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Abstract

The paper provides a proof of the Herman-Światec Theorem [Her] that a C^3 critical circle homeomorphism with one non-flat critical point and irrational rotation number θ is quasi-symmetrically conjugate to the corresponding rigid rotation, if and only if θ is Diophantine of exponent 2. Moreover the Herman-Światec Theorem is applied to prove there exist quadratic Siegel polynomials, $P_\theta(z) = e^{i2\pi\theta}z + z^2$, with θ of unbounded type, for which the Julia set is locally connected.

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Abstract

The paper provides a proof of the Herman-Światec Theorem [Her] that a C^3 critical circle homeomorphism with one non-flat critical point and irrational rotation number θ is quasi-symmetrically conjugate to the corresponding rigid rotation, if and only if θ is Diophantine of exponent 2. Moreover the Herman-Światec Theorem is applied to prove there exist quadratic Siegel polynomials, $P_\theta(z) = \mathrm{e}^{i2\pi\theta}z + z^2$, with θ of unbounded type, for which the Julia set is locally connected.

1 Introduction

Let f be a C^3, orientation preserving circle homeomorphism with irrational rotation number θ. We suppose moreover that f has exactly one critical point a_0, which is non flat. We shall identify the circle with $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ and give $\mathbb{T}$ the induced orientation. Moreover by abuse of notation we shall not distinguish f and lifts of f to $\mathbb{R}$. Technically it suffices that :

Hypothesis 1.1 *The homeomorphism f is C^1, has a critical point a_0 and there exists a neighbourhood $W \subseteq \mathbb{T}$ of a_0 such that :*

1. *f is C^3 on W and has negative Schwarzian derivative Sf on $W \setminus a_0$.*
2. *There exists a constant $A > 0$ and $l \in \mathbb{N}$ such that*

$$\forall x \in W : A|x - a_0|^{2l} < f'(x) < 2A|x - a_0|^{2l}$$

3. *The variation of $\log f'$ on $\mathbb{T} \setminus W$ is bounded by some number, say ρ.*

4. *f has irrational rotation number $\theta \in \mathbb{T}$*

In this article we shall prove the following Theorem:

Theorem 1.2 (Herman-Swiatec, 86) *Let f be a circle homeomorphism with rotation number θ and satisfying the hypotheses above. Then f is c-quasi symmetrically conjugate to the rigid rotation (of angle θ), if and only if θ has constant type. Moreover the constant $c > 1$ depends only on the bound N on the coefficients of the continued fraction expansion for θ.*

The proof is based partially on a more extensive, but unpublished manuscript by J.-C. Yoccoz, [Yoc].

For $|\theta| = 1$ let c_θ be the complex number for which the quadratic polynomial $Q_{c_\theta}(z) = z^2 + c_\theta$ has a fixed point with multiplier $\exp(i2\pi\theta)$. Herman has proved that there exists irrational numbers θ not of constant type for which the quadratic polynomial Q_{c_θ} has a Siegel disk, whose boundary is a Jordan curve containing the critical point 0. Here we take his proof one step further and prove:

Theorem 1.3 *There exist irrational θ not of constant type, such that the Julia set of the quadratic polynomial Q_{c_θ} is locally connected.*

Following the established conventions of the subject, we shall freely use the letter c to denote any constant, which depends only on Df. The obtained constants are by no means optimal. It should be noted however that unless stated explicitly otherwise *the constants c depend only on those macroscopic properties of Df, stipulated in the first three items of the above hypotheses.* In particular the constants do not depend on the irrational rotation number θ of f. Thus given some f then for any $\eta \in \mathbb{T}$ such that $f_\eta = f + \eta$ have irrational rotation number, the constants only depend on f.

2 Notation and prerequisites

Notation 2.1 *We shall use freely the following notations:*

1. *The sequence of irreducible rational numbers $\left\{\frac{p_n}{q_n}\right\}_{n\geq 0}$ will denote the convergents of θ. Moreover the sequence $\{b_n\}_{n\in\mathbb{N}}$ denotes the coefficients of the continued fraction expansion of θ, so that $p_n = b_n p_{n-1} + p_{n-2}$ and $q_n = b_n q_{n-1} + q_{n-2}$. The number θ is Diophantine of exponent 2*

or equivalently, has constant type, if and only if the sequence b_n is bounded.

2. *For each $i \in \mathbb{Z}$ define $a_i = f^{-i}(a_0)$, (note the critical orbit is indexed time reversedly).*
3. *For $a < b < c < d < a+1$ in $\mathbb{R}$ we define the cross-ratio :*

$$[a,b,c,d] = \frac{b-a}{c-a}\frac{d-c}{d-b}$$

4. *For $n \geq 0$ we define*

$$I_n(x) = \begin{cases} [x, f^{-q_n}(x)], & n \text{ odd}, \\ [f^{-q_n}(x), x], & n \text{ even}. \end{cases}$$

$$m_n(x) = |f^{q_n}(x) - x|,$$

$$K_n = \{a_i \mid 0 \leq i < q_n\}, \qquad K_n^* = K_n - \{a_{q_n}\}$$

Let us recall some basic properties of circle homeomorphism with irrational rotation numbers in general and critical such circle homeomorphism in particular. The first property is the Poincaré semi-conjugation Theorem.

Theorem 2.2 (Poincaré semi-conjugation Theorem) *Let f be any orientation preserving circle homeomorphism with irrational rotation number θ. Then f is semi conjugate to the rigid rotation $R_\theta(z) = z + \theta$ with angle θ, i.e. there exists a continuous map $\phi : \mathbb{T} \longrightarrow \mathbb{T}$ such that $\phi \circ f = R_\theta \circ \phi$.*

The Poincaré semi-conjugation Theorem implies that the combinatorial orbit structure of any orientation preserving circle homeomorphism is the same as that for the rigid rotation.

Remark 2.3 *Recall the following properties of circle homeomorphisms:*

1. *For even n (respectively for odd n) the point of K_n^* following (respectively preceding) a point a_i for $0 \leq i < q_n$ is a_j with*

$$j = \begin{cases} i + q_{n-1}, & \text{if } 0 \leq i < q_n - q_{n-1} \\ i + q_{n-1} - q_n, & \text{if } q_n - q_{n-1} < i < q_n \end{cases}$$

2. *For any $x \in \mathbb{T}$ and any $n \geq 1$ all the intervals $I_n(f^j(x))$ for $0 \leq j < q_{n+1}$ and $I_{n+1}(f^j(x))$ for $0 \leq j < q_n$ have mutually disjoint interiors.*

3. *For any $x \in \mathbb{T}$*

$$\mathbb{T} = \bigcup_{0 \leq j < q_{n+1}+q_n} I_n(f^j(x)) = \bigcup_{0 \leq j < q_{n+1}} I_n(f^j(x)) \cup \bigcup_{0 \leq j < q_n} I_{n+1}(f^j(x))$$

4. *Let $a < b < c < d < a+1$ in $\mathbb{R}$ be arbitrary and let $N \geq 1$. If the open interval $]a,d[$ does not contain any of the points a_j, $0 \leq j < N$ and if $Sf^N \leq 0$ on $]a,d[$, then*

$$[f^N(a), f^N(b), f^N(c), f^N(d)] \leq [a,b,c,d].$$

Here as elsewhere Sf denotes the Schwarzian derivative of f and is given by

$$\begin{aligned} Sf &= D^2 \log Df - \frac{1}{2}(D \log Df)^2 \\ &= (D^3 Df - \frac{3}{2}(D^2 f)^2)(Df)^{-2} \\ &= -2(Df)^{-\frac{1}{2}} \cdot D^2(Df)^{-\frac{1}{2}}. \end{aligned}$$

Theorem 2.4 (Światec) *There exists a constant $c > 1$ such that for any quadruple $a < b < c < d < a+1$ in $\mathbb{R}$ and any $N > 0$ one has*

$$[f^N(a), f^N(b), f^N(c), f^N(d)] \leq c^m \cdot [a,b,c,d],$$

where m is the covering number :

$$m = \max_{x \in \mathbb{T}} \#\{j | 0 \leq j < N \text{ and } x \in]f^j(a), f^j(d)[\}.$$

Proof : This is the *Cross-Ratio Inequality* of Światec [Swi, p.112]. **q.e.d.**

Combining Remark 2.3.4. and Theorem 2.4 we have

Corollary 2.5 *Suppose f is C^3 and $Sf(x) \leq 0$, $\forall x \in \mathbb{T} - \{a_0\}$. Then there exists a constant $c > 1$ such that for any $N > 0$ and any $a < b < c < d < a+1$ in $\mathbb{R}$ one has*

$$[f^N(a), f^N(b), f^N(c), f^N(d)] \leq c^m \cdot [a,b,c,d],$$

where $m = \#\{j | 0 \leq j < N$ and $a_0 \in]f^j(a), f^j(d)[\}$.

3 The a priori bounds

Let us fix $n \geq 0$, say even to fix the ideas. Set $q = q_n$, $m(x) = m_n(x)$, $I(x) = I_n(x)$ for $x \in \mathbb{T}$ and define

$$\Delta(x) = [f^{-2q}(x), f^{-q}(x), x, f^q(x)] \qquad \text{for } x \in \mathbb{T}.$$

Lemma 3.1 *There exists a constant $c > 1$ such that $\forall x \in \mathbb{T}$ and for all $z \in I(x)$*

$$c^{-1} \cdot \min\{m(z), m(f^q(z))\} \leq m(x) \leq m(z) + m(f^q(z)).$$

Proof : The right-hand inequality is a useful but trivial observation. Towards the left hand inequality, let us prove that, there exists a constant $c_1 > 1$ such that

$$m(x) \geq c_1^{-1} \cdot \min\{m(f^q(x)), m(f^{-q}(x))\} \qquad \forall x \in \mathbb{T}. \tag{1}$$

For any $x \in \mathbb{T}$ the inequality $m(x) \leq \min\{m(f^q(x)), m(f^{-q(x)})\}$ implies $\Delta(x) \geq \frac{1}{4}$. Write $r(x) \cdot m(x) = \min\{m(f^q(x)), m(f^{-q}(x))\}$, and assume $r(x) > 1$ then

$$\begin{aligned}\Delta(f^{-q}(x)) &= \frac{m(f^{-2q}(x))}{m(f^{-2q}(x)) + m(f^{-q}(x))} \cdot \frac{m(x)}{m(x) + m(f^{-q}(x))} \\ &\leq \frac{m(x)}{m(x) + m(f^{-q}(x))} \leq \frac{1}{1 + r(x)}.\end{aligned}$$

On the other hand the intervals $f^j(]f^{-3q}(x), x[)$, $0 \leq j < q$ cover any point

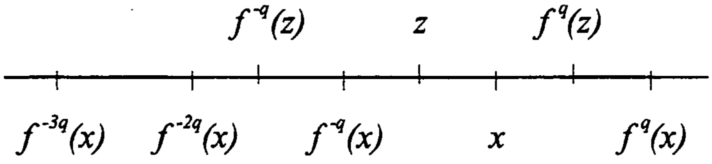

Figure 1:

$y \in \mathbb{T}$ at most 3 times. Let $c_0 > 1$ be the constant of the Światec Theorem (2.4), then

$$\frac{1}{4} \leq \Delta(x) \leq c_0^3 \cdot \Delta(f^{-q}(x)) \leq \frac{c_0^3}{1 + r(x)},$$

thus $r(x) \leq 4 \cdot c_0^3 - 1$.

Let $c = 4 \cdot c_0^3$, so that the constant $c_1 = c - 1$ applies in (1), by arbitrariness of $x \in \mathbb{T}$. Let $x \in \mathbb{T}$ and $z \in I(x)$ be arbitrary. Then by (1) either

$$m(z) < m(f^{-q}(x)) + m(x) \leq (c_1 + 1) \cdot m(x) = c \cdot m(x) \qquad \text{or}$$
$$m(f^q(z)) < m(f^q(x)) + m(x) \geq (c_1 + 1) \cdot m(x) = c \cdot m(x).$$

q.e.d.

Proposition 3.2 *There exists a constant $c > 1$ such that for all $x \in \mathbb{T}$ and $z \in I(x)$ one has*

$$c^{-1} \cdot m(x) \leq m(z) \leq c \cdot m(x).$$

Proof : By the above Lemma 3.1 it suffices to prove that there exists $c_1 > 1$ such that for all $z \in \mathbb{T}$

$$c_1^{-1} \cdot m(z) \leq m(f^{-q}(z)) \leq c_1 \cdot m(z) \tag{2}$$

Let $y \in \mathbb{T}$ be a point where the continuous function $z \mapsto m(z)$ attains its minimum, and let $y_j = f^{-j}(y)$ for all $j \in \mathbb{Z}$. Given any $z \in \mathbb{T}$ let j be the minimal non negative integer with $y_j \in I_n(f^{2q}(z))$. Then $0 \leq j < q_{n+1} + q_n$ by Remark 2.3.2. Thus by the Światec Theorem there exists $c_1 > 1$ such that $c_1^{-1} \leq \Delta(y_j)$, because the intervals $]y_{i+2q}, y_{i-q}[$, $0 < i \leq j$ cover any point of the circle at most three times. Furthermore again by the Światec Theorem we can suppose $\Delta(y_{j+q}), \Delta(j + 2q) \geq c_1^{-1}$ increasing c_1 if necessary. Writing out the definitions of $\Delta(y_j), \Delta(y_{j+q}), \Delta(y_{j+2q})$ we obtain for $c_2 = c_1 + 1$ that:

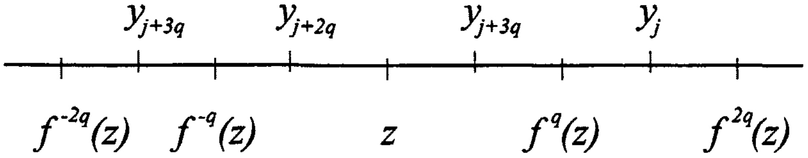

Figure 2:

$$c_1^{-1} \leq \Delta(y_j) \Rightarrow m(y_j) \leq c_2 m(y_{j+q})$$
$$c_1^{-1} \leq \Delta(y_j) \Rightarrow \begin{cases} m(y_{j+q}) \leq c_2 m(y_j) \\ m(y_{j+q}) \leq c_2 m(y_{j+2q}) \end{cases}$$
$$c_1^{-1} \leq \Delta(y_{j+2q}) \Rightarrow m(y_{j+2q}) \leq c_2 m(y_{j+q})$$

So that

$$c_2^{-1} m(y_{j+q}) \leq m(y_j), m(y_{j+2q}) \leq c_2 m(y_{j+q}) \tag{3}$$

Let c_3 be the constant from Lemma 3.1 then

$$c_3^{-1} \min\{y_{j+2q}, y_{j+q}\} \leq m(z) \leq y_{j+2q} + y_{j+q} \tag{4}$$
$$c_3^{-1} \min\{y_j, y_{j+q}\} \leq m(f^q(z)) \leq y_j + y_{j+q} \tag{5}$$

Combining the estimates (3) with the pair of estimates (4) and (5) we obtain (2), and the Proposition follows. **q.e.d.**

Corollary 3.3 *The circle homeomorphism f is minimal, i.e every orbit is dense and any Poincaré semi conjugacy is a homeomorphism and true conjugacy.*

Proof : Suppose to the contrary that f is a Denjoy counter example with invariant Cantor set L. Let z be a left endpoint of some interval I of the complement of L. Then

$$\lim_{n\to\infty} m_{2n}(z) = 0,$$
$$\lim_{n\to\infty} m_{2n}(f^{q_{2n}}(z)) = |I| > 0,$$

which contradicts Proposition 3.2. Here $|I|$ denotes the length of the interval I. **q.e.d.**

Theorem 3.4 *There exists a constant $c > 1$ such that*

$$\forall x \in K_n : m_{n-1}(x) \leq c \cdot m_n(x).$$

Proof : To fix the ideas let us suppose n is even and define for $i \in \mathbb{Z}$

$$\begin{aligned}\Sigma_i &= [a_{i+q_n}, a_i, a_{i+q_{n-1}}, a_{i+2q_{n-1}}] \\ &= \frac{m_n(a_i)}{m_n(a_i) + m_{n-1}(a_i)} \cdot \frac{m_{n-1}(a_{i+2q_{n-1}})}{m_{n-1}(a_{i+q_{n-1}}) + m_{n-1}(a_{i+2q_{n-1}})}\end{aligned}$$

and

$$u_i = \frac{m_n(a_i)}{m_{n-1}(a_i)} = \frac{a_i - a_{i+q_n}}{a_{i+q_{n-1}} - a_i}.$$

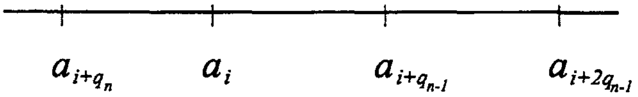

Figure 3:

then by Proposition 3.2 there exists $c_1 > 1$ such that

$$c_1^{-1}\frac{u_i}{1+u_i} \le \Sigma_i \le u_i \tag{6}$$

Thus it suffices to prove that there exists $c > 1$ such that

$$c^{-1} \le \Sigma_i \qquad \text{for } 0 \le i \le q_n. \tag{7}$$

Moreover by the Światec-Theorem it suffices to prove (7) in the case $i = 0$, because the intervals $]a_{i+q_n}, a_{i+2q_{n-1}}[$, $0 < i \le q_n$ cover any point of the circle at most three times.

Comparing interval by interval and using Proposition 3.2 we find that there exists $c_2 > 1$ such that

$$\Sigma_0 \le c_2 \cdot \Sigma_{-q_n}. \tag{8}$$

Increasing n if necessary we can suppose $I_{n-1}(a_0), I_n(a_0) \subset W$ so that (by integrating f') there exists $c_3 > 1$ with

$$\Sigma_{-1} \le u_{-1} \le c_3 u_0^{2l+1}.$$

Invoking Theorem 2.4 again (from Σ_{-1} to Σ_{-q_n}) we can suppose, increasing c_3 if necessary that

$$\Sigma_{-q_n} \le c_3 \cdot u_0^{2l+1}. \tag{9}$$

Suppose $u_0 < 1$ we then obtain from (6), (8) and (9)

$$\frac{u_0}{2c_1} \le \Sigma_0 \le c_2 \cdot c_3 \cdot u_0^{2l+1}$$

This shows that both u_0 and Σ_0 are bounded from below and completes the proof. **q.e.d.**

The above Theorem/or the following Corollary is often referred to as a-priori real bounds for critical circle maps. They are used as a bootstrap to control the geometry and topology of holomorphic maps, which restricts to critical circle homeomorphisms. See for example [dF], [dFdM], [dFaWdM], [GS], [Pet] and [Yam]. The papers [GS] and [dFdM] also contain proofs of the a-priori real bounds, not much different from the one presented here.

Corollary 3.5 (a-priori real bounds) *There exists a constant* $c > 1$ *such that*

$$\forall\, n \geq 2 \ : \ |a_0 - a_{q_{n-1}}| \leq c \cdot |a_{q_n} - a_0|.$$

4 Quasi-symmetric conjugacies

Corollary 4.1 *Suppose* θ *has constant type. Then there exists a constant* $c > 1$ *depending only on* Df *and on the bound* N *on the coefficients* b_n *so that*

$$\forall z \in \mathbb{T}, \forall n \in \mathbb{N} : \frac{m_n(z)}{m_{n+1}(z)} \leq c.$$

Proof : Let $z \in \mathbb{T}$ be arbitrary. Given $n \in \mathbb{N}$ there exists $x \in K_n$ such that either $z \in I_{n-1}(z)$ or $z \in I_{n-1}(f^{-q_{n-1}}(z))$ by Remark 2.3.2. For this x there exists $k \leq N+1$ such that $z \in I_n(f^{\pm k q_n})$. Let c_1 be the constant from Proposition 3.2 and let c_2 be the constant from Proposition 3.4 then we obtain

$$\frac{m_{n-1}(z)}{m_n(z)} \leq c_2 c_1^{k+1} \leq c_2 c_1^{N+2}.$$

q.e.d.

For $n \in \mathbb{N}$ let $M_n = d(\mathbb{Z}, q_n\theta) = |z - R_\theta^{q_n}(z)|$, where the last equality holds for any $z \in \mathbb{T}$ exactly because R_θ is a rigid rotation. Then for any $n \in \mathbb{N}$

$$a_{n+1} M_n < M_{n-1} < (1 + a_{n+1}) M_n.$$

Corollary 4.2 *Suppose* θ *has constant type. Then there exists a constant* $c > 1$ *depending only on* Df *and on the bound* N *on the coefficients* b_n *so that* f *is c-quasi-symmetrically conjugate to the rigid rotation* R_θ.

Proof : By Corollary 3.3 there exists a homeomorphism $h : \mathbb{T} \longrightarrow \mathbb{T}$ conjugating the rigid rotation R_θ to f. Let $z \in \mathbb{T}$ and $0 < \delta < \frac{1}{2}$ be arbitrary. Choose $n \in \mathbb{N}$ such that $M_{n+1} < \delta \leq M_n$. Let us suppose n is even to fix the ideas (n odd being similar) then

$$m_{n+1}(h(z)) \leq |h([z, z+\delta])| \leq m_n(f^{q_n}(h(z))) \tag{10}$$
$$m_{n+1}(f^{q_{n+1}}(h(z))) \leq |h([z-\delta, z])| \quad \leq m_n(h(z)). \tag{11}$$

Let c_1 be the constant from Proposition 3.2 and let c_2 be the constant from

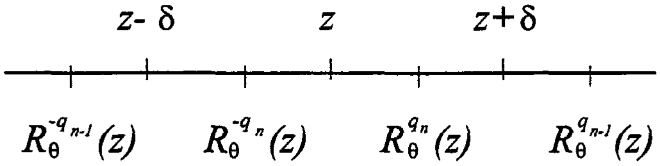

Figure 4:

Proposition 4.1 then we obtain

$$\frac{1}{c_2} \leq \frac{|h([z, z+\delta])|}{|h([z-\delta, z])|} \leq c_1^2 c_2.$$

q.e.d.

Corollary 4.3 *Suppose θ does not have constant type. Then f is not quasi symmetrically conjugate to the rigid rotation.*

Proof : It suffices to prove that a homeomorphism $h : \mathbb{T} \longrightarrow \mathbb{T}$ conjugating f to the rigid rotation R_θ is not quasi symmetric. To this end choose a subsequence $\{b_{n_k}\}_{k \in \mathbb{N}}$ diverging to ∞. Then

$$\frac{h(I_{n_k-1}(a_0))}{h(I_{n_k-2}(a_0))} = \frac{M_{n_k-1}}{M_{n_k-2}} \leq \frac{1}{a_{n_k}},$$

which contradicts that h is quasi symmetric, because $I_{n_k-1}(a_0)$ and $I_{n_k-2}(a_0)$ are commensurable by Corollary 3.4. **q.e.d.**

5 Local connectivity of Julia sets

Let $f_0 : \overline{\mathbb{C}} \longrightarrow \overline{\mathbb{C}}$ denote the Blaschke product

$$f_0(z) = z^2 \frac{z-3}{1-3z}.$$

For each irrational $\theta \in]0,1[$ let λ_θ be the unique unimodular constant for which the restriction $f_\theta = \lambda_\theta \cdot f_0 : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$ has rotation number θ. Let J_{f_θ} denote the Julia set of f_θ and let $J_\theta \subset J_{f_\theta}$ denote the boundary of the immediate attracted basin $\Lambda_\theta(\infty)$ for ∞. It was proved in [Pet] that

Theorem 5.1 *For every irrational θ the subset J_θ and the full Julia set J_{f_θ} are locally connected.*

We shall prove the following Theorem which combined with Theorem 5.1 implies Theorem 1.3:

Theorem 5.2 *There exists irrational θ of unbounded type for which the Julia set J_{c_θ} for Q_{c_θ} is homeomorphic to J_θ.*

The equivalent of Theorem 5.2 in the case of constant type θ was proven simultaneously by Douady, Ghys, Herman, Hubbard and Shishikura, who all used the following procedure:
Suppose the number θ is of constant type. Let $h : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$ denote the conjugacy, between f_θ and the rigid rotation R_θ. As θ has constant type, the Herman-Światec Theorem 1.2 implies that the map h is c-quasi-symmetric, with a constant $c > 1$, which only depends on the bound N on the coefficients of the continued fraction expansion for θ. Let $\psi_\theta : \overline{\mathbb{D}} \longrightarrow \overline{\mathbb{D}}$ be a K-quasiconformal extension of h (with a $K > 1$ which only depends on c and hence on N) and define $F_\theta : \overline{\mathbb{C}} \longrightarrow \overline{\mathbb{C}}$ by

$$F_\theta(z) = \begin{cases} f_\theta(z) & \text{if } z \in \overline{\mathbb{C}} \setminus \mathbb{D} \\ \psi_\theta^{-1} \circ R_\theta \circ \psi_\theta & \text{if } z \in \overline{\mathbb{D}}. \end{cases}$$

Let σ_0 denote the standard almost complex structure on $\overline{\mathbb{C}}$, and let σ_θ denote the F_θ invariant almost complex structure given by

$$\sigma_\theta(z) = \begin{cases} \psi^*(\sigma_0)(z) & \text{if } z \in \mathbb{D} \\ ((\psi \circ F_\theta^n)^*(\sigma_0)(z) & \text{if } F^n(z) \in \mathbb{D} \\ \sigma_0(z) & \text{othervice.} \end{cases}$$

Finally let $\phi_\theta : \overline{\mathbb{C}} \longrightarrow \overline{\mathbb{C}}$ be the integrating map, the quasi-conformal homeomorphism for which $\sigma_\theta = \phi_\theta^*(\sigma_0)$, normalized so that the conjugate map $\phi_\theta \circ F_\theta \circ \phi_\theta^{-1}$ equals the quadratic polynomial $Q_\theta(z) = z^2 + c_\theta$, with an indifferent fixed point of multiplier $e^{i2\pi\theta}$.

The conjugacy $h : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$ between f_θ and R_θ, normalized by $h(1) = 1$ depends continuously on θ in the C^0-topology, because f_θ and R_θ depend continuously on θ in the C^0-topology.

Thus by choosing the quasi-conformal extension ψ_θ in some canonical way, say by using the Ahlfors-Beurling extension, [LV, Th. 6.3] or the Douady-Earle extension, [DE] we can suppose that also the quasi-conformal extensions ψ_θ depend continuously on θ in the C^0-topology. To fix the ideas we choose to use say the Douady-Earle extension for every θ.

Lemma 5.3 *Let (θ_n) be a sequence of irrationals converging to some irrational θ_0. Suppose the θ_n have constant type with a uniform constant type bound N. Then θ_0 has constant type with bound N and the two sequences (ϕ_{θ_n}) and $\phi_{\theta_n}^{-1}$ converges to ϕ_{θ_0} respectively $\phi_{\theta_0}^{-1}$ in the C^0-topology.*

Proof : The coefficients of θ_n converge to those of θ_0 so that θ_0 has constant type with bound N. We shall prove that ϕ_{θ_n} converges uniformly (C^0) to ϕ_{θ_0}, from which the Lemma follows.

There exists $K(N) \geq 1$, such that each map ϕ_{θ_n} is K-quasi conformal. Thus extracting a subsequence, if necessary we can assume the sequence converge C^0 to a K quasi-conformal homeomorphism ϕ_0. We shall prove that

$$\phi_0 = \phi_{\theta_0}. \tag{12}$$

As (F_{θ_n}) converges C^0 to F_{θ_0} and $(Q_{c_{\theta_n}})$ converges C^0 to $Q_{c_{\theta_0}}$ the map ϕ_0 conjugates F_{θ_0} to Q_{θ_0} and the restriction of ϕ_0 to the immediate attracted basin of ∞ is biholomorphic. Thus (12) holds on the closure of the immediate attracted basin of ∞, by uniqueness of the holomorphic conjugacy. Moreover (12) also holds on the 'Siegel-disk' $\mathbb{D}$, because the Douady-Earle extension depends continuously on the boundary data h_θ. Finally it holds on the grand orbit of $\mathbb{D}$, because both ϕ_0 and ϕ_{θ_0} conjugates dynamics. Thus any (C^0) limit function of the ϕ_{θ_n} equals ϕ_{θ_0}. Combining this with the precompactness of the sequence (ϕ_{θ_n}) and the compactness of $\overline{\mathbb{C}}$ completes the proof. **q.e.d.**

Proof of Theorem 5.2: Let (ϵ_n) be a summable sequence of strictly positive numbers. Let (d_n) be any unbounded sequence of natural numbers and let θ_1 be any irrational of constant type. Given a natural number k_1 let for each $n \geq 1$ the irrational number $\theta_{1,n}$ be obtained from θ_1 by replacing the $(n+k_1)$-th coefficient b_{n+k_1} in the continued fraction expansion of θ_1 with d_1. Then the sequence $(\theta_{1,n})$ converges to θ_1. Hence by Lemma 5.3 there exists n such that

$$d_{C^0}(\phi_{\theta_1}, \phi_{\theta_{1,n}}) \leq \epsilon_1 \qquad \text{and} \qquad d_{C^0}(\phi_{\theta_1}^{-1}, \phi_{\theta_{1,n}}^{-1}) \leq \epsilon_1$$

Let $\theta_2 = \theta_{1,n}$ and let $k_2 = k_1 + n$. Then we may restart the process using θ_2, k_2, d_2 and ϵ_2 to obtain an irrational θ_3 of constant type, but with $b_{k_2} = d_1$ and $b_{k_3} = d_2$. Proceeding inductively we find a sequence (θ_n) of irrational numbers of constant type converging to some irrational of non constant type θ_0 and a Cauchy sequence (in the C^0-norm) of (quasi-conformal) homeomorphisms (ϕ_{θ_n}), such that also the the sequence of inverse maps is a Cauchy sequence. Let ϕ_0 denote the limit of the sequence (ϕ_{θ_n}), then ϕ_0 is a homeomorphism conjugating F_{θ_0} to Q_{c_0}. In particular $J_{c_{\theta_0}} = \phi_0(J_{\theta_0})$. **q.e.d.**

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