_\lambda & \longrightarrow & 0 \\ & & & & \uparrow i_+ & & & & \\ & & & & L_+ & & & & \end{array}$$

the range of i_+ is dense in $\boldsymbol{\beta}_+$ and $\dim\boldsymbol{\beta}_+/F_\lambda<+\infty$. So, the map $p\circ i_+$ is surjective. We therefore have

$$\boldsymbol{\beta}_+=F_\lambda+i_+(L_+) \tag{1.8}$$

and $\ker(p\circ i_+)=i_+^{-1}(F_\lambda)$. It follows that

$$L_+/i_+^{-1}(F_\lambda)\cong\big(F_\lambda+i_+(L_+)\big)\,/\,F_\lambda=\boldsymbol{\beta}_+\,/\,F_\lambda\,. \tag{1.9}$$

This implies that the closed subspace $i_+^{-1}(F_\lambda)$ of L_+ is of finite codimension.

Now let $x, y \in i_+^{-1}(F_\lambda)$ and set $a := i_+(x)$ and $b := i_+(y)$. Then

$$\begin{aligned}\omega_L(x+\varphi_\lambda(x), y+\varphi_\lambda(y)) &= \omega_L(x,\varphi_\lambda(y)) + \omega_L(\varphi_\lambda(x), y)\\ &= \omega_{\boldsymbol\beta}(a, f_\mu(b)) + \omega_{\boldsymbol\beta}(f_\mu(a), b)\\ &= \omega_{\boldsymbol\beta}(a+f_\mu(a), b+f_\mu(b)) \;=\; 0\end{aligned}$$

by the compatibility condition (1.3) and the isotropy of μ. So $\operatorname{graph}\varphi_\lambda$ is isotropic. Moreover, we have

$$\dim\bigl(\operatorname{graph}(\varphi_\lambda)^0 / \operatorname{graph}(\varphi_\lambda)\bigr) = 2\dim\bigl(L_+/i_+^{-1}(F_\lambda)\bigr) = 2\dim\bigl(\boldsymbol\beta_+/F_\lambda\bigr).$$

Here $\operatorname{graph}(\varphi_\lambda)^0$ denotes the annihilator of $\operatorname{graph}(\varphi_\lambda)$ with respect to the symplectic form ω_L. Clearly,

$$i_-(\lambda\cap\boldsymbol\beta_-) \subset \operatorname{graph}(\varphi_\lambda)^0 \quad\text{and}\quad i_-(\lambda\cap\boldsymbol\beta_-)\cap\operatorname{graph}(\varphi_\lambda) = \{0\}.$$

Further, we have

$$\dim i_-(\lambda\cap\boldsymbol\beta_-) = \dim(\lambda\cap\boldsymbol\beta_-) = \dim\bigl(\boldsymbol\beta_+/F_\lambda\bigr),$$

since the Fredholm index of the Lagrangian pair $(\lambda, \boldsymbol\beta_-)$ vanishes. From this dimension examination we see that adding $i_-(\lambda\cap\boldsymbol\beta_-)$ to $\operatorname{graph}(\varphi_\lambda)$ makes $\tau(\lambda)$ a Lagrangian subspace of L. It also follows that $\tau(\lambda)\cap L_- = i_-(\lambda\cap\boldsymbol\beta_-)$ is of finite dimension and that $L_- + \tau(\lambda) = L_- + \operatorname{graph}(\varphi_\lambda)$ is of finite codimension, hence $(\tau(\lambda), L_-)$ is a Fredholm pair in L. □

REMARK 1.3. From equation (1.7) it particularly follows that

$$\tau\bigl(\mathcal{M}_{\boldsymbol\beta_-}(\boldsymbol\beta)\bigr) \;\subset\; \mathcal{M}_{L_-}(L)\,.$$

1.2. The continuity of τ. We fix a closed $W \subset L_-$ with $\dim L_-/W < +\infty$ and choose a Lagrangian subspace θ of $\boldsymbol\beta$ with $\theta \pitchfork \boldsymbol\beta_+$ and $W_{\boldsymbol\beta} \subset \theta \subset W_{\boldsymbol\beta}^0$, where $W_{\boldsymbol\beta} := i_-^{-1}(W)$. Here "$\pitchfork$" means that the two subspaces intersect transversally. We notice that

$$W^0 = L_- \dot{+} (L_+ \cap W^0)$$

and, correspondingly,

$$W_{\boldsymbol\beta}^0 = \boldsymbol\beta_- \dot{+} (\boldsymbol\beta_+ \cap W_{\boldsymbol\beta}^0).$$

Next we exploit that the injection $i_- : \boldsymbol\beta_- \to L_-$ has a dense range. Similar to the short exact sequence at the beginning of the proof of Lemma 1.2, we have

$$\begin{array}{ccccccccc} 0 & \longrightarrow & W & \hookrightarrow & L_- & \xrightarrow{q} & L_-/W & \longrightarrow & 0 \\ & & & & \uparrow i_- & & & & \\ & & & & \boldsymbol\beta_- & & & & \end{array}$$

We deduce, as above,

$$L_- = W + i_-(\boldsymbol\beta_-) \quad\text{and}\quad \dim\bigl(\boldsymbol\beta_-/W_{\boldsymbol\beta}\bigr) = \dim\bigl(L_-/W\bigr).$$

Moreover, we have

$$i_+(W^0\cap L_+) = W_{\boldsymbol\beta}^0 \cap \boldsymbol\beta_+\,. \tag{1.10}$$

To show the inclusion "$\subset$", we consider any $x \in W^0 \cap L_+$. Then $\omega_L(x,y) = 0$ for all $y \in W$, and hence also for $y = i_-(z)$ for any $z \in W_\beta = i_-^{-1}(W)$. Now, by (1.3),

$$0 = \omega_L(x, i_-(z)) = \omega_\beta(i_+(x), z).$$

So,

$$i_+(x) \in W_\beta^0 \cap \beta_+ .$$

We deduce the identity "=" by the following dimension examination:

$$2\dim(W^0 \cap L_+) = \dim(W^0/W) = \dim(W_\beta^0/W_\beta) = 2\dim(W_\beta^0 \cap \beta_+).$$

Equation (1.10) permits us to rewrite

$$W_\beta^0 = i_+(L_+ \cap W^0) \dot{+} \beta_-$$

and to define

$$\begin{array}{cccc} i_W : & W_\beta^0 & \longrightarrow & W^0 \\ & i_+(x) + z & \mapsto & x + i_-(z) \end{array}$$

for $x \in L_+ \cap W^0$ and $z \in \beta_-$. We obtain a new splitting of the symplectic Hilbert spaces:

PROPOSITION 1.4. *The space $\eta := i_W(\theta) + W$ is a Lagrangian subspace of L and the mapping $i_W|_\theta : \theta \to \eta$ has a dense image. Further, we have new direct sum decompositions*

$$\beta = \beta_+ \dot{+} \theta \quad \text{and} \quad L = L_+ \dot{+} \eta$$

which are compatible with regard to the symplectic forms ω_L and ω_β (similar to (1.3)).

PROOF. Clearly

$$W \subset \eta = i_W(\theta) + W \subset W^0 \quad \text{and} \quad \dim W^0/W < +\infty,$$

hence η is also closed and isotropic. We notice that the mapping $q \circ i_W|_\theta$ is surjective with $\ker q \circ i_W|_\theta = W_\beta$, where $q : \eta \to \eta/W$ denotes the projection. So,

$$W_\beta^0/W_\beta \supset \theta/W_\beta \cong \eta/W \subset W^0/W.$$

Then, from the dimension examination

$$\dim(W_\beta^0/W_\beta) = \dim(W^0/W),$$

it follows that η is a Lagrangian subspace of L.

To see that the range of $i_W|_\theta$ is dense in η we recall that the mapping $i_W : W_\beta^0 \to W^0$ is an isomorphism $W_\beta^0 \cap \beta_+ \cong W^0 \cap L_+$ on the first finite–dimensional component, and it is equal to the dense embedding $i_- : \beta_- \to L_-$ on the second component, hence its restriction to θ has a dense range in η.

The new direct sum decompositions and the compatibility of the symplectic forms follow at once. □

The preceding proposition permits a further simplification of the graph representation of $\tau(\lambda)$, obtained in (1.7):

COROLLARY 1.5. *For any* $\lambda \in \mathcal{FL}_{\beta_-}(\boldsymbol{\beta})$ *with* $\lambda \pitchfork \theta$, *we have*

$$\tau(\lambda) = \operatorname{graph}(i_W \circ f_\lambda \circ i_+)\,,$$

where $f_\lambda : \boldsymbol{\beta}_+ \to \theta$ *such that* $\operatorname{graph} f_\lambda = \lambda$.

Now we can prove the continuity of τ and the invariance of the Maslov index under τ. We set

$$\mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)} := \{\mu \in \mathcal{FL}_{W_\beta}(\boldsymbol{\beta}) \mid \mu \cap W_\beta = \{0\}\}.$$

Then we have an open covering

$$\bigcup_W \mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)} = \mathcal{FL}_{\beta_-}(\boldsymbol{\beta})\,,$$

where the union is taken over all closed subspaces $W \subset L_-$ of finite codimension. It follows that $\tau(\mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)}) \subset \mathcal{FL}_W(L)^{(0)}$. Thus, we have established restrictions

$$\tau_W : \mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)} \longrightarrow \mathcal{FL}_W(L)^{(0)}$$

of τ for each W.

We fix a W. We denote the space of Lagrangian subspaces in a finite-dimensional symplectic space by $\operatorname{Lag}(\cdot)$ and define the reduction map

$$\begin{array}{rccc} \rho_{W_\beta}: & \mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)} & \longrightarrow & \operatorname{Lag}\big({W_\beta}^0/W_\beta\big) \\ & \lambda & \mapsto & \big(\lambda \cap {W_\beta}^0 + W_\beta\big)\,/\,W_\beta \end{array}$$

Then, for each Lagrangian subspace θ in $\boldsymbol{\beta}$ with $\theta \supset W_\beta$ and $\theta \pitchfork \boldsymbol{\beta}_+$ (as before), the set

$$U(W_\beta,\theta) := \{\Lambda \in \operatorname{Lag}(W_\beta^0/W_\beta) \mid \Lambda \pitchfork \theta/W_\beta\}$$

is open in $\operatorname{Lag}(W_\beta^0/W_\beta)$ and we have an open covering

$$\bigcup_{\theta \pitchfork \boldsymbol{\beta}_+} \rho_{W_\beta}^{-1}\Big(U\big(W_\beta,\theta\big)\Big) \;=\; \mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)}\,.$$

By Corollary 1.5, the mapping τ is continuous on each $\rho_{W_\beta}^{-1}(U(W_\beta,\theta))$, and so it is continuous on the whole $\mathcal{FL}_{\beta_-}(\boldsymbol{\beta})$. This concludes the proof of (a) of Theorem 1.1.

Moreover, we have

$$\tau\big(\rho_{W_\beta}^{-1}(U(W_\beta,\theta))\big) \subset \rho_W^{-1}\Big(U\big(W,\tau_W(\theta)\big)\Big),$$

where the reduction map $\rho_W : \mathcal{FL}_W(L)^{(0)} \to \operatorname{Lag}\big(W^0/W\big)$ is defined correspondingly to ρ_{W_β}. We denote by

$$\overline{\tau_W} : \operatorname{Lag}\big(W_\beta^0\,/\,W_\beta\big) \longrightarrow \operatorname{Lag}\big(W^0/W\big)$$

the mapping naturally induced by τ_W. Then we have the following finite-dimensional symplectic reduction:

PROPOSITION 1.6. *The diagram*

$$
\begin{tikzcd}
\mathcal{FL}_{\beta_-}(\boldsymbol{\beta}) \arrow[d, "\tau"] & \mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)} \arrow[l, hook'] \arrow[r, "\rho_{W_\beta}"] \arrow[d, "\tau_W"] & \mathrm{Lag}(W_\beta^0/W_\beta) \arrow[d, "\cong\ \overline{\tau_W}"] \\
\mathcal{FL}_{L_-}(L) & \mathcal{FL}_W(L)^{(0)} \arrow[l, hook'] \arrow[r, "\rho_W"'] & \mathrm{Lag}(W^0/W)
\end{tikzcd} \tag{1.11}
$$

is commutative.

For any continuous curve $\{\lambda_t\}_{t\in[0,1]} \in \mathcal{FL}_{\beta_-}(\boldsymbol{\beta})$, we can find a closed subspace $W \subset L_-$ of finite codimension such that the whole curve is contained in $\mathcal{FL}_{W_\beta}(\boldsymbol{\beta})^{(0)}$, respectively $\{\tau(\lambda_t)\}_{t\in[0,1]}$ is contained in $\mathcal{FL}_W(L)^{(0)}$. By finite symplectic reduction we obtain at once:

COROLLARY 1.7. *The Maslov index coincides under the transformations τ, τ_W, and $\overline{\tau_W}$ for loops and also for paths.*

NOTE. For paths, the Maslov index depends on the choice of the Maslov cycle. In $\mathrm{Lag}(W^0/W)$, the Maslov index is taken with respect to the Maslov cycle $\mathcal{M}_{L_-/W}(W^0/W)$. Correspondingly, the Maslov index in $\mathrm{Lag}(W_\beta^0/W_\beta)$ is taken with respect to the Maslov cycle $\mathcal{M}_{\beta_-/W_\beta}(W_\beta^0/W_\beta)$. Note that the specified Maslov cycle in $\mathrm{Lag}(W_\beta^0/W_\beta)$ is mapped onto the specified Maslov cycle in $\mathrm{Lag}(W^0/W)$ by $\overline{\tau_W}$.

Proposition 1.6 and Corollary 1.7 together with Remark 1.3 give the proof of (b) and (c) of Theorem 1.1.

2. The General Spectral Flow Formula

Let $\mathcal{H}$ be a real separable Hilbert space and A an (unbounded) closed symmetric operator defined on the domain $D_{\min}$ which is supposed to be dense in $\mathcal{H}$. Let A^* denote its adjoint operator with domain $D_{\max}$. We have that $A^*|_{D_{\min}} = A$ and that A^* is the maximal closed extension of A in $\mathcal{H}$.

We form the space $\boldsymbol{\beta}$ of *natural boundary values* with the *natural trace map* γ in the following way:

$$
\begin{array}{ccc}
D_{\max} & \xrightarrow{\gamma} & D_{\max}/D_{\min} =: \boldsymbol{\beta} \\
x & \mapsto & \gamma(x) = [x] := x + D_{\min}\,.
\end{array}
$$

The space $\boldsymbol{\beta}$ becomes a symplectic Hilbert space with the scalar product induced by the graph norm

$$(x,y)_G := (x,y) + (\mathrm{A}\,x, \mathrm{A}\,y) \tag{2.1}$$

and the symplectic form given by Green's form

$$\omega([x],[y]) := (\mathrm{A}\,x, y) - (x, \mathrm{A}\,y) \qquad \text{for } [x],[y] \in \boldsymbol{\beta}. \tag{2.2}$$

We define the *natural Cauchy data space* $\Lambda := \gamma(\ker \mathrm{A}^*)$. It is a Lagrangian subspace of $\boldsymbol{\beta}$ under the assumption that A admits at least one self-adjoint Fredholm extension A_D. Actually, we shall assume a little more, namely that

A has a self–adjoint extension A_D with compact resolvent. Then $(\Lambda, \gamma(D))$ is a Fredholm pair of subspaces of $\boldsymbol{\beta}$, i.e. $\Lambda \in \mathcal{FL}_{\gamma(D)}(\boldsymbol{\beta})$.

We consider a continuous curve $\{C_t\}_{t\in[0,1]}$ in the space of bounded self–adjoint operators on $\mathcal{H}$ and assume that the operators $A^* + C_t - r$ have no 'inner solutions', i.e. satisfy the *weak unique continuation property* (UCP)

$$\ker(A^* + C_t - r) \cap D_{\min} = \{0\}$$

for $t \in [0,1]$ and $|r| < \varepsilon_0$ with $\varepsilon_0 > 0$.

Clearly, the domains $D_{\max}$ and $D_{\min}$ are unchanged by the perturbation C_t for any t. So, $\boldsymbol{\beta}$ does not depend on the parameter t. Moreover, the symplectic form ω is invariantly defined on $\boldsymbol{\beta}$ and so also independent of t. It follows (see [**2**], Theorem 3.9) that the curve $\{\Lambda_t := \gamma\big(\ker(A^* + C_t)\big)\}$ is continuous in $\mathcal{FL}_{\gamma(D)}(\boldsymbol{\beta})$.

Given this, the family $\{A_D + C_t\}$ can be considered at the same time in the spectral theory of self–adjoint operators, defining a spectral flow, and in the symplectic category, defining a Maslov index. Under the preceding assumptions, the main result obtainable at that level is the following general spectral flow formula (proved in [**2**], Theorem 5.1):

THEOREM 2.1. *Let A_D be a self–adjoint extension of A with compact resolvent and let $\{A_D + C_t\}$ be a family satisfying the UCP assumption. Then*

$$\mathbf{sf}\{A_D + C_t\} \quad = \quad \mathbf{mas}\ (\{\Lambda_t\}, \gamma(D))\,.$$

3. A Proof of the Yoshida–Nicolaescu Formula

Let M be a closed connected smooth Riemannian manifold and let $\Sigma \subset M$ be a hypersurface. Let

$$\big\{\mathrm{A}_t := \mathrm{A}_0 + C_t : C^\infty(M;S) \longrightarrow C^\infty(M;S)\big\}_{0\le t\le 1}$$

be a continuous family of symmetric elliptic differential operators of first order with the same principal symbol, acting on sections of a real bundle S over M. The variation of the fixed operator A_0 is given by a continuous family $\{C_t\}_{t\in[0,1]}$ of smooth symmetric bundle homomorphisms. We assume that the normal bundle of the hypersurface Σ is orientable and that all metric structures of M and S are *product* in a collar neighbourhood $\mathcal{N} = (-1,1) \times \Sigma$ of Σ. We have

$$\mathrm{A}_t = \sigma\Big(\frac{\partial}{\partial\tau} + B_t\Big) \quad \text{on } \mathcal{N}, \tag{3.1}$$

where τ denotes the normal coordinate, σ is orthogonal (assuming $\sigma^2 = -\mathrm{Id}$), $\sigma B_t = -B_t\sigma$, and B_t is a self–adjoint elliptic differential operator over Σ, called the *tangential operator*. Here the point of the product structure is that then σ and B_t do not depend on the normal variable.

We cut the manifold at Σ and attach a copy of Σ to each side. So, we obtain a new manifold $M_\sharp$ with boundary $\Sigma_0 \sqcup \Sigma_1 = (-\Sigma) \sqcup \Sigma$. Then the $\boldsymbol{\beta}$–space of

$M_\sharp$, being a $C^\infty(\partial M_\sharp)$–module, splits according to the connected components of $\partial M_\sharp$ and we obtain two symplectic Hilbert spaces

$$\boldsymbol{\beta} := \boldsymbol{\beta}_0 \dot{+} \boldsymbol{\beta}_1 \qquad \text{and} \qquad L := L_0 \dot{+} L_1$$

with

$$\gamma = \gamma_0 \oplus \gamma_1 : D_{\max}(M_\sharp) \longrightarrow \boldsymbol{\beta}_0 \dot{+} \boldsymbol{\beta}_1$$

and $L_j = L^2(\pm\Sigma; S|_{\pm\Sigma})$ for $j = 0, 1$. We note that L_0 and L_1 are naturally identified as Hilbert spaces, but their symplectic forms have opposite signs. In $\boldsymbol{\beta}$ we have a Lagrangian subspace

$$\Delta_{\boldsymbol{\beta}} := \{(x, x) \in \boldsymbol{\beta}_0 \dot{+} \boldsymbol{\beta}_1 \mid \gamma_0(s) = x = \gamma_1(s) \text{ for } s \in H^1(M)\}, \tag{3.2}$$

where $H^1(M)$ denotes the first order Sobolev space. Since M is closed, the operator A_t on $H^1(M)$ is a self–adjoint Fredholm operator. It can be identified with the operator $A^\sharp_{t,D} = A^\sharp_{0,D} + C_t$ over the new manifold $M_\sharp$ with domain

$$D := \{s \in D_{\max}(M_\sharp) \mid \gamma_0(s) = \gamma_1(s)\},$$

i.e. it can be considered as a global self–adjoint boundary problem with $\gamma(D) = \Delta_{\boldsymbol{\beta}}$. (Actually, D coincides with $H^1(M)$).

We set $\boldsymbol{\beta}_- := \Delta_{\boldsymbol{\beta}}$. In L, we have a Lagrangian subspace

$$L_- := \text{ the diagonal of } L^2(-\Sigma; S|_{-\Sigma}) \dot{+} L^2(\Sigma; S|_\Sigma). \tag{3.3}$$

Clearly, the embedding $\boldsymbol{\beta}_- \hookrightarrow L_-$ is bounded and dense.

To define suitable complementary Lagrangian subspaces $\boldsymbol{\beta}_+$ and L_+, we must fall back on a spectral resolution $\{\varphi_k, \lambda_k\}$ of $L^2(\Sigma)$ by eigensections of B_0. (Here and in the following we do not mention the bundle S). For simplicity, we assume $\ker B_0 = \{0\}$. Otherwise we must decompose the finite–dimensional symplectic vector space $\ker B_0$ into two Lagrangian subspaces and add these spaces to the half parts defined by the spectral cut at 0.

Following [3], Proposition 7.15 (for related results see also [5] and [**11**]), we decompose

$$\boldsymbol{\beta}_0 = \boldsymbol{\beta}^0_- \dot{+} \boldsymbol{\beta}^0_+ \ \text{ and } \ \boldsymbol{\beta}_1 = \boldsymbol{\beta}^1_- \dot{+} \boldsymbol{\beta}^1_+,$$

where

$$\boldsymbol{\beta}^1_- := \overline{[\{\varphi_k\}_{k<0}]}^{H^{\frac12}(\Sigma)} \ \text{ and } \ \boldsymbol{\beta}^1_+ := \overline{[\{\varphi_k\}_{k>0}]}^{H^{-\frac12}(\Sigma)}$$

and

$$\boldsymbol{\beta}^0_- := \overline{[\{\varphi_k\}_{k<0}]}^{H^{-\frac12}(\Sigma)} \ \text{ and } \ \boldsymbol{\beta}^0_+ := \overline{[\{\varphi_k\}_{k>0}]}^{H^{\frac12}(\Sigma)}.$$

In a similar way, we decompose

$$L_0 = L^0_- \dot{+} L^0_+ \ \text{ and } \ L_1 = L^1_- \dot{+} L^1_+$$

with

$$L^1_- := \overline{[\{\varphi_k\}_{k<0}]}^{L^2(\Sigma)} \ \text{ and } \ L^1_+ := \overline{[\{\varphi_k\}_{k>0}]}^{L^2(\Sigma)}$$

and

$$L^0_- := \overline{[\{\varphi_k\}_{k<0}]}^{L^2(\Sigma)} \text{ and } L^0_+ := \overline{[\{\varphi_k\}_{k>0}]}^{L^2(\Sigma)} .$$

Rewriting

$$\boldsymbol{\beta} = \boldsymbol{\beta}_0 \dot{+} \boldsymbol{\beta}_1 = \boldsymbol{\beta}^0_- \dot{+} \boldsymbol{\beta}^0_+ \dot{+} \boldsymbol{\beta}^1_- \dot{+} \boldsymbol{\beta}^1_+ , \tag{3.4}$$

we obtain

$$\boldsymbol{\beta}_- = \Delta_{\boldsymbol{\beta}} = \{(a,b,a,b) \mid (a,b) = \gamma_0(s) = \gamma_1(s) \text{ with } s \in H^1(M)\}.$$

Correspondingly, we define

$$\boldsymbol{\beta}_+ := \{(x,0,0,y) \mid x \in \boldsymbol{\beta}^0_-,\ y \in \boldsymbol{\beta}^1_+\}$$

and

$$L_+ := \{(u,0,0,v) \mid u \in L^0_-,\ v \in L^1_+\}.$$

Clearly, we have $L = L_- \dot{+} L_+$, a dense bounded embedding $L_+ \hookrightarrow \boldsymbol{\beta}_+$, and $\Delta_{\boldsymbol{\beta}} \cap \boldsymbol{\beta}_+ = \{0\}$. From the decomposition (3.4) we have also that $\Delta_{\boldsymbol{\beta}} + \boldsymbol{\beta}_+ = \boldsymbol{\beta}$. Then, as explained above in Section 2, the $\boldsymbol{\beta}$–theory gives the continuity of the family of Cauchy data spaces $\{\Lambda_t\}$ (of the continuous operator family $\{A_t\}$, considered over $M_\sharp$). They are all Lagrangian subspaces of $\boldsymbol{\beta}$ and make Fredholm pairs with $\boldsymbol{\beta}_- = \Delta_{\boldsymbol{\beta}} = \gamma(H^1(M))$. From the General Spectral Flow Formula (here Theorem 2.1) and by the criss–cross reduction of the Maslov index (Theorem 1.1), we obtain

$$\mathbf{sf}\,\{A_0 + C_t\} = \mathbf{sf}\,\{A^\sharp_{0,D} + C_t\} \tag{3.5}$$

$$\overset{\text{Th.2.1}}{=} \mathbf{mas}\ (\{\Lambda_t\}, \Delta_{\boldsymbol{\beta}}) \tag{3.6}$$

$$\overset{\text{Th.1.1}}{=} \mathbf{mas}\ (\{\Lambda_t \cap L\}, L_-)\,. \tag{3.7}$$

Note that here

$$\Lambda_t \cap L = \{s|_{(-\Sigma)\sqcup\Sigma} \mid s \in H^{\frac12}(M_\sharp) \text{ and } \mathrm{A}_t(s) = 0 \text{ in } M_\sharp \setminus \partial M_\sharp\},$$

i.e. it coincides with the 'conventional' L^2–definition of the Cauchy data spaces.

In the particular case of a *partitioned* manifold

$$M = M_0 \cup_\Sigma M_1 \text{ with } \Sigma = \partial M_0 = \partial M_1 = M_0 \cap M_1 ,$$

not only $\boldsymbol{\beta}$ and L split but also the Cauchy data spaces split

$$\Lambda_t = \Lambda^0_t \dot{+} \Lambda^1_t ,$$

according to the splitting of M into two parts. So, we obtain from (3.7) the true Yoshida–Nicolaescu Formula, though without any assumptions about the regularity at the endpoints or the differentiability of the curve:

THEOREM 3.1.

$$\begin{aligned}\mathbf{sf}\,\{A_0 + C_t\} &= \mathbf{mas}\ \left(\{\Lambda^0_t \cap L^2(-\Sigma) \dot{+} \Lambda^1_t \cap L^2(\Sigma)\}, L_-\right)\\ &=: \mathbf{mas}\ \left(\{\Lambda^0_t \cap L^2(-\Sigma)\}, \{\Lambda^1_t \cap L^2(\Sigma)\}\right),\end{aligned}$$

where the last expression is just a rewriting of the Maslov index of Fredholm pairs of two curves.

REMARK 3.2. (*a*) We notice that the subspaces $\boldsymbol{\beta}_-$ and L_- are defined independently of a reference tangential operator, here B_0, but the choice of any other reference operator B_t would have given a different decomposition, though not a different result.
(*b*) We would like to emphasize that the Yoshida–Nicolaescu formulas in the literature always assume product structures near Σ, whereas the general spectral flow formulas, as proved in [**2**] and expressed in [**3**] in $\boldsymbol{\beta} \subset H^{-1/2}(\Sigma)$, do not require product structures near Σ. However, we also need product structures near Σ, to apply our criss–cross reduction theorem and to transform the general spectral flow formulas, expressed in 'natural' distribution spaces, into 'conventional' L^2–formulas. Possibly, the criss–cross reduction theorem may provide one explanation for the need of product structures for L^2–formulas.
(*c*) We also want to point to a misprint in [**3**], p. 74 (after Equation (7.13)), where it must read that '$\gamma(S)\cap L^2(\widehat{\Sigma})$ is not closed in $L^2(\widehat{\Sigma})$' instead of '$\gamma(S)\cap L^2(\Sigma)$ is not closed in $L^2(\Sigma)$'.

Appendix A. Addendum and Corrections to [2]

Let $\mathcal{L}$ denote the space of all Lagrangian subspaces of a real separable symplectic Hilbert space $\mathcal{H}$, and let $\mathcal{L}^{\mathbf{C}}$ denote the space of all complex Lagrangian subspaces of $\mathcal{H}\otimes\mathbf{C}$. As correctly observed in [**2**], the full group $\mathcal{U}(\mathcal{H})$ of unitary operators of $\mathcal{H}$ does *not* act on $\mathcal{FL}_{\lambda_0}(\mathcal{H})$, but the reduced group $\mathcal{U}_c(\mathcal{H})$ does. It consists of unitary operators of the form $\mathrm{Id}+K$, where K is a compact operator. In [**2**], below on p. 5, we claimed that the reduced group acts transitively on $\mathcal{FL}_{\lambda_0}(\mathcal{H})$, i.e. the mapping

$$\begin{array}{rccc} \rho: & \mathcal{U}_c(\mathcal{H}) & \longrightarrow & \mathcal{FL}_{\lambda_0}(\mathcal{H}) \\ & U & \mapsto & U(\lambda_0^{\perp}) \end{array}$$

is surjective. This claim is erroneous, since the difference of the orthogonal projections onto a Fredholm pair of closed subspaces of $\mathcal{H}$ need not be of the form $\mathrm{Id}+K$, but can become an arbitrary Fredholm operator as proved in [**3**], Appendix.

However, if we replace $\mathcal{U}_c(\mathcal{H})$ by $\mathcal{U}(\mathcal{H})$, $\mathcal{FL}_{\lambda_0}(\mathcal{H})$ by $\mathcal{L}$, $\mathcal{U}_c(\mathcal{H}\otimes\mathbf{C})$ by $\mathcal{U}(\mathcal{H}\otimes\mathbf{C})$, and $\mathcal{FL}^{\mathbf{C}}_{\lambda_0\otimes\mathbf{C}}(\mathcal{H}\otimes\mathbf{C})$ by $\mathcal{L}^{\mathbf{C}}$, we have the same results as in [**2**], §§ 1.1-1.2:

The group $\mathcal{U}(\mathcal{H})$ (resp. $\mathcal{U}(\mathcal{H}\otimes\mathbf{C})$) acts transitively on $\mathcal{L}$ (resp. on $\mathcal{L}^{\mathbf{C}}$). So let

$$\begin{array}{rccc} \rho: & \mathcal{U}(\mathcal{H}) & \longrightarrow & \mathcal{L} \\ & U & \mapsto & U(\lambda_0^{\perp}) \end{array} \tag{A.1}$$

and

$$\begin{array}{rccc} \rho^{\mathbf{C}}: & \mathcal{U}(\mathcal{H}\otimes\mathbf{C}) & \longrightarrow & \mathcal{L}^{\mathbf{C}} \\ & g & \mapsto & g(\lambda_0^{\perp}\otimes\mathbf{C}) \end{array} \tag{A.2}$$

denote the mappings defined by these actions. We obtain a commutative diagram

$$\text{(A.3)} \qquad \begin{array}{ccc} \mathcal{U}(\mathcal{H}) & \xrightarrow{\tilde{\tau}} & \mathcal{U}(\mathcal{H}\otimes \mathbf{C}) \\ \downarrow{\scriptstyle \rho} & & \downarrow{\scriptstyle \rho^{\mathbf{C}}} \\ \mathcal{L} & \xrightarrow{\tau} & \mathcal{L}^{\mathbf{C}}, \end{array}$$

where $\tilde{\tau}$ denotes the complexification $U \mapsto U \otimes \mathrm{Id}$.

Correspondingly to [**2**], Proposition 1.3 we have

PROPOSITION A.1. *The mapping*

$$\rho^{\mathbf{C}} \circ \Phi : \mathcal{U}(\mathcal{H}) \longrightarrow \mathcal{L}^{\mathbf{C}}$$

is a homeomorphism, where Φ *is defined in the same way as in* [**2**], p. 7.

Now we must add the following lemma to [**2**], following Definition 1.4:

LEMMA A.2. *Let* $\lambda \in \mathcal{L}$ *and* $\lambda = U(\lambda_0^{\perp})$. *Then we have that*

$$\lambda \in \mathcal{FL}_{\lambda_0}(\mathcal{H}) \iff U\overline{U}^{-1} + \mathrm{Id} \textit{ is a Fredholm operator.}$$

PROOF. By identifying $\mathcal{H} \cong \lambda_0 \otimes \mathbf{C} \cong \lambda_0 \oplus \sqrt{-1}\,\lambda_0$, we represent U as

$$U = X + \sqrt{-1}\,Y$$

with $X, Y : \lambda_0 \to \lambda_0$. Since

$$U\overline{U}^{-1} + \overline{U}\,\overline{U}^{-1} = (U + \overline{U})\,\overline{U}^{-1} = 2X\overline{U}^{-1},$$

we have that $U\overline{U}^{-1} + \mathrm{Id}$ is a Fredholm operator, if and only if X is a Fredholm operator.

First we assume $\lambda \in \mathcal{FL}_{\lambda_0}(\mathcal{H})$. Then we know already by [**2**], Equation (1.1) that

$$\text{(A.4)} \qquad \ker X = \lambda \cap \lambda_0,$$

and we have also

$$\text{(A.5)} \qquad \lambda + \lambda_0 = \{-Y(x) + \sqrt{-1}\,X(x) + y \mid x, y \in \lambda_0\}.$$

So

$$\text{(A.6)} \qquad \mathcal{H}/(\lambda + \lambda_0) \cong \lambda_0/\operatorname{range}(X),$$

hence $\operatorname{range}(X)$ is closed and of finite codimension in λ_0.

Conversely, if X is a Fredholm operator, then also by (A.4), (A.5), and (A.6) we have $\lambda \in \mathcal{FL}_{\lambda_0}(\mathcal{H})$. □

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by: Mette Olufsen(Math-Tech), Finn Nielsen (RISØ National Laboratory), Per Føge Jensen (Herlev University Hospital), Stig Andur Pedersen (Roskilde University)

279/94 Description of a method of measuring the shear modulus of supercooled liquids and a comparison of their thermal and mechanical response functions.
af: Tage Christensen

280/94 A Course in Projective Geometry
by Lars Kadison and Matthias T. Kromann

281/94 Modellering af Det Cardiovaskulære System med Neural Pulskontrol
Projektrapport udarbejdet af:
Stefan Frello, Runa Ulsøe Johansen, Michael Poul Curt Hansen, Klaus Dahl Jensen
Vejleder: Viggo Andreasen

282/94 Parallelle algoritmer
af: Erwin Dan Nielsen, Jan Danielsen, Niels Bo Johansen

283/94 Grænser for tilfældighed
(en kaotisk talgenerator)

af: Erwin Dan Nielsen og Niels Bo Johansen

284/94 Det er ikke til at se det, hvis man ikke lige ve' det!
Gymnasiematematikkens begrundelsesproblem

En specialerapport af Peter Hauge Jensen og Linda Kyndlev

Vejleder: Mogens Niss

285/94 Slow coevolution of a viral pathogen and its diploid host
by: Viggo Andreasen and Freddy B. Christiansen

286/94 The energy master equation: A low-temperature approximation to Bässler's random walk model
by: Jeppe C. Dyre

287/94 A Statistical Mechanical Approximation for the Calculation of Time Auto-Correlation Functions
by: Jeppe C. Dyre

288/95 PROGRESS IN WIND ENERGY UTILIZATION
by: Bent Sørensen

289/95 Universal Time-Dependence of the Mean-Square Displacement in Extremely Rugged Energy Landscapes with Equal Minima
by: Jeppe C. Dyre and Jacob Jacobsen

290/95 Modellering af uregelmæssige bølger
Et 3.modul matematik projekt
af: Anders Marcussen, Anne Charlotte Nilsson, Lone Michelsen, Per Mørkegaard Hansen
Vejleder: Jesper Larsen

291/95 1st Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM
an example of using methods developed for the OECD/IEA and the US/EU fuel cycle externality study
by: Bent Sørensen

292/95 Fotovoltaisk Statusnotat 3
af: Bent Sørensen

293/95 Geometridiskussionen - hvor blev den af?
af: Lotte Ludvigsen & Jens Frandsen
Vejleder: Anders Madsen

294/95 Universets udvidelse - et metaprojekt
Af: Jesper Duelund og Birthe Friis
Vejleder: Ib Lundgaard Rasmussen

295/95 A Review of Mathematical Modeling of the Controled Cardiovascular System
By: Johnny T. Ottesen

296/95 RETIKULER den klassiske mekanik
af: Peder Voetmann Christiansen

297/95 A fluid-dynamical model of the aorta with bifurcations
by: Mette Olufsen and Johnny Ottesen

298/95 Mordet på Schrödingers kat - et metaprojekt om to fortolkninger af kvantemekanikken
af: Maria Hermannsson, Sebastian Horst, Christina Specht

Vejledere: Jeppe Dyre og Peder Voetmann Christiansen

299/95 ADAM under figenbladet - et kig på en samfundsvidenskabelig matematisk model
Et matematisk modelprojekt
af: Claus Dræby, Michael Hansen, Tomas Højgård Jensen
Vejleder: Jørgen Larsen

300/95 Scenarios for Greenhouse Warming Mitigation
by: Bent Sørensen

301/95 TOK Modellering af træers vækst under påvirkning af ozon
af: Glenn Møller-Holst, Marina Johannessen, Birthe Nielsen og Bettina Sørensen

Vejleder: Jesper Larsen

302/95 KOMPRESSORER - Analyse af en matematisk model for aksialkompressorer
Projektrapport sf: Stine Bøggild, Jakob Hilmer, Pernille Postgaard

Vejleder: Viggo Andreasen

303/95 Masterlignings-modeller af Glasovergangen
Termisk-Mekanisk Relaksation
Specialerapport udarbejdet af:

Johannes K. Nielsen, Klaus Dahl Jensen

Vejledere: Jeppe C. Dyre, Jørgen Larsen

304a/95 STATISTIKNOTER Simple binomialfordelingsmodeller
af: Jørgen Larsen

304b/95 STATISTIKNOTER Simple normalfordelingsmodeller
af: Jørgen Larsen

304c/95 STATISTIKNOTER Simple Poissonfordelingsmodeller
af: Jørgen Larsen

304d/95 STATISTIKNOTER Simple multinomialfordelingsmodeller
af: Jørgen Larsen

304e/95 STATISTIKNOTER Mindre matematisk-statistisk opslagsvær indeholdende bl.a. ordforklaringer, resuméer og tabeller
af: Jørgen Larsen

305/95 The Maslov Index:
A Functional Analytical Definition
And The Spectral Flow Formula.

By: B. Booss-Bavnbek, K. Furutani

306/95 Goals of mathematics teaching

Preprint of a chapter for the forth-comming International Handbook of Mathematics Education (Alan J.Bishop, ed)

By: Mogens Niss

307/95 Habit Formation and the Thirdness of Signs

Presented at the semiotic symposium

The Emergence of Codes and Intensions as a Basis of Sign Processes

By: Peder Voetmann Christiansen

308/95 Metaforer i Fysikken

af: Marianne Wilcken Bjerregaard, Frederik Voetmann Christiansen, Jørn Skov Hansen, Klaus Dahl Jensen Ole Schmidt

Vejledere: Peder Voetmann Christiansen og Petr Viscor

309/95 Tiden og Tanken
En undersøgelse af begrebsverdenen Matematik udført ved hjælp af en analogi med tid

af: Anita Stark og Randi Petersen

Vejleder: Bernhelm Booss-Bavnbek

310/96 Kursusmateriale til "Lineære strukturer fra algebra og analyse" (E1)

af: Mogens Brun Heefelt

311/96 2nd Annual Report from the project LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM

by: Hélène Connor-Lajambe, Bernd Kuemmel, Stefan Krüger Nielsen, Bent Sørensen

312/96 Grassmannian and Chiral Anomaly

by: B. Booss-Bavnbek, K.P.Wojciechowski

313/96 THE IRREDUCIBILITY OF CHANCE AND THE OPENNESS OF THE FUTURE
The Logical Function of Idealism in Peirce's Philosophy of Nature

By: Helmut Pape, University of Hannover

314/96 Feedback Regulation of Mammalian Cardiovascular System

By: Johnny T. Ottesen

315/96 a + b "Rejsen til tidens indre" - Udarbejdelse af et manuskript til en fjernsynsudsendelse + manuskript

af: Gunhild Hune og Karina Goyle

Vejledere: Peder Voetmann Christiansen og Bruno Ingemann

316/96 Plasmaoscillation i natriumklynger

Specialerapport af: Peter Meibom, Mikko Østergård

Vejledere: Jeppe Dyre & Jørn Borggreen

317/96 Poincaré og symplektiske algoritmer

af: Ulla Rasmussen

Vejleder: Anders Madsen

318/96 Modelling the Respiratory System

by: Tine Guldager Christiansen, Claus Dræby

Supervisors: Viggo Andreasen, Michael Danielsen

319/96 Externality Estimation of Greenhouse Warming Impacts

by: Bent Sørensen

320/96 Grassmannian and Boundary Contribution to the -Determinant

by: K.P.Wojciechowski et al.

321/96 Modelkompetencer - udvikling og afprøvning af et begrebsapparat

Specialerapport af: Nina Skov Hansen, Christine Iversen, Kristin Troels-Smith

Vejleder: Morten Blomhøj

322/96 OPGAVESAMLING
Bredde-Kursus i Fysik 1976 - 1996

323/96 Structure and Dynamics of Symmetric Diblock Copolymers
PhD Thesis

by: Christine Maria Papadakis

324/96 Non-linearity of Baroreceptor Nerves

by: Johnny T. Ottesen

325/96 Retorik eller realitet ?
Anvendelser af matematik i det danske Gymnasiums matematikundervisning i perioden 1903 - 88

Specialerapport af Helle Pilemann

Vejleder: Mogens Niss

326/96 Bevisteori
Eksemplificeret ved Gentzens bevis for konsistensen af teorien om de naturlige tal

af: Gitte Andersen, Lise Mariane Jeppesen, Klaus Frovin Jørgensen, Ivar Peter Zeck

Vejledere: Bernhelm Booss-Bavnbek og Stig Andur Pedersen

327/96 NON-LINEAR MODELLING OF INTEGRATED ENERGY SUPPLY AND DEMAND MATCHING SYSTEMS

by: Bent Sørensen

328/96 Calculating Fuel Transport Emissions

by: Bernd Kuemmel

329/96 The dynamics of cocirculating influenza strains conferring partial cross-immunity

and

A model of influenza A drift evolution

by: Viggo Andreasen, Juan Lin and Simon Levin

330/96 LONG-TERM INTEGRATION OF PHOTOVOLTAICS INTO THE GLOBAL ENERGY SYSTEM

by: Bent Sørensen

331/96 Viskøse fingre

Specialerapport af:

Vibeke Orlien og Christina Specht

Vejledere: Jacob M. Jacobsen og Jesper Larsen

332/97 ANOMAL SWELLING AF LIPIDE DOBBELTLAG

Specialerapport af:

Stine Sofia Korremann

Vejleder: Dorthe Posselt

333/97 Biodiversity Matters

an extension of methods found in the literature on monetisation of biodiversity

by: Bernd Kuemmel

334/97 LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM

by: Bernd Kuemmel and Bent Sørensen

335/97 Dynamics of Amorphous Solids and Viscous Liquids

by: Jeppe C. Dyre

336/97 PROBLEM-ORIENTATED GROUP PROJECT WORK AT ROSKILDE UNIVERSITY

by: Kathrine Legge

337/97 Verdensbankens globale befolkningsprognose - et projekt om matematisk modellering

af: Jørn Chr. Bendtsen, Kurt Jensen, Per Pauli Petersen

Vejleder: Jørgen Larsen

338/97 Kvantisering af nanolederes elektriske ledningsevne

Første modul fysikprojekt

af: Søren Dam, Esben Danielsen, Martin Niss, Esben Friis Pedersen, Frederik Resen Steenstrup

Vejleder: Tage Christensen

339/97 Defining Discipline

by: Wolfgang Coy

340/97 Prime ends revisited - a geometric point of view -

by: Carsten Lunde Petersen

341/97 Two chapters on the teaching, learning and assessment of geometry

by Mogens Niss

342/97 LONG-TERM SCENARIOS FOR GLOBAL ENERGY DEMAND AND SUPPLY

A global clean fossil scenario discussion paper prepared by Bernd Kuemmel

Project leader: Bent Sørensen

343/97 IMPORT/EKSPORT-POLITIK SOM REDSKAB TIL OPTIMERET UDNYTTELSE AF EL PRODUCERET PÅ VE-ANLÆG

af: Peter Meibom, Torben Svendsen, Bent Sørensen

344/97 Puzzles and Siegel disks

by Carsten Lunde Petersen

345/98 Modeling the Arterial System with Reference to an Anestesia Simulator

Ph.D. Thesis

by: Mette Sofie Olufsen

346/98 Klyngedannelse i en hulkatode-forstøvningsproces

af: Sebastian Horst

Vejledere: Jørn Borggren, NBI, Niels Boye Olsen

347/98 Verificering af Matematiske Modeller - en analyse af Den Danske Eulerske Model

af: Jonas Blomqvist, Tom Pedersen, Karen Timmermann, Lisbet Øhlenschlæger

Vejleder: Bernhelm Booss-Bavnbek

348/98 Case study of the environmental permission procedure and the environmental impact assessment for power plants in Denmark

by: Stefan Krüger Nielsen

Project leader: Bent Sørensen

349/98 Tre rapporter fra FAGMAT - et projekt om tal og faglig matematik i arbejdsmarkedsuddannelserne

af: Lena Lindenskov og Tine Wedege

350/98 OPGAVESAMLING - Bredde-Kursus i Fysik 1976 - 1998

Erstatter teksterne 3/78, 261/93 og 322/96

351/98 Aspects of the Nature and State of Research in Mathematics Education

by: Mogens Niss

352/98 The Herman-Swiatec Theorem with applications
by: Carsten Lunde Petersen

353/98 Problemløsning og modellering i en almendannende matematikundervisning
Specialerapport af: Per Gregersen og Tomas Højgaard Jensen
Vejleder: Morten Blomhøj

354/98 A GLOBAL RENEWABLE ENERGY SCENARIO
by: Bent Sørensen and Peter Meibom

355/98 Convergence of rational rays in parameter spaces
by: Carsten Lunde Petersen and Gustav Ryd

356/98 Terrænmodellering
Analyse af en matematisk model til konstruktion af terrænmodeller
Modelprojekt af: Thomas Frommelt, Hans Ravnkjær Larsen og Arnold Skimminge
Vejleder: Johnny Ottesen

357/98 *Cayleys Problem*
En historisk analyse af arbejdet med Cayley problem fra 1870 til 1918
Et matematisk videnskabsfagsprojekt af: Rikke Degn, Bo Jakobsen, Bjarke K.W. Hansen, Jesper S. Hansen, Jesper Udesen, Peter C. Wulff
Vejleder: Jesper Larsen

358/98 *Modeling of Feedback Mechanisms which Control the Heart Function in a View to an Implementation in Cardiovascular Models*
Ph.D. Thesis by: Michael Danielsen

359/99 *Long-Term Scenarios for Global Energy Demand and Supply Four Global Greenhouse Mitigation Scenarios*
by: Bent Sørensen

360/99 ***SYMMETRI I FYSIK***
En Meta-projektrapport af: Martin Niss, Bo Jakobsen & Tune Bjarke Bonné
Vejleder: Peder Voetmann Christiansen

361/99 *Symplectic Functional Analysis and Spectral Invariants*
by: Bernhelm Booss-Bavnbek, Kenro Furutani

362/99 *Er matematik en naturvidenskab? - en udspænding af diskussionen*
En videnskabsfagsprojekt-rapport af Martin Niss
Vejleder: Mogens Nørgaard Olesen

363/99 ***EMERGENCE AND DOWNWARD CAUSATION***
by: Donald T. Campbell, MArk H. Bickhard and Peder V. Christiansen

364/99 ***Illustrationens kraft***
Visuel formidling af fysik
Integreret speciale i fysik og kommunikation
af: Sebastian Horst
Vejledere: Karin Beyer, Søren Kjørup

365/99 ***To know - or not to know - mathematics, that is a question of context***
by: Tine Wedege

366/99 ***LATEX FOR FORFATTERE***
En introduktion til LATEX og IMFUFA-LATEX
af: Jørgen Larsen

367/99 Boundary Reduction of Spectral invariants and Unique Continuation Property
by Bernhelm Booss-Bavnbek

368/99 Kvartalsrapport for projektet
SCENARIER FOR SAMLET UDNYTTELSE AF BRINT SOM ENERGIBÆRER I DANMARKS FREMTIDIGE ENERGISTYSTEM
Projektleder: Bent Sørensen

369/99 DYNAMICS OF Complex Quadratic Correspondences
by: Jacob Jalving

370/99 OPGAVESAMLING
Bredde-Kursus i Fysik 1976 - 1999
(erstatter tekst nr. 350/98

370/99 Bevisets stilling
- beviser og bevisførelse i en gymnasial matematikundervisning
Matematikspeciale af: Maria Hermannsson
Vejleder: Mogens Niss