TEKST NR 355 **1998**

Convergence of rational rays in parameter spaces

Carsten Lunde Petersen and Gustav Ryd

TEKSTER fra

IMFUFA **ROSKILDE UNIVERSITETSCENTER**
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES
FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, ROSKILDE UNIVERSITY, POSTBOX 260, DK-4000 ROSKILDE
DENMARK.

Convergence of rational rays in parameter spaces

by

Carsten Lunde Petersen, IMFUFA and

Gustav Ryd [1], Dept. of Mathematics, KTH, 100 44 Stockholm, Sweden

IMFUFA-text no 355/98 10 pages ISSN 0106-6242

Abstract

We give an elementary proof of the landing Theorem for rational external rays of the Mandelbrot set and related connectedness loci for the one-parameter families of polynomials $\{P_c(z) = z^d + c\}_{c\in\mathbb{C}}$, $d \geq 2$.

---

[1] present address : Research & Trade, 103 95 Stockholm, Sweden.

Convergence of rational rays in parameter spaces

Carsten Lunde Petersen and Gustav Ryd

Abstract

We give an elementary proof of the landing Theorem for rational external rays of the Mandelbrot set and related connectedness loci for the one-parameter families of polynomials $\{P_c(z) = z^d + c\}_{c\in\mathbb{C}}$, $d \geq 2$.

Throughout this paper the integer $d \geq 2$ will be arbitrary but fixed, and used without further notice. It will be the degree of our polynomials. Let $P_c(z) = P_c = z^d + c$, $c \in \mathbb{C}$, denote the family of monic degree d polynomials with a degree $(d-1)$ critical point at the origin. For each c let J_c denote the Julia set for P_c and define the domain of attraction to infinity

$$\Lambda_c = \Lambda_c(\infty) = \{z \in \mathbb{C} | P_c^n(z) \longrightarrow \infty, \text{as } n \to \infty\}.$$

Let $\phi_c : \widetilde{\Lambda}_c \longrightarrow (\mathbb{C} \setminus \overline{\mathbb{D}})_c$ be the maximal univalent 'radial' (on the image side) extension of the unique Bötcher coordinate tangent to the identity at infinity.

Define $M_d = \{c \in \mathbb{C} | c \notin \Lambda_c\}$ and let $\Phi_d : \overline{\mathbb{C}} \setminus M_d \longrightarrow \overline{\mathbb{C}} \setminus \overline{\mathbb{D}}$ denote the unique Riemann map tangent to the identity at infinity, then $\Phi_d(c) = \phi_c(c)$. Given $\theta \in \mathbb{T} = \mathbb{R}/\mathbb{Z}$ and $c \in \mathbb{C}$: The dynamical (external) ray $R_c(\theta)$ of J_c is the analytic arc $\phi_c^{-1}(\{\exp(s + i2\pi\theta) | s > 0\} \cap (\overline{\mathbb{C}} \setminus \overline{\mathbb{D}})_c)$. It starts at ∞ and ends either at a precritical point $z_0 \in \cup_{n\geq 0} P_c^{-n}(c)$ or on the Julia set J_c. The parameter (external) ray $R_{M_d}(\theta)$ of M_d is the analytic arc $\Phi_d^{-1}(\{\exp(s + i2\pi\theta) | s > 0\})$. A ray is called a rational ray if θ is rational, i.e. $\theta \in \mathbb{Q}/\mathbb{Z}$. A ray R is said to land or converge if $\overline{R} \setminus R$ is a singleton subset of J (if it is a dynamical ray) or M_d (if it is a parameter ray).

On the boundary of M_d we distinguish two particular and different types of parameters, parabolic parameters and Misiurewicz parameters. A parameter c_0 is called parabolic, if P_{c_0} has a parabolic cycle, i.e., there exists a positive integer n and a periodic point p such that $P_{c_0}^n(p) = p$ and $(P_{c_0}^n)'(p) = 1$.

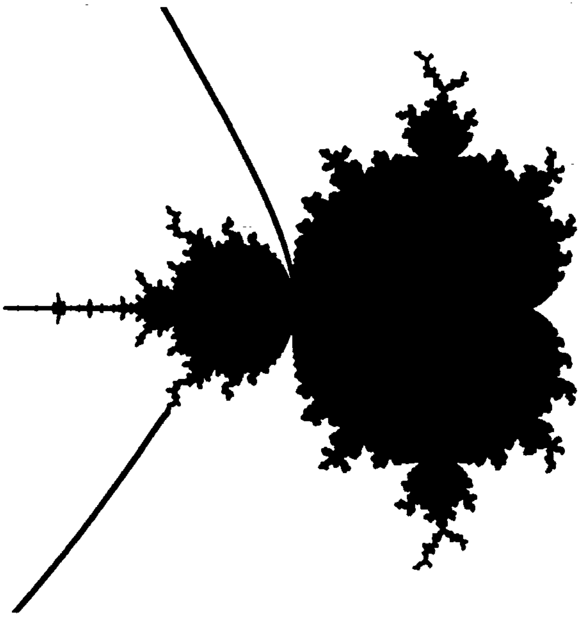

Figure 1: The Mandelbrot set with a periodic and a strictly preperiodic ray.

A parameter c_0 is called a Misiurewicz parameter, if the orbit of c_0 is finite and ends in a repelling periodic orbit.

In this paper we give an elementary proof of the landing Theorem for rational external rays of M_d, for any integer $d \geq 2$:

Theorem 1 *Given $\theta \in \mathbb{T}$ with $d^l\theta \equiv d^{l+q}\theta \mod 1$, for some minimal integers $l \geq 0$ and $q \geq 1$. Then the parameter ray $R_{M_d}(\theta)$ lands on a parameter $c_0 \in \partial M_d$. Furthermore,*

1. *if $l = 0$ then c_0 is a parabolic parameter, the (unique) parabolic orbit for P_{c_0} has one cycle of immediate attracted basins. This cycle contains both 0 and c and its exact period is q. Moreover the dynamic ray $R_{c_0}(\theta)$ lands on the parabolic point to which c_0 is attracted under iteration by $P_{c_0}^q$.*
2. *if $l > 0$ then c_0 is a Misiurewicz parameter, $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$ and the dynamic ray $R_{c_0}(\theta)$ land on c_0.*

This Theorem is well-known, at least for $d = 2$, since [DH]. The original proof is however rather involved. There are other proofs, such as [HS], which

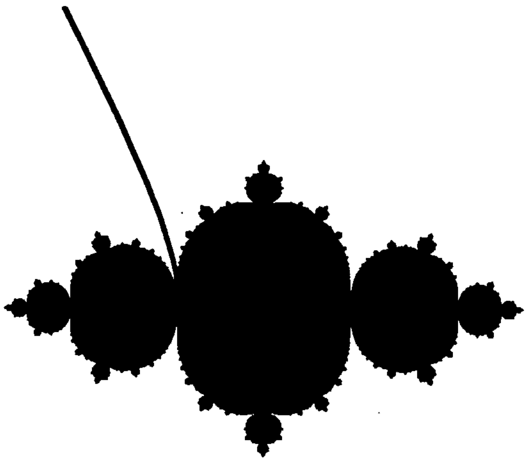

Figure 2: The Julia set for a quadratic polynomial with a period-2 ray landing at a parabolic fixed point.

uses iteration on Teichmüller spaces, and [Mil], [Sch], which both depend on global counting arguments and a priori analysis of the sets M_d. In [CG] this theorem is claimed, but there seems to be a part missing. This paper has evolved from [Pet] and [Ryd], which both were extending Theorem 1 to different settings but using similar techniques.

Our proof differs from the original proof in [DH] mainly in our direct approach to convergence and uses only elementary analytical means. Moreover it has easy generalizations to many other settings. We want to make our proof available to the public, because we believe that our elementary approach will be of use also to others.

We will frequently use $g_c : \mathbb{C} \longrightarrow \mathbb{R}_+$, the Green's function for J_c with pole at ∞, which is the subharmonic function which is harmonic on Λ_c, coincides with $\log|\phi_c(z)|$ on $\widetilde{\Lambda}_c$ and which is identically zero on $\mathbb{C} \setminus \Lambda_c$. Similarly, the Green's function for M_d with pole at ∞, $G_{M_d} : \mathbb{C} \longrightarrow \mathbb{R}_+$ is the subharmonic function, which is 0 on M_d and which coincides with $\log|\Phi_{M_d}|$ on $\mathbb{C} \setminus M_d$. Note that an external ray is a gradient line for the appropriate Green's function.

We first prove that the end of the external ray gives rise to a sequence of

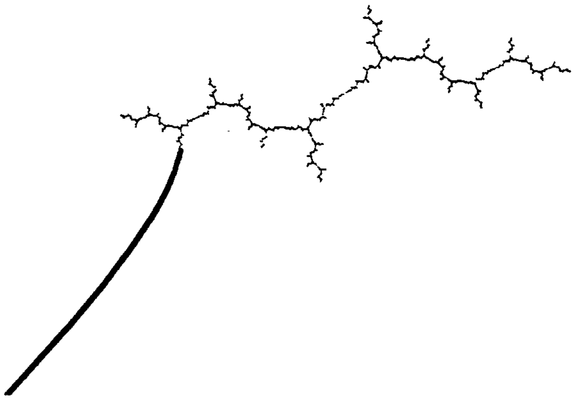

Figure 3: The Julia set for a quadratic polynomial with a preperiodic ray landing at a Misiurewicz point.

limit maps which relates the dynamics in the filled Julia set, K_{c_0}, with the dynamics in Λ_{c_0}.

Given $c_0 \in \mathbb{C}$ and $\phi \in \mathbb{T}$, for $0 < \epsilon$ and $w \in R_c(\phi)$ we define a 'rectangle'

$$V(w,\epsilon) = \{\exp(t + i2\pi\eta) | g_{c_0}(w) - \epsilon < t < d^q g_{c_0}(w) + \epsilon \text{ and } |\eta - \phi| < \epsilon\}.$$

Proposition 1.1 *Given $\theta \in \mathbb{T}$ with $d^l\theta \equiv d^{l+q}\theta \mod 1$, for some minimal $l \geq 0$ and $q \geq 1$. Let $c_0 \in \partial M_d$ be a limit point of the ray $R_{M_d}(\theta)$. Then there exists $w \in R_{c_0}(d^l\theta)$, $\epsilon > 0$ and a sequence of maps $\{\psi_n : V \longrightarrow K_{c_0}\}_{n \geq 0}$, $V = V(w,\epsilon)$ such that*

1. *For all $0 \leq n : \psi_n(w) = P^n_{c_0}(c_0)$ and $\psi_{n+1} = P_{c_0} \circ \psi_n$.*
2. *For all $l \leq n : \psi_n \circ P^q_{c_0} = P^q_{c_0} \circ \psi_n$, wherever defined and in particular $\psi_n(P^q_{c_0}(w)) = P^{n+q}_{c_0}(c_0) = \psi_{n+q}(w)$.*
3. *One of the following mutually exclusive cases occur*
   - (a) *The ψ_n are univalent and for all $0 \leq n \neq m$ with either $n < l$ or $|n - m| \neq q : \psi_n(V) \cap \psi_m(V) = \emptyset$.*

(b) All ψ_n are constants.

Proof : Let $\phi = d^l\theta \mod 1$ be the first periodic angle in the orbit of θ. And let $\{c_j\}_{j\in\mathbb{N}} \subset R_{M_d}(\theta) \cap \{c|G_{M_d}(c) < d^{-l}\}$ be a sequence converging to c_0. For each j define N_j by $P_{c_j}^{N_j}(c_j) := w_j \in R_{c_j}(\phi) \cap \{z|1 \leq g_{c_j}(z) < d^q\}$. Passing to a subsequence if necessary, we can suppose by relative compactness that $P_{c_j}^{N_j}(c_j) \longrightarrow w \in R_{c_0}(\phi)$ with $1 \leq g_{c_0}(w) \leq d^q$.

Fix $0 < \epsilon < 1/2$ and define a simply connected domain $V = V(w, \epsilon)$. We can assume that $w_j \in V$. For each j and $0 \leq n \leq N_j$ let $\psi_{n,j} : V \longrightarrow \mathbb{C}$ be the unique branch of $P_{c_j}^{-N_j+n}$ which maps w_j to $P_{c_j}^n(c_j)$. Then for each fixed j the uniformly bounded univalent maps $\psi_{n,j}$ satisfy

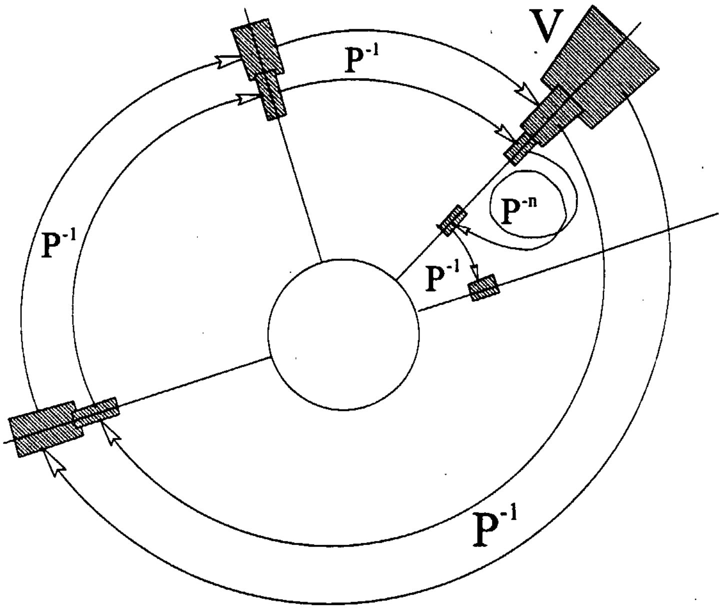

Figure 4: The domain V and the image domains $\psi_{n,j}(V)$ viewed in the Bötcher coordinate at infinity.

$$\forall\, 0 \leq n \leq N_j : \qquad \psi_{n,j}(w_j) = P_{c_j}^n(c_j) \quad \text{and} \quad \psi_{n+1,j} = P_{c_j} \circ \psi_{n,j} \tag{1.1}$$

$$\forall\, l \leq n \leq N_j - q : \qquad \psi_{n,j} \circ P_{c_j}^q = P_{c_j}^q \circ \psi_{n,j}. \tag{1.2}$$

$$\forall\, 0 \leq n \neq m \leq N_j \text{ with } n < l \text{ or } |n-m| \neq q : \qquad \psi_{n,j}(V) \cap \psi_{m,j}(V) = \emptyset \tag{1.3}$$

Here the disjointness property (1.3) holds if ϵ is small enough. Note that $N_j \longrightarrow \infty$ as $j \to \infty$. As the univalent maps $\psi_{n,j}$ are uniformly bounded on V, we can suppose, using the Cantor diagonal principle to extract a subsequence if necessary, that for each $n \in \mathbb{N}$

$$\psi_{n,j} \underset{j\to\infty}{\rightrightarrows} \psi_n$$

where $\psi_n : V \longrightarrow \mathbb{C}$ is some holomorphic map, which is either univalent or constant by Hurwitz Theorem. Note that $g_{c_0} = 0$ on $\psi_n(V)$ so $\psi_n(V) \subset K_{c_0}$. By uniform convergence the properties (1.1), (1.2) and (1.3) also holds for the ψ_n (with P_{c_0}) so Properties 1. and 2. follows. By (1.1) if one ψ_n is constant they all are, so Property 3. follows. **q.e.d.**

Proposition 1.2 *Let θ and c_0 be as in Proposition 1.1. If $c_0 \in J_{c_0}$ then 3b. holds, $P^l_{c_0}(c_0) = P^{l+q}_{c_0}(c_0)$, $l > 0$ and the dynamic ray $R_{c_0}(\theta)$ lands on c_0.*

Proof : Let V and ψ_n be as in the conclusion of Proposition 1.1. The image $\psi_n(V) \subset K_{c_0}$. Thus if $c_0 = \psi_0(w) \in J_{c_0} = \partial K_{c_0}$ then ψ_0 cannot be open thus all ψ_n are constant. Combining Properties 1. and 2. we have $P^l_{c_0}(c_0) = P^{l+q}_{c_0}(c_0)$. If $l = 0$ then c_0 is q-periodic, but then also the critical point 0 is q-periodic, because it is the only preimage of c_0. This contradicts however the fact that $c_0 \in \partial M_d$. Hence $l > 0$ and c_0 is a Misiurewicz parameter with $P^l_{c_0}(c_0) = P^{l+q}_{c_0}(c_0)$.

To complete the proof we show that the ray $R_{c_0}(\theta)$ lands on c_0. To this end let $\{c_j\}_{j\in\mathbb{N}} \subset R_{M_d}(\theta)$ be a sequence converging to c_0 as in the proof of Proposition 1.1. Define $R^*_{c_j}(\theta) = ((R_{c_j}(\theta) \cap \{z | g_{c_j}(z) \geq g_{c_j}(c_j)\}) \cup \infty)$. Then the sequence $\{R^*_{c_j}\}_{j\in\mathbb{N}}$ is a sequence of compact sets in $\overline{\mathbb{C}}$. Thus passing to a subsequence, if necessary, we can suppose the sequence converges to some compact subset $R \subset \overline{\mathbb{C}}$ containing c_0 and ∞. Moreover $R \cap \Lambda_{c_0} = R_{c_0}(\theta)$ by continuity with respect to the parameter of the Bötcher coordinates ϕ_c. Let $z \in R \cap J_{c_0}$ be arbitrary (note that $J_{c_0} = \partial\Lambda_{c_0} = \mathbb{C} \setminus \Lambda_{c_0}$) and let $z_j \in R^*_{c_j}(\theta)$ be a sequence with $z_j \longrightarrow z$ as $j \to \infty$. Arguing as in the proof of Proposition 1.1 we find a point $\widehat{w} \in R_{c_0}(d^l\theta)$, an open set $\widehat{V}$ and a sequence of holomorphic maps $\widehat{\psi}_n : \widehat{V} \longrightarrow \mathbb{C}$ satisfying the properties 1., 2. and 3. of Proposition 1.1, except that c_0 is replaced by z. As above case 3b. in 3. holds, because $z \in J_{c_0}$. But then $P^l_{c_0}(z) = P^{l+q}_{c_0}(z)$. The set $\{z | P^l_{c_0}(z) = P^{l+q}_{c_0}(z)\}$ is however finite and hence $R \cap J_{c_0}$ is a singleton containing c_0. **q.e.d.**

Proposition 1.3 *Let θ and c_0 be as in Proposition 1.1. If c_0 belongs to a Fatou component U for P_{c_0}, then 3a. holds. Furthermore, there exists a sequence of Jordan arcs $\gamma_n : [0,1] \longrightarrow P_{c_0}^n(U)$ satisfying*

1'. For all $0 \leq n : \gamma_n(0) = P_{c_0}^n(c_0)$ and $\gamma_{n+1} = P_{c_0} \circ \gamma_n$.

2'. For all $l \leq n : \gamma_n \cap \gamma_{n+q} = \gamma_n(1) = \gamma_{n+q}(0) = P_{c_0}^{n+q}(0)$.

3'. For all $0 \leq n \neq m$ with either $n < l$ or $|n-m| \neq q : \gamma_n \cap \gamma_m = \emptyset$.

Proof : Let V and ψ_n be as in the conclusion of Proposition 1.1. If the ψ_n were constants then by combining Property 1. and Property 2., $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$. Hence the unique critical point is preperiodic, but then it either belongs to the Julia set or is superattracting. The first possibility is excluded by hypothesis and in the second case c_0 belongs to the interior of M_d, which is impossible. Thus ψ_l is not constant so Property 3a. holds.

Let w be as in the proof of Proposition 1.1 and define an arc $\Gamma \subset V$ by

$$\Gamma = R_{c_0}(\phi) \cap \{z | g_{c_0}(w) \leq g_{c_0}(z) \leq d^q g_{c_0}(w)\}.$$

The arc Γ is naturally parmetrised linearly in the potential by the interval $[0,1]$. Define arcs $\gamma_n = \psi_n \circ \Gamma : [0,1] \longrightarrow P_{c_0}^n(U)$.

Properties 1'. and 3'. follows immediately from Properties 1. and 3a. To see also Property 2'. define

$$\widehat{V} = \{\exp(t + i2\pi\eta) | g_{c_0}(w) - \epsilon < t < d^{2q} g_{c_0}(w) + \epsilon \text{ and } |\eta - \phi| < \epsilon\},$$
$$\widehat{\Gamma} = R_{c_0}(\phi) \cap \{z | g_{c_0}(w) \leq g_{c_0}(z) \leq d^{2q} g_{c_0}(w)\}$$

and note that the ψ_n have univalent extensions $\widehat{\psi}_n$ to $\widehat{V}$. Moreover for $n \geq l : \widehat{\psi}_n(\widehat{\Gamma}) = \gamma_n \cup \gamma_{n+q}$ is an arc. Thus also the interior disjointness $\gamma_n \cap \gamma_{n+q} = P_{c_0}^{n+q}(c_0)$ follows. **q.e.d.**

Proposition 1.4 *Let θ, c_0 and U be as in Proposition 1.3. Then U is an immediate parabolic basin of exact period q and the preperiod $l = 0$.*

Proof : Let $U_l = P_{c_0}^l(U)$. Then $\gamma_l \in U_l$ and U_l is q-periodic, because both $P_{c_0}^l(c_0) = \gamma_l(0)$ and $P_{c_0}^{l+q}(c_0) = \gamma_l(1) \in U_l$. But then it is either a hyperbolic, a parabolic or a rotation domain, however the first implies c_0 is in the interior of M_d and the last requires at least one critical point in the Julia set and are hence excluded. Thus U_l is an immediate parabolic basin and so is U, because the cycle of such immediate basins should contain a critical point and value. Thus U is q-periodic. Let the (exact) period of U be p. Then $\gamma_p \in U$. Let $\Phi : U \longrightarrow \mathbb{C}$ be a Fatou coordinate for $P^p_{c_0}$ on U normalized by $\Phi(c_0) = 0$ and $\Phi(P^p_{c_0}(c_0)) = 1$. The map Φ has a univalent inverse branch

$$\Psi : \{z = x + iy | y > -1\} \longrightarrow \Omega$$

with $\Psi(0) = c_0$. By compactness there exists $n \geq 0$ such that γ_{pn} and $\gamma_{p(n+1)}$ are contained in Ω. Let γ'_{pn} and $\gamma'_{p(n+1))}$ denote the corresponding images under Φ. The map $z \rightarrow \exp(2\pi i p/q)$ maps these two arcs onto two simple closed curves which are rotations (around the origin) of each other. Thus they intersect and so do γ'_{pn} and $\gamma'_{p(n+1)}$. Since γ_{pn} and $\gamma_{p(n+1)}$ are contained in Ω they also intersect. But then $p = q$ by Property 3'.

We prove that $l = 0$. Choose n such that $\gamma_{nq} \subset \Omega$ so that $\gamma_{(n+m)q} \subset \Omega$ for all $m \geq 0$. Then $\Phi(\gamma_{nq})$ satisfies the hypothesis of the isotopy Lemma 1.5 below, because of Properties 2'. and 3'. Let $\widetilde{\Gamma}$ be the corresponding isotopy of Lemma 1.5. Increasing n, if necessary, we can assume $\widetilde{\Gamma}([0,1] \times [0,1]) \subset \Phi(\Omega)$, because of Property 1'. Define $\Gamma_n : [0,1] \times [0,1] \longrightarrow \Omega$ as the isotopy $\Gamma_n = \Psi \circ \widetilde{\Gamma}$. Define isotopies $\Gamma_k : [0,1] \times [0,1] \longrightarrow U$, $n \geq k \geq 0$ recursively by Γ_{k-1} is the lift of Γ_k to $P^q_{c_0}$ with $\Gamma_{k-1}(1,[0,1]) = \Gamma_k(0,[0,1])$. An easy induction proof shows that for each $n \geq k \geq 0$ and each $0 \leq t \leq 1$: $\Gamma_k(t,0) = \Psi(k+t)$. In particular $\Gamma_k(0,[0,1]) = P^{kq}_{c_0}(c_0)$. But then $\gamma_0 \cap \gamma_q = P^q_{c_0}(c_0)$, which contradicts Property 3', when $l > 0$. **q.e.d.**

The proof of the following isotopy Lemma is left to the reader.

Lemma 1.5 *Suppose* $\gamma_1 : [0,1] \longrightarrow \mathbb{C}$ *is an arc with* $\gamma_1(0) = 0, \gamma_1(1) = 1$ *and with* $\gamma_1 \cap (\gamma_1 + n) = \emptyset$ *for* $|n| > 1$ *and with* $\gamma_1 \cap (\gamma_1 + 1) = 1$, $\gamma_1 \cap (\gamma_1 - 1) = 0$. *Then there exists an isotopy of arcs* $\Gamma : [0,1] \times [0,1] \longrightarrow \mathbb{C}$ *with* $\Gamma(t,1) = \gamma_1(t)$, $\Gamma(t,0) = t := \gamma_0(t)$, $\Gamma(0,[0,1]) = 0$, $\Gamma(1,[0,1]) = 1$ *and* $\Gamma(]0,1[,[0,1]) \subset \mathbb{C} \setminus \mathbb{Z}$.

Proposition 1.6 *Let* θ, c_0 *and* U *be as in Proposition 1.3. Then the dynamic ray* $R_{c_0}(\theta)$ *lands on the parabolic point on the boundary of* U.

Proof : Let R be as in the second part of the proof of Proposition 1.2. Let α denote the parabolic point to which c_0 is iterated by $P^q_{c_0}$. Then $\alpha \in R$ by closedness and forward invariance of R. We claim that $R \cap J_{c_0} = \alpha$. To prove this let Ω be any bounded Fatou component with $R \cap \Omega \neq \emptyset$. We shall prove that $R \cap \partial\Omega$ is a single q-periodic parabolic point, because Ω contains at most one critical point for $P_{c_0}^q$. If the degree is 2, (the critical point is simple) there is nothing to prove, because a degree two basin with locally connected boundary contains precisely one periodic orbit with period dividing that of the basin. If the degree is higher, or one does not want to rely on local connectivity, one may procede as follows: Let $z_0 \in R \cap K_{c_0}$ be arbitrary. Redoing the proof of Proposition 1.1 but with c_0 replaced by z_0 we find $w' \in R_{c_0}(\theta)$, $V' = V'(w', \epsilon)$ and holomorphic maps $\{\psi_{z_0,n} : V' \longrightarrow K_{c_0}\}_{n \geq 0}$ satisfying the conclusions 1.–3. of that Proposition (with c_0 replaced by z_0). If $z_0 \in R \cap J_{c_0}$ then case 3.3b. occurs and z_0 is q-periodic. If z_0 belongs to the Fatou set then case 3.3a. occurs and in particular $\psi_{z_0,0} : V' \longrightarrow U$ is univalent with $\psi_{z_0,0}(w') = z_0$, $\psi_{z_0,0}(P_{c_0}^q(w')) = P_{c_0}^q(z_0)$ and

$$\psi_{z_0,0}(\{z \in R_{c_0}(\theta) | g_{c_0}(z) \geq g_{c_0}(w')\}) \subsetneq R \cap U \subseteq \psi_{z_0,0}(R_{c_0}(\theta) \cap V').$$

Suppose there exists a bounded Fatou component Ω for which $L = R \cap \Omega$ connects two distinct q-periodic points α', β on the boundary of Ω. By the above any point $z \in L$ is connected to $P_{c_0}^q(z)$ in L through an analytic arc contained in L. Hence we can assume that α' is parabolic. Let $\phi : \Omega \longrightarrow \mathbb{D}$ be a Riemann map say mapping α' to 1. Let $\widehat{L} = \phi(L)$ and define $\widehat{P}$ as the conjugate Blaschke product $\widehat{P} = \phi \circ P_{c_0}^q \circ \phi^{-1}$. Then $\widehat{L}$ connects the parabolic point 1 for $\widehat{P}$ with a repelling fixed point $\widehat{\beta} \neq 1$. In fact $\widehat{L}$ is easily seen to contain an analytic arc $\widehat{\gamma}$, which connects $\widehat{\beta}$ to 1, and which is parametrized by an analytic diffeomorphism $\eta : \mathbb{R} \longrightarrow \widehat{\gamma}$ with $\eta(t+1) = \widehat{P}(\eta(t))$.

Let $U_1, U_2 \subset \mathbb{D}$ be the two complimentary components of $\mathbb{D} \setminus \eta$. Consider say U_1. It contains a unique connected component U_1' of $P^{-1}(U_1)$, because the dynamics of P on η is diffeomorphically conjugate to translation by 1. The degree of the restriction $p : U_1' \longrightarrow U_1$ is at least 2 because $\overline{U_1} \cap \mathbb{S}^1$ is covered at least twice by $\overline{U_1'} \cap \mathbb{S}^1$. Thus U_1 contains a critical point. And simillarly so do U_2. This contradicts that P has only one critical point. It follows that R contains precisely one q-periodic point. The parabolic one to which c_0 is iterated under $P_{c_0}^q$. **q.e.d.**

Remark 1.7 *Consider the family* $Q_c(z) = z^4 - 2z^3 + z^2 + z + c$, $c \in \mathbb{C}$ *of quartic polynomials. For the polynomial* Q_0 *the real axis contains one critical point* $\omega \in]-\frac{1}{3}, 0[$, *whose critical value* v_0 *is contained in the same interval and is attracted to the parabolic fixed point* 0. *The two other critical points are* *complex conjugate and attracted to the parabolic fixed point* 1. *Moreover the external ray of argument* 0 *equals* $[1, \infty]$. *For any sequence* $\{c_n\}_{n \in \mathbb{N}} \subset]0, \infty$ *converging to* 0 *the Hausdorff limit* R *of the segments of external rays* $[v_n, \infty]$ *equals* $[v_0, \infty]$. *In this case the segment* $]0, 1[$ *is like the arc* L *in the proof of the above theorem.*

Proof of Theorem 1: The cluster set $Cl_{M_d}(\theta) = \overline{R_{M_d}(\theta)} \setminus R_{M_d}(\theta) \subseteq \partial M_d$ is a continuum and in particular connected. Let $c \in Cl_{M_d}(\theta)$. By Propositions 1.2 and 1.4, c has either parabolic or Misiurewicz dynamics. Since the period q is fixed, this implies that c varies in a finite set and is thus unique, which proves convergence. The dichotomy follows from Proposition 1.2 and Proposition 1.4. The rest of the statements 1. and 2. follow by combining Proposition 1.2, Proposition 1.4 and Proposition 1.6. **q.e.d.**

References

[CG] L. Carleson and T. Gamelin. *Complex Dynamics*. Springer-Verlag, 1993.

[DH] A. Douady and J.H. Hubbard. Etude Dynamique des Polynômes Complexes. Publications Mathématique d'Orsay, 1985.

[HS] J. H. Hubbard and D Schleicher. The Spider Algorithm. In *Proceedings of Symposia in Applied Mathematics*, volume 49, 1994.

[Mil] J. Milnor. Periodic orbits, external rays and the Mandelbrot set; an expository account. Preprint, 1995.

[Pet] C. L. Petersen. Convergence of (pre)periodic rays in parameter spaces. Manuscript, 1997.

[Ryd] G. Ryd. *Iterations of one parameter families of complex polynomials*. PhD thesis, Department of Mathematics, KTH Stockholm, Sweden, 1997.

[Sch] D. Schleicher. Rational Parameter Rays of the Mandelbrot Set. November 1997, IMS Preprint 1997/13 SUNY StonyBrook.

Liste over tidligere udkomne tekster
tilsendes gerne. Henvendelse herom kan
ske til IMFUFA's sekretariat

tlf. 46 74 22 63

217/92 "Two papers on APPLICATIONS AND MODELLING IN THE MATHEMATICS CURRICULUM"
by: Mogens Niss

218/92 "A Three-Square Theorem"
by: Lars Kadison

219/92 "RUPNOK - stationær strømning i elastiske rør"
af: Anja Boisen, Karen Birkelund, Mette Olufsen
Vejleder: Jesper Larsen

220/92 "Automatisk diagnosticering i digitale kredsløb"
af: Bjørn Christensen, Ole Møller Nielsen
Vejleder: Stig Andur Pedersen

221/92 "A BUNDLE VALUED RADON TRANSFORM, WITH APPLICATIONS TO INVARIANT WAVE EQUATIONS"
by: Thomas P. Branson, Gestur Olafsson and Henrik Schlichtkrull

222/92 On the Representations of some Infinite Dimensional Groups and Algebras Related to Quantum Physics
by: Johnny T. Ottesen

223/92 THE FUNCTIONAL DETERMINANT
by: Thomas P. Branson

224/92 UNIVERSAL AC CONDUCTIVITY OF NON-METALLIC SOLIDS AT LOW TEMPERATURES
by: Jeppe C. Dyre

225/92 "HATMODELLEN" Impedansspektroskopi i ultrarent en-krystallinsk silicium
af: Anja Boisen, Anders Gorm Larsen, Jesper Varmer, Johannes K. Nielsen, Kit R. Hansen, Peter Bøggild og Thomas Hougaard
Vejleder: Petr Viscor

226/92 "METHODS AND MODELS FOR ESTIMATING THE GLOBAL CIRCULATION OF SELECTED EMISSIONS FROM ENERGY CONVERSION"
by: Bent Sørensen

227/92 "Computersimulering og fysik"
af: Per M.Hansen, Steffen Holm, Peter Maibom, Mads K. Dall Petersen, Pernille Postgaard, Thomas B.Schroder, Ivar P. Zeck
Vejleder: Peder Voetmann Christiansen

228/92 "Teknologi og historie"
Fire artikler af:
Mogens Niss, Jens Høyrup, Ib Thiersen, Hans Hedal

229/92 "Masser af information uden betydning"
En diskussion af informationsteorien i Tor Nørretranders' "Mærk Verden" og en skitse til et alternativ basseret på andenordens kybernetik og semiotik.
af: Søren Brier

230/92 "Vinklens tredeling - et klassisk problem"
et matematisk projekt af
Karen Birkelund, Bjørn Christensen
Vejleder: Johnny Ottesen

231A/92 "Elektrondiffusion i silicium - en matematisk model"
af: Jesper Voetmann, Karen Birkelund, Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B.Hansen

231B/92 "Elektrondiffusion i silicium - en matematisk model" Kildetekster
af: Jesper Voetmann, Karen Birkelund, Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B.Hansen

232/92 "Undersøgelse om den simultane opdagelse af energiens bevarelse og især deles om de af Mayer, Colding, Joule og Helmholtz udførte arbejder"
af: L.Arleth, G.I.Dybkjær, M.T.Østergård
Vejleder: Dorthe Posselt

233/92 "The effect of age-dependent host mortality on the dynamics of an endemic disease and Instability in an SIR-model with age-dependent susceptibility
by: Viggo Andreasen

234/92 "THE FUNCTIONAL DETERMINANT OF A FOUR-DIMENSIONAL BOUNDARY VALUE PROBLEM"
by: Thomas P. Branson and Peter B. Gilkey

235/92 OVERFLADESTRUKTUR OG POREUDVIKLING AF KOKS
- Modul 3 fysik projekt -
af: Thomas Jessen

236a/93 INTRODUKTION TIL KVANTE HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

236b/93 STRØMSSAMMENBRUD AF KVANTE HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

237/93 The Wedderburn principal theorem and Shukla cohomology

af: Lars Kadison

238/93 SEMIOTIK OG SYSTEMEGENSKABER (2)

Vektorbånd og tensorer

af: Peder Voetmann Christiansen

239/93 Valgsystemer - Modelbygning og analyse

Matematik 2. modul

af: Charlotte Gjerrild, Jane Hansen, Maria Hermannsson, Allan Jørgensen, Ragna Clauson-Kaas, Poul Lützen

Vejleder: Mogens Niss

240/93 Patologiske eksempler.
Om sære matematiske fisks betydning for den matematiske udvikling

af: Claus Dræby, Jørn Skov Hansen, Runa Ulsøe Johansen, Peter Meibom, Johannes Kristoffer Nielsen

Vejleder: Mogens Niss

241/93 FOTOVOLTAISK STATUSNOTAT 1

af: Bent Sørensen

242/93 Brovedligeholdelse - bevar mig vel

Analyse af Vejdirektoratets model for optimering af broreparationer

af: Linda Kyndlev, Kare Fundal, Kamma Tulinius, Ivar Zeck

Vejleder: Jesper Larsen

243/93 TANKEEKSPERIMENTER I FYSIKKEN

Et 1.modul fysikprojekt

af: Karen Birkelund, Stine Sofia Korremann

Vejleder: Dorthe Posselt

244/93 RADONTRANSFORMATIONEN og dens anvendelse i CT-scanning

Projektrapport

af: Trine Andreasen, Tine Guldager Christiansen, Nina Skov Hansen og Christine Iversen

Vejledere: Gestur Olafsson og Jesper Larsen

245a+b
/93 Time-Of-Flight målinger på krystallinske halvledere
Specialerapport

af: Linda Szkotak Jensen og Lise Odgaard Gade

Vejledere: Petr Viscor og Niels Boye Olsen

246/93 HVERDAGSVIDEN OG MATEMATIK
- LÆREPROCESSER I SKOLEN

af: Lena Lindenskov, Statens Humanistiske Forskningsråd, RUC, IMFUFA

247/93 UNIVERSAL LOW TEMPERATURE AC CONDUCTIVITY OF MACROSCOPICALLY DISORDERED NON-METALS

by: Jeppe C. Dyre

248/93 DIRAC OPERATORS AND MANIFOLDS WITH BOUNDARY

by: B. Booss-Bavnbek, K.P.Wojciechowski

249/93 Perspectives on Teichmüller and the Jahresbericht Addendum to Schappacher, Scholz, et al.

by: B. Booss-Bavnbek

With comments by W.Abikoff, L.Ahlfors, J.Cerf, P.J.Davis, W.Fuchs, F.P.Gardiner, J.Jost, J.-P.Kahane, R.Lohan, L.Lorch, J.Radkau and T.Söderqvist

250/93 EULER OG BOLZANO - MATEMATISK ANALYSE SET I ET VIDENSKABSTEORETISK PERSPEKTIV

Projektrapport af: Anja Juul, Lone Michelsen,
Tomas Højgård Jensen

Vejleder: Stig Andur Pedersen

251|93 Genotypic Proportions in Hybrid Zones

by: Freddy Bugge Christiansen, Viggo Andreasen and Ebbe Thue Poulsen

252|93 MODELLERING AF TILFÆLDIGE FÆNOMENER

Projektrapport af: Birthe Friis, Lisbeth Helmgaard, Kristina Charlotte Jakobsen, Marina Mosbæk Johannessen, Lotte Ludvigsen, Mette Hass Nielsen

253/93 Kuglepakning

Teori og model

af: Lise Arleth, Kåre Fundal, Nils Kruse

Vejleder: Mogens Niss

254/93 Regressionsanalyse

Materiale til et statistikkursus

af: Jørgen Larsen

255/93 TID & BETINGET UAFHÆNGIGHED

af: Peter Harremoës

256/93 Determination of the Frequency Dependent Bulk Modulus of Liquids Using a Piezoelectric Spherical Shell (Preprint)

by: T. Christensen and N.B.Olsen

257/93 Modellering af dispersion i piezoelektriske keramikker

af: Pernille Postgaard, Jannik Rasmussen, Christina Specht, Mikko Østergård

Vejleder: Tage Christensen

258/93 Supplerende kursusmateriale til

"Lineære strukturer fra algebra og analyse"

af: Mogens Brun Heefelt

259/93 STUDIES OF AC HOPPING CONDUCTION AT LOW TEMPERATURES

by: Jeppe C. Dyre

260/93 PARTITIONED MANIFOLDS AND INVARIANTS IN DIMENSIONS 2, 3, AND 4

by: B. Booss-Bavnbek, K.P.Wojciechowski

261/93 OPGAVESAMLING
Bredde-kursus i Fysik
Eksamensopgaver fra 1976-93

262/93 Separability and the Jones Polynomial
by: Lars Kadison

263/93 Supplerende kursusmateriale til
"Lineære strukturer fra algebra
og analyse" II
af: Mogens Brun Heefelt

264/93 FOTOVOLTAISK STATUSNOTAT 2
af: Bent Sørensen

265/94 SPHERICAL FUNCTIONS ON ORDERED SYMMETRIC SPACES
To Sigurdur Helgason on his sixtyfifth birthday
by: Jacques Faraut, Joachim Hilgert and Gestur Olafsson

266/94 Kommensurabilitets-oscillationer i laterale supergitre
Fysikspeciale af: Anja Boisen, Peter Bøggild, Karen Birkelund
Vejledere: Rafael Taboryski, Poul Erik Lindelof, Peder Voetmann Christiansen

267/94 Kom til kort med matematik på Eksperimentarium - Et forslag til en opstilling
af: Charlotte Gjerrild, Jane Hansen
Vejleder: Bernhelm Booss-Bavnbek

268/94 Life is like a sewer ...
Et projekt om modellering af aorta via en model for strømning i kloakrør
af: Anders Marcussen, Anne C. Nilsson, Lone Michelsen, Per M. Hansen
Vejleder: Jesper Larsen

269/94 Dimensionsanalyse en introduktion
metaprojekt, fysik
af: Tine Guldager Christiansen, Ken Andersen, Nikolaj Hermann, Jannik Rasmussen
Vejleder: Jens Højgaard Jensen

270/94 THE IMAGE OF THE ENVELOPING ALGEBRA AND IRREDUCIBILITY OF INDUCED REPRESENTATIONS OF EXPONENTIAL LIE GROUPS
by: Jacob Jacobsen

271/94 Matematikken i Fysikken.
Opdaget eller opfundet
NAT-BAS-projekt
vejleder: Jens Højgaard Jensen

272/94 Tradition og fornyelse
Det praktiske elevarbejde i gymnasiets fysikundervisning, 1907-1988
af: Kristian Hoppe og Jeppe Guldager
Vejledning: Karin Beyer og Nils Hybel

273/94 Model for kort- og mellemdistanceløb
Verifikation af model
af: Lise Fabricius Christensen, Helle Pilemann, Bettina Sørensen
Vejleder: Mette Olufsen

274/94 MODEL 10 - en matematisk model af intravenøse anæstetikas farmakokinetik
3. modul matematik, forår 1994
af: Trine Andreasen, Bjørn Christensen, Christine Green, Anja Skjoldborg Hansen. Lisbeth Helmgaard
Vejledere: Viggo Andreasen & Jesper Larsen

275/94 Perspectives on Teichmüller and the Jahresbericht
2nd Edition
by: Bernhelm Booss-Bavnbek

276/94 Dispersionsmodellering
Projektrapport 1. modul
af: Gitte Andersen, Rehannah Borup, Lisbeth Friis, Per Gregersen, Kristina Vejrø
Vejleder: Bernhelm Booss-Bavnbek

277/94 PROJEKTARBEJDSPÆDAGOGIK - Om tre tolkninger af problemorienteret projektarbejde
af: Claus Flensted Behrens, Frederik Voetmann Christiansen, Jørn Skov Hansen, Thomas Thingstrup
Vejleder: Jens Højgaard Jensen

278/94 The Models Underlying the Anaesthesia Simulator Sophus
by: Mette Olufsen(Math-Tech), Finn Nielsen (RISØ National Laboratory), Per Føge Jensen (Herlev University Hospital), Stig Andur Pedersen (Roskilde University)

279/94 Description of a method of measuring the shear modulus of supercooled liquids and a comparison of their thermal and mechanical response functions.
af: Tage Christensen

280/94 A Course in Projective Geometry
by Lars Kadison and Matthias T. Kromann

281/94 Modellering af Det Cardiovaskulære System med Neural Pulskontrol
Projektrapport udarbejdet af:
Stefan Frello, Runa Ulsøe Johansen, Michael Poul Curt Hansen, Klaus Dahl Jensen
Vejleder: Viggo Andreasen

282/94 Parallelle algoritmer
af: Erwin Dan Nielsen, Jan Danielsen, Niels Bo Johansen

283/94 Grænser for tilfældighed
(en kaotisk talgenerator)

af: Erwin Dan Nielsen og Niels Bo Johansen

284/94 Det er ikke til at se det, hvis man ikke lige ve' det!
Gymnasiematematikkens begrundelsesproblem

En specialerapport af Peter Hauge Jensen
og Linda Kyndlev

Vejleder: Mogens Niss

285/94 Slow coevolution of a viral pathogen and its diploid host

by: Viggo Andreasen and
Freddy B. Christiansen

286/94 The energy master equation: A low-temperature approximation to Bässler's random walk model

by: Jeppe C. Dyre

287/94 A Statistical Mechanical Approximation for the Calculation of Time Auto-Correlation Functions

by: Jeppe C. Dyre

288/95 PROGRESS IN WIND ENERGY UTILIZATION

by: Bent Sørensen

289/95 Universal Time-Dependence of the Mean-Square Displacement in Extremely Rugged Energy Landscapes with Equal Minima

by: Jeppe C. Dyre and Jacob Jacobsen

290/95 Modellering af uregelmæssige bølger

Et 3.modul matematik projekt

af: Anders Marcussen, Anne Charlotte Nilsson,
Lone Michelsen, Per Mørkegaard Hansen

Vejleder: Jesper Larsen

291/95 1st Annual Report from the project

LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM

an example of using methods developed for the OECD/IEA and the US/EU fuel cycle externality study

by: Bent Sørensen

292/95 Fotovoltaisk Statusnotat 3

af: Bent Sørensen

293/95 Geometridiskussionen - hvor blev den af?

af: Lotte Ludvigsen & Jens Frandsen

Vejleder: Anders Madsen

294/95 Universets udvidelse - et metaprojekt

Af: Jesper Duelund og Birthe Friis

Vejleder: Ib Lundgaard Rasmussen

295/95 A Review of Mathematical Modeling of the Controled Cardiovascular System

By: Johnny T. Ottesen

296/95 RETIKULER den klassiske mekanik

af: Peder Voetmann Christiansen

297/95 A fluid-dynamical model of the aorta with bifurcations

by: Mette Olufsen and Johnny Ottesen

298/95 Mordet på Schrödingers kat - et metaprojekt om
to fortolkninger af kvantemekanikken

af: Maria Hermannsson, Sebastian Horst,
Christina Specht

Vejledere: Jeppe Dyre og Peder Voetmann Christiansen

299/95 ADAM under figenbladet - et kig på en samfunds-videnskabelig matematisk model

Et matematisk modelprojekt

af: Claus Dræby, Michael Hansen, Tomas Højgård Jensen

Vejleder: Jørgen Larsen

300/95 Scenarios for Greenhouse Warming Mitigation

by: Bent Sørensen

301/95 TOK Modellering af træers vækst under påvirkning af ozon

af: Glenn Møller-Holst, Marina Johannessen, Birthe
Nielsen og Bettina Sørensen

Vejleder: Jesper Larsen

302/95 KOMPRESSORER - Analyse af en matematisk model for aksialkompressorer

Projektrapport sf: Stine Bøggild, Jakob Hilmer,
Pernille Postgaard

Vejleder: Viggo Andreasen

303/95 Masterlignings-modeller af Glasovergangen

Termisk-Mekanisk Relaksation

Specialerapport udarbejdet af:

Johannes K. Nielsen, Klaus Dahl Jensen

Vejledere: Jeppe C. Dyre, Jørgen Larsen

304a/95 STATISTIKNOTER Simple binomialfordelingsmodeller

af: Jørgen Larsen

304b/95 STATISTIKNOTER Simple normalfordelingsmodeller

af: Jørgen Larsen

304c/95 STATISTIKNOTER Simple Poissonfordelingsmodeller

af: Jørgen Larsen

304d/95 STATISTIKNOTER Simple multinomialfordelingsmodeller

af: Jørgen Larsen

304e/95 STATISTIKNOTER Mindre matematisk-statistisk opslagsværk indeholdende bl.a. ordforklaringer, resuméer og tabeller

af: Jørgen Larsen

305/95 The Maslov Index:
A Functional Analytical Definition
And The Spectral Flow Formula

By: B. Booss-Bavnbek, K. Furutani

306/95 Goals of mathematics teaching

Preprint of a chapter for the forthcomming International Handbook of Mathematics Education (Alan J.Bishop, ed)

By: Mogens Niss

307/95 Habit Formation and the Thirdness of Signs

Presented at the semiotic symposium

The Emergence of Codes and Intensions as a Basis of Sign Processes

By: Peder Voetmann Christiansen

308/95 Metaforer i Fysikken

af: Marianne Wilcken Bjerregaard,
Frederik Voetmann Christiansen,
Jørn Skov Hansen, Klaus Dahl Jensen
Ole Schmidt

Vejledere: Peder Voetmann Christiansen og
Petr Viscor

309/95 Tiden og Tanken
En undersøgelse af begrebsverdenen Matematik udført ved hjælp af en analogi med tid

af: Anita Stark og Randi Petersen

Vejleder: Bernhelm Booss-Bavnbek

310/96 Kursusmateriale til "Lineære strukturer fra algebra og analyse" (E1)

af: Mogens Brun Heefelt

311/96 2nd Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM

by: Hélène Connor-Lajambe, Bernd Kuemmel,
Stefan Krüger Nielsen, Bent Sørensen

312/96 Grassmannian and Chiral Anomaly

by: B. Booss-Bavnbek, K.P.Wojciechowski

313/96 THE IRREDUCIBILITY OF CHANCE AND THE OPENNESS OF THE FUTURE
The Logical Function of Idealism in Peirce's Philosophy of Nature

By: Helmut Pape, University of Hannover

314/96 Feedback Regulation of Mammalian Cardiovascular System

By: Johnny T. Ottesen

315/96 "Rejsen til tidens indre" - Udarbejdelse af
a + b et manuskript til en fjernsynsudsendelse
+ manuskript

af: Gunhild Hune og Karina Goyle

Vejledere: Peder Voetmann Christiansen og
Bruno Ingemann

316/96 Plasmaoscillation i natriumklynger

Specialerapport af: Peter Meibom, Mikko Østergård

Vejledere: Jeppe Dyre & Jørn Borggreen

317/96 Poincaré og symplektiske algoritmer

af: Ulla Rasmussen

Vejleder: Anders Madsen

318/96 Modelling the Respiratory System

by: Tine Guldager Christiansen, Claus Dræby

Supervisors: Viggo Andreasen, Michael Danielsen

319/96 Externality Estimation of Greenhouse Warming Impacts

by: Bent Sørensen

320/96 Grassmannian and Boundary Contribution to the -Determinant

by: K.P.Wojciechowski et al.

321/96 Modelkompetencer - udvikling og afprøvning af et begrebsapparat

Specialerapport af: Nina Skov Hansen,
Christine Iversen, Kristin Troels-Smith

Vejleder: Morten Blomhøj

322/96 OPGAVESAMLING

Bredde-Kursus i Fysik 1976 - 1996

323/96 Structure and Dynamics of Symmetric Diblock Copolymers

PhD Thesis

by: Christine Maria Papadakis

324/96 Non-linearity of Baroreceptor Nerves

by: Johnny T. Ottesen

325/96 Retorik eller realitet ?
Anvendelser af matematik i det danske Gymnasiums matematikundervisning i perioden 1903 - 88

Specialerapport af Helle Pilemann

Vejleder: Mogens Niss

326/96 Bevisteori
Eksemplificeret ved Gentzens bevis for konsistensen af teorien om de naturlige tal

af: Gitte Andersen, Lise Mariane Jeppesen,
Klaus Frovin Jørgensen, Ivar Peter Zeck

Vejledere: Bernhelm Booss-Bavnbek og
Stig Andur Pedersen

327/96 NON-LINEAR MODELLING OF INTEGRATED ENERGY SUPPLY AND DEMAND MATCHING SYSTEMS

by: Bent Sørensen

328/96 Calculating Fuel Transport Emissions

by: Bernd Kuemmel

329/96 The dynamics of cocirculating influenza strains conferring partial cross-immunity

and

A model of influenza A drift evolution

by: Viggo Andreasen, Juan Lin and
Simon Levin

330/96 LONG-TERM INTEGRATION OF PHOTOVOLTAICS INTO THE GLOBAL ENERGY SYSTEM

by: Bent Sørensen

331/96 Viskøse fingre

Specialerapport af:

Vibeke Orlien og Christina Specht

Vejledere: Jacob M. Jacobsen og Jesper Larsen

332/97 ANOMAL SWELLING AF LIPIDE DOBBELTLAG

Specialerapport af:

Stine Sofia Korremann

Vejleder: Dorthe Posselt

333/97 Biodiversity Matters

an extension of methods found in the literature on monetisation of biodiversity

by: Bernd Kuemmel

334/97 LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM

by: Bernd Kuemmel and Bent Sørensen

335/97 Dynamics of Amorphous Solids and Viscous Liquids

by: Jeppe C. Dyre

336/97 PROBLEM-ORIENTATED GROUP PROJECT WORK AT ROSKILDE UNIVERSITY

by: Kathrine Legge

337/97 Verdensbankens globale befolkningsprognose - et projekt om matematisk modellering

af: Jørn Chr. Bendtsen, Kurt Jensen,
Per Pauli Petersen

Vejleder: Jørgen Larsen

338/97 Kvantisering af nanolederes elektriske ledningsevne

Første modul fysikprojekt

af: Søren Dam, Esben Danielsen, Martin Niss,
Esben Friis Pedersen, Frederik Resen Steenstrup

Vejleder: Tage Christensen

339/97 Defining Discipline

by: Wolfgang Coy

340/97 Prime ends revisited - a geometric point of view -

by: Carsten Lunde Petersen

341/97 Two chapters on the teaching, learning and assessment of geometry

by Mogens Niss

342/97 LONG-TERM SCENARIOS FOR GLOBAL ENERGY DEMAND AND SUPPLY

A global clean fossil scenario discussion paper prepared by Bernd Kuemmel

Project leader: Bent Sørensen

343/97 IMPORT/EKSPORT-POLITIK SOM REDSKAB TIL OPTIMERET UDNYTTELSE AF EL PRODUCERET PÅ VE-ANLÆG

af: Peter Meibom, Torben Svendsen, Bent Sørensen

344/97 Puzzles and Siegel disks

by Carsten Lunde Petersen

345/98 Modeling the Arterial System with Reference to an Anestesia Simulator

Ph.D. Thesis

by: Mette Sofie Olufsen

346/98 Klyngedannelse i en hulkatode-forstøvningsproces

af: Sebastian Horst
Vejledere: Jørn Borggren, NBI, Niels Boye Olsen

347/98 Verificering af Matematiske Modeller - en analyse af Den Danske Eulerske Model

af: Jonas Blomqvist, Tom Pedersen, Karen Timmermann,
Lisbet Øhlenschlæger

Vejleder: Bernhelm Booss-Bavnbek

348/98 Case study of the environmental permission procedure and the environmental impact assessment for power plants in Denmark

by: Stefan Krüger Nielsen

Project leader: Bent Sørensen

349/98 Tre rapporter fra FAGMAT - et projekt om tal og faglig matematik i arbejdsmarkedsuddannelserne

af: Lena Lindenskov og Tine Wedege

350/98 OPGAVESAMLING - Bredde-Kursus i Fysik 1976 - 1998

Erstatter teksterne 3/78, 261/93 og 322/96

351/98 Aspects of the Nature and State of Research in Mathematics Education

by: Mogens Niss