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Global Dimension, Tower of Algebras, and Jones Index of Split Separable Subalgebras with Unitality Condition

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In Memory of Oscar Goldman, Mathematician and Family Friend

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I wish to extend warm thanks to teachers, family, and colleagues Daniel Kastler and Peter Seibt at C.P.T., C.N.R.S.-Luminy, and Jesus Gonzalo of Universidad Autonoma de Madrid. *My current address: Department of Mathematics, University of California, Berkeley, CA, 94720 U.S.A.*

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1 Introduction

Let M and N be finite factors, i.e., the set of values of the trace on projections are finite, and the center is trivial, with N an subfactor of M where $1_N = 1_M$. Let $E : M \to N$ denote the (unique trace-preserving) conditional expectation of M onto N, i.e., E is an N-N bimodule morphism of the obvious bimodule structures on M and N, and E(n) = n $\forall n \in N$.

The basic construction builds a finite factor M_1 containing M as a subfactor with properties among which we mention:

(1) M_1 is singly generated as an M-M bimodule by a projection e_1.

(2) $e_1 m e_1 = E(m)e_1 = e_1 E(m)$.

(3) Given $m \in M$, $me_1 = 0$ or $e_1 m = 0 \Rightarrow m = 0$.

(4) The unique trace-preserving conditional expectation $E_1 : M_1 \to M$ is given by $E_1(e_1) = \tau 1$ for some positive number τ.

The Jones index [M:N] of N in M is τ^{-1}, a real number between 1 and ∞ with interesting restrictions on its value [12]. Iterating the basic construction leads to a tower of algebras $N \subseteq M \subseteq \cdots \subseteq M_i \subseteq M_{i+1} \subseteq \cdots$ and a family of projections $\{e_i\}_{i=1}^{\infty}$ in the direct limit M_∞ (also a finite factor). These projections satisfy relations that permit representations of the infinite braid group and certain infinite Hecke algebras:

(i) $e_i e_{i+1} e_i = \tau e_i$,

(ii) $e_{i+1} e_i e_{i+1} = \tau e_{i+1}$

(iii) $e_i e_j = e_j e_i \quad i - j \geq 2$.

The trace of M_∞ plays a critical role in defining the Jones polynomial invariant of knot theory [13].

The next theorem is crucial to this paper.

Theorem 1.1 (Pimsner,Popa [26]) *If $[M : N] < \infty$ with n the integer part of [M:N], then there exists a family $\{m_j\}_{j=1}^{n+1}$ of elements in M satisfying the properties:*

(a) $E(m_j^* m_k) = 0, \quad j \neq k$,

(b) $E(m_j^* m_j) = 1, \quad 1 \leq j \leq n$;

(c) $E(m_{n+1}^* m_{n+1})$ is a projection in N of trace [M:N] - n.

(d) $\sum_{j=1}^{n+1} m_j e_1 m_j^* = 1$;

(e) $\sum_{j=1}^{n+1} m_j m_j^* = [M : N]$.

It follows from the theorem that M_N is a finitely generated projective right (or left, see section 4) N-module with dual basis $\{m_j\}_{j=1}^{n+1}$ in M and $\{E(m_j^* -)\}_{j=1}^{n+1}$ in $Hom_N(M, N)$. After giving several applications of their theorem, Pimsner and Popa propose that N and M share many properties for an unspecified reason related to their theorem [26, p. 67]. In this paper we abstract algebraic properties from their theorem that ensures that numerous algebraic properties are indeed shared by subalgebra N and algebra M, though N and M are neither isomorphic nor Morita equivalent.

We place three conditions on a (not necessarily involutive but unital) subalgebra S of A over a commutative ring k that ensures that properties of an algebra definable in terms of module categories (with certain mild restrictions) are shared by both S and A. In addition, from S and A we obtain the basic construction, the Jones index, tower of algebras with Morita equivalence of every other algebra in the tower, and countably many idempotents satisfying braid relations. The conditions we impose on the subalgebra S of A, sometimes referred to in the reverse as the extension A of S, are the following three with labels for purposes of reference:

(I)**Relative separability.** A is a separable extension of S, i.e., A has relative cohomological dimension zero over S [10], or equivalently, there exists a (separating) element e in $A \otimes_S A$ satisfying i) $\mu_S(e) = 1$ where $\mu_S : A \otimes_S A \to A$ is the multiplication map; ii) ae = ea $\forall a \in A$ [16].

(II) **Split extension.** S is a direct summand of A as S-S bimodules, i.e., a conditional expectation $E : A \to S$ exists.

(III) **Unitality.** There exists a conditional expectation $E : A \to S$ and separating element $e = \tau \sum_{i=1}^{n} x_i \otimes y_i$ with $\tau \in k$ such that

$$\sum_{i=1}^{n} E(x_i) y_i = 1 = \sum_{i=1}^{n} x_i E(y_i)$$

If the subalgebra S of A satisfies (I), (II), *and* (III) we refer to S as a split separable subalgebra of A, or A as a split separable extension of S. What condition (I) has to do with the finite factors is given by joint work with D. Kastler:

Proposition 1.1 ([18]) *M is a separable extension of N with separating element*

$$\frac{1}{[M:N]} \sum_{j=1}^{n+1} m_j \otimes_S m_j^*.$$

Together with the unique trace-preserving conditional expectation $E : M \to N$, it is an exercise in applying (d) and (2) above to show that property (III) is satisfied. The author and Kastler have also shown that type III subfactors of finite Kosaki index are relatively separable as well. Numerous examples of split separable extensions exist in algebra, most notably subgroups of finite index and the group algebra extensions they generate (or twisted, smashed versions thereof).

A convenience to our paper is the language of change of rings, which we review. For a unital ring homomorphism $R \to S$, we say an S-module *restricts* to the R-module obtained by using the arrow, while an R-module M *induces* to the S-module $S \otimes_R M$. The arrow $R \to S$ is referred to as a projective, flat or finitely generated change of rings in case S is projective, flat, or finitely generated, respectively, in the natural R-module structure induced from the arrow. A property of modules is an assignment to each ring R, of a subclass

Φ_R of all R-modules, **R-Mod.** A property is said to induce or restrict in case every module in Φ_R induces to one in Φ_S, or every module in Φ_S restricts to one in Φ_R. Note that projectivity or flatness always induce but only restrict under projective or flat change of rings.

Now it is a tautology that a subalgebra S shares property X with A means

- S has property X if and only if A has property X.

Call a property X of modules, $\Phi_R \subseteq \mathbf{R-Mod}$, *nice* if it induces and restricts under a finitely generated projective change of rings, and direct summands of members of Φ_R are again members for each ring R. For example, flatness and projectivity are nice properties. Then call a property Y of rings *homological* if R has property Y is equivalent to two nice properties of modules coinciding, $\Phi_R = \Psi_R$. A simple example of a homological property is von Neumann regularity for rings, which is definable as rings all of whose modules are flat.

For some homological properties such as (weak) global dimension 0, the forward implication in • is given by relative separability alone. The relevant module properties of relative separability is the following:

Proposition 1.2 *A is a separable extension of S iff for every k-algebra C and C-A bimodule N, N is a direct summand of $N \otimes_S A$ as C-A bimodules.*

As we have seen, projectives and flats do not restrict in general, so it turns out that few properties pass in the reverse implication of • where A is a split extension of S. However, there is a dual to the last proposition that is also useful.

Proposition 1.3 *A is a split extension of S iff for every k-algebra B and B-S bimodule N, N is a direct summand of $N \otimes_S A$ as B-S bimodules.*

A limited application of split and [1] separable extensions using restricted and induced modules was given in the elegant

Higman's Theorem [9]. Let F be a field of characteristic p and G a finite group. Then the group algebra F[G] has finitely many indecomposable representations up to isomorphism (has f.r.t.) iff each sylow p-subgroup is cyclic.

This theorem may be proven conceptually by first noting that F[G] is a separable extension (though not F-separable!) of F[H] for any of its sylow p-subgroups H, secondly that F[G] is a split extension of F[H], and thirdly proving by means of propositions 1.2 and 1.3

Jans's Theorem [11]. Let A and S be Artinian algebras, S a subalgebra of A such that A is finitely generated over S as a right module. If A is a split extension of S, then A has f.r.t. implies S has f.r.t. If A is a separable extension of S, then S has f.r.t. implies A has f.r.t.

Now we find adding the unitality condition (III) to split and separable extensions a bonanza for several reasons. First, it is unrestrictive: consider the many

[1] notice our use of "and" to distinguish the absence of property (III): "we bend the grammar to make the mathematics come out right" [19].

examples of split and separable extensions, including Higman's group algebras, that can be shown to satisfy the unitality condition (cf. section 3). Second, in the notation of (III), $\sum_{i=1}^{n} x_i \otimes y_i$ is the identity with respect to an algebra structure [2] on $A \otimes_S A$ given by

$$(a_1 \otimes_S a_2)(a_3 \otimes_S a_4) = a_1 E(a_2 a_3) \otimes_S a_4 \quad (a_i \in A)$$

We show that $A \otimes_S A$ is a split separable extension of A, $A \otimes_S A$ is Morita equivalent to S, whence one implication alone in • may be proven in showing A and S share a property X, since Morita equivalent rings have equivalent categories of modules. Third, we show that A is a finite projective over S from either side, as in the Pimsner-Popa theorem: then projectives and flats restrict and homological properties properties pass down to the subalgebra while resolutions of flats and projectives induce up. As a consequence, statement • above becomes a metatheorem for homological properties X and split separable extensions A of S. Our main theorem is valid for weak, left, or right global dimension D(-) of rings:

Theorem 1.2 *Suppose A is a split separable extension of S. Then D(A) = D(S).*

In terms of noncommutative topology, theorem 1.2 may be viewed as an algebraic analogue of the fact that the dimension of a space is the same as any of its quotients by a free action of a finite group of homeomorphisms (cf. example 3.1.4). It is also a generalization of the Serre extension theorem (with a certain restriction in the characteristic p case cf. section 5,[24], [29]) and scattered results in commutative ring theory (an extension of a Dedekind ring is Dedekind, etc.). Other properties that are necessarily shared by a subalgebra and algebra include that of being a left Noetherian ring, left perfect ring, left coherent ring, quasi-Frobenius ring, or a polynomial identity ring.

In spite of their homological similarities, A and S are not Morita equivalent: e.g., the real quaternions are a split separable extension of the reals but not Morita equivalent to them. We clarify the relation of Morita equivalence with split separability by studying a generalization of both we call split separable equivalence of rings. This equivalence is obtained by first weakening the bimodule isomorphism condition of Morita equivalence to only requiring split surjectivity; second, fully symmetrizing the unitality condition.

Our paper proceeds with showing that a split separable subalgebra S of A possesses a Jones tower of algebras with index. Indeed, a generalization of the Jones index of subfactors is given by

Definition 1.1 *With e and E satisfying property (III) of split separable extensions and τ invertible in the ground ring k, define the index of S in A, [A:S] =* τ^{-1}.

[2] whose multiplication is mentioned in [14] and implicit in [15] where an identification of M_1 with $M \otimes_N M$ as M-M bimodules is made in the finite factor case

Fixing the conditional expectation (there is usually a canonical one that one wants to retain), the index [A:S] is independent of the choice of separating element e so long as (III) is satisfied. Also, A inside the algebra $A \otimes_S A$ has the same index. We label $A \otimes_S A$ the basic construction since it is isomorphic to M_1 where A and S are the finite factors M and N, respectively (cf. [15]). The expectation map $E_1 : A \otimes_S A \to A$ is then $\tau\mu_S$, where μ_S is the multiplication map, and the idempotent $e_1 = 1 \otimes 1$ satisfies properties (1)-(4) above (where A replace M, $A \otimes_S A$ replaces M_1). The basic construction may be iterated to obtain a tower of algebras over A and S with countable family of idempotents $\{e_i\}_{i=1}^{\infty}$ satisfying the relations (i)-(iii) above. We conclude the paper with a condition on the ground ring k that permits an explicit representation of the braid group on n letters in the n'th algebra of the tower.

The paper is organized as follows:

Contents

1 Introduction 2

2 Relative Separability 6

3 Split Separable Extensions 10

4 The Basic Construction 12

5 Global Dimension of Algebra and Subalgebra 16

6 A Metatheorem for Split Separably Equivalent Rings 18

7 The Jones Index and Tower of Algebras 21

Note. I learned from Susan Montgomery after writing this preprint of Y. Watatani, Index of C^*-algebras, Memoirs A.M.S. 83, no. 492 (1990), in which there is some overlap. Late versions of this paper should reflect improvements gleened from a reading of Watatani.

2 Relative Separability

Throughout this section, let k be a commutative ring with unit, A a k-algebra with subalgebra S with $1_A \in S$.

Definition 2.1 *A is said to be a separable extension of S iff there exists an $e \in A \otimes_S A$ (which we call a separating element) such that*

(i) $\mu_S(e) = 1$ *where* $\mu_S : A \otimes_S A \to A$ *is the multiplication map,* $a \otimes b \longmapsto ab$.

$$(ii) \quad ae = ea \quad \forall a \in A.$$

Example 2.1 This example is known in representation theory of finite groups under a different guise [7]. Suppose H is subgroup of finite index [G:H] = n of a possibly infinite group G. If n is invertible in k, then the group algebra k[G] is a separable extension of k[H]. If $\{g_i\}_{i=1}^n$ is a cross-section of the left cosets of H in G, then

$$e = \frac{1}{[G:H]} \sum_{i=1}^{n} g_i \otimes_{k[H]} g_i^{-1}$$

is a separating element. k may be replaced by any k-algebra A with G embedded in its automorphism group, then twisted or quantum group versions of this example are similarly shown to be separable extensions.

Proposition 2.1 *The following conditions on $A \supseteq S$ are equivalent:*

1. *A is a separable extension of S;*
2. *A has relative Hochschild cohomological dimension 0 over S;*
3. *A is an S^e-relative projective A^e-module (where B^e denotes $B \otimes_k B^{op}$ for general k-algebras B);*
4. *The universal derivation $d : A \to A \otimes_S A$ is inner;*
5. *the module condition in Proposition 1.2*
6. *Every S^e-split epi is A^e-split.*

Proof. The equivalence of conditions 1),2),3),4), and 5) may be found in [16] or [2] (terminology from [10] and [20]). We next show the equivalence of 1) with 6). First assume 6). The multiplication map μ_S is a epimorphism of A^e-modules, S^e-split by the map sending $a \longmapsto a \otimes_S 1$. Hence there exists an A^e-splitting $\eta : A \to A \otimes_S A$, and $e = \eta(1)$ is checked to be a separating element. The equivalence of 1) with 5) is is Proposition 2.2 below.

Two applications of the next lemma makes an S^e-split epi of A^e-modules into a split epi to show that 1) implies 6). □

Lemma 2.1 (Trace Lemma) *If A is a separable extension of S, and C is an arbitrary unital k-algebra, then a C-S split epi of A-A bimodules is C-A split. Also, an S-C split epi of A-A bimodules is A-C split. Hence, A has relative global dimension zero over S: short exact sequences of A-modules that split over S can be made to split over A.*

Proof. [3] Let $e = \sum_{i=1}^n x_i \otimes_S y_i$ be a separating element, $\sigma : N \to M$ an epi of C-A bimodules with splitting $f \in Hom_{C-S}(M, N)$. We now apply a trace operator to alter f to a C-A module morphism γ satisfying $\sigma\gamma = 1$.

[3] The idea of proof is a generalization of the trace argument proving Maschke's theorem on semisimplicity of finite group algebras.

The trace operator $Tr_{(-)}(-) : Hom_{C-S}(M,N)\otimes_k A\otimes_S A \longrightarrow Hom_{C-k}(M,N)$ is given by

$$Tr_x(g)(m) = \sum_{i=1}^{N} g(mz_i)w_i \quad where \quad x = \sum_{i=1}^{N} z_i \otimes w_i \quad and \quad g \in Hom_{C-S}(M,N).$$

Clearly, $Tr_x(g)$ is C-k linear. That $\gamma = Tr_e(f)$ is C-A linear follows from property ii) of separating elements:

$$Tr_e(f)(ma) = Tr_{ae}(f)(m) = Tr_{ea}(f)(m) = Tr_e(f)(m)a.$$

By property i) we have $\sigma \circ \gamma(m) = \sum_{i=1}^{n} \sigma\gamma(mx_i)y_i = m\sum x_iy_i = m$, so σ is indeed C-A split.

The argument reversing sides for C and S carries through in perfect analogy. If C = k we obtain that short exact sequences of A-modules split over S can be made split over A through an application of Tr_e to the section.□

Remark 2.1 It is a consequence of the proof that $Tr_e(g) \in Hom_{C-A}(M,N)$ for any $g \in Hom_{C-S}(M,N)$, and we will refer to this and the corresponding fact where right and left are transposed also as the trace lemma.

Notation. *If an R-module M contains a submodule N as a direct summand, we write $N|M$.*

Proposition 2.2 (= 1.2) *A is a separable extension of S iff for every k-algebra B and A-B bimodule N, N is a direct summand of $A \otimes_S N$ as A-B bimodules. Similarly, every B-A bimodule N is a direct summand of $N \otimes_S A$.*

Proof. For the back implication, take N = A, and note that $A \otimes_S A$ is an S-relative projective right or left A-module by Frobenius reciprocity [10]. Then A is also an S-relative projective module, so the S-A split map μ_S (split by $s \longmapsto s \otimes_S 1$) has an A-A splitting. It follows routinely that a separating element is given as the image of 1 under an A-A splitting of μ_S.

For the forward implication, the multiplication map $\mu : A \otimes_S N \longrightarrow N$ given by $a \otimes_S n \longmapsto n$ is split by the S-B bimodule map $n \longmapsto 1 \otimes n$. By the trace lemma we obtain an A-B splitting of μ. Hence, N is a direct summand of $A \otimes_S A$. The statement for B-A bimodules is proven similarly. □

Remark 2.2 Relative separability has been referred to as "generic relative global dimension zero" by G. Bergman [2]. We have just seen in the trace lemma that relative separability is stronger than the condition, relative global dimension zero: it is so in very favorable ways which we now discuss (though not central to this paper). Among several interesting classes of separable extensions are ring epimorphisms such as a ring inside a localization [2], a ring R with elements a and b such that ab = 1 but ba ≠ 1 over the subring S generated by 1 and bRa [2], and matrix rings over an arbitrary algebra [10]. We have already mentioned the group algebra and von Neumann factor examples above. In addition, separable k-algebras are separable extensions of their unit subalgebra k1. The terminology "separable extension of algebras" is probably due to R. Pierce on the grounds that a finite separable extensions of fields $F_1 \subseteq F_2$ is a separable F_1-algebra [25].

The class of separable algebras has closure properties even nicer than those of separable k-algebras. Suppose A_1 is a separable extension of S_1, and A_2 is a separable extension of S_2. Then separable extension is closed under direct sum, tensor product, and homomorphic image: i.e., $A_1 \oplus A_2$ is a separable extension of $S_1 \oplus S_2$, $A_1 \otimes_k A_2$ is a separable extension of $S_1 \otimes_k S_2$, and if f is an algebra homomorphism with domain A then $f(A_1)$ is a separable extension of $f(S_1)$[17]. In particular, f may be an automorphism of A_1 so that relative separability is a conjugacy invariant property of a subalgebra in the sense of [22]. In addition, if I is an ideal in an algebra A such that $A = A_1 \oplus I$, then A is separable extension of $S = S_1 \oplus I$ [17].

In addition, if $S \subseteq T \subseteq A$ is an intermediate algebra with A a separable extension of T, and T a separable extension of S, then A is a separable extension of S; if A is a separable extension of S only, then A is a separable extension of any intermediate algebra T [25]. Then a separable k-algebra A may separably extend an algebra S that is not semisimple: e.g., A = $M_2(k)$ and S = a triangular algebra inside A. This shows that the converse to the next proposition is not true.

Proposition 2.3 *Suppose A is a separable extension of S. If S is semisimple (von Neumann regular), then A is semisimple (respectively, von Neumann regular).*

Proof. Recall that for any subalgebra S of A, an A-module N may be restricted to an S-module ${}_SN$ while an S-module P may be induced to an A-module $A \otimes_S P$. Inducing always takes flat modules to flat modules and projectives to projectives. Also recall that a ring R is semisimple (von Neumann regular) iff every left R-module is projective (resp., flat).

Begin with any A-module M. Its restriction ${}_SM$ is projective (flat) since S is semisimple (resp., von Neumann regular). Then the induced module $A \otimes_S M$ is projective (resp., flat). But $M | A \otimes_S M$ as noted, whence inherits projectivity (resp., flatness). Hence, A is semisimple (resp., von Neumann regular). □

Remark 2.3 A may not be projective over S, as the separable Z-algebra Q. Extra conditions on separable extensions are needed for this to happen: conditions(II) and (III) of section 1. Then projective or flat resolutions induce up. We will prove for split separable extensions $A \supseteq S$, that S has global dimension n iff A has global dimension n (weak, right, or left). What happens to injectives under separable extension is the subject of the next proposition.

Proposition 2.4 *Let A be a separable extension of S. Then every A-module that restricts to an injective S-module is itself injective. As a consequence, if A is an integral domain with S a Dedekind domain, then A is a Dedekind domain.*

Proof. Any A-module M has injective envelope Q. Since ${}_SM$ is injective, it follows that the inclusion $M \longrightarrow Q$ is S-split, therefore A-split by the trace lemma. Therefore, $M|Q$, so M is injective.

Let A be a domain with S a Dedekind domain. By a well-known theorem, it will suffice to consider a divisible A-module M and show it is injective [28]. But its restriction ${}_SM$ is trivially divisible, therefore injective, so M is injective. □

3 Split Separable Extensions

In this section we fix the following notation and concepts. Let k be a unital commutative ring, A a k-algebra containing a subalgebra S where S $\ni 1_A$. The next definition refers to the natural S-S bimodule structures on S and A resulting from multiplication.

Definition 3.1 *A is a split extension of S iff as S-S bimodules, S is a direct summand of A.*

Proposition 3.1 *The following conditions on a subalgebra S of A are equivalent:*

1. *A is split extension of S;*
2. *There exists a conditional expectation $E : A \longrightarrow S$, i.e., $E|_S = Id_S$ and E is an S-S bimodule morphism;*
3. *For every k-algebra B and S-B bimodule N, N is a direct summand of $A \otimes_S N$. Similarly, $N|N \otimes_S A$ for every B-S bimodule N. (Proposition 1.3.)*

Proof. (1) iff (2) results from noting that the inclusion $S \longrightarrow A$ splits: E is a choice of splitting. (2) implies (3): Note that $\iota : N \longrightarrow A \otimes_S N$ defined by $n \longmapsto 1 \otimes_S n$ is split as S-B bimodule maps by $E \otimes_S Id_N$ under the obvious identification of $S \otimes_S N$ with N. (3) implies (1): Take N = S, B = S, and make the identification of $A \otimes_S S$ with A. □

Example 3.1 1. Let H be a subgroup of a group G. Then

$$k[G] = k[H] \oplus \sum_{y \in G-H} kx$$

as k[H]-bimodules. An arbitrary k-algebra B may replace k in this example, since tensoring preserves direct sum, or introduce a twist given by G acting as automorphisms of B.

2. Any algebra A over a field F is a split extension over F1. A von Neumann algebra factor is a split extension over any of its subfactors.

3. Any matrix algebra over k is a split extension of k1. Polynomial rings and power series rings are split extensions over the coefficient ring.

The next proposition gives an inequality of right (left, or weak) global dimension D(-) of S and A which brings into play the projective dimension of A as an S-module.

Proposition 3.2 *If A is a split extension of S, then $D(S) \leq D(A) + pr.dim.A_S$.*

Proof. Let M be a right S-module. By proposition 3.1, $M | M \otimes_S A$, whence pr. dim. $M_S \leq$ pr. dim. $(M \otimes_S A)_S$. By two well-known change of rings spectral sequences in tor and ext functors, we have pr. dim. $(M \otimes_S A)_S \leq$ pr. dim. $(M \otimes_S A)_A$ + pr. dim. A_S. Then D(S) = sup pr. dim. $M_S \leq$ D(A) + pr. dim.A_S. This argument works for left modules, or weak global dimension by replacing pr. dim with flat dimension of modules. □

Definition 3.2 *A is a split separable extension of S iff the following three conditions are met:*

(I) A is a separable extension of S;

(II) A is a split extension of S;

(III) There exists a separating element $e \in A \otimes_S A$ and conditional expectation $E : A \to S$ such that

$$\mu_S(Id \otimes_S E)e = \tau 1_A = \mu_S(E \otimes_S Id)e$$

for some nonzero scalar $\tau \in k$. We call this the unitality condition.

Example 3.2 In each example below k is a commutative ring where τ is invertible.

1. The group algebra example in section 2. Take $E : k[G] \to k[H]$ to be $E(\sum_{g \in G} a_g g) = \sum_{g \in H} a_g g$. Then

$$\mu_S(E \otimes_S 1)e = \frac{1}{[G:H]} \sum_{i=1}^{n} E(g_i) g_i^{-1} = \frac{1}{[G:H]} = \mu_S(1 \otimes E)e.$$

   Twisted tensor products with an algebra or smash products with a Hopf subalgebra with left integral and antipode may also be checked to be split separable extensions.

2. Von Neumann algebra finite factors $N \subseteq M$, N a subfactor of M of finite Jones index are split separable extensions (cf. section 1 and [18]).

3. The full matrix extension $M_n(A)$ of any k-algebra A is a split separable extension with separating element

$$e = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} E_{ij} \otimes_A E_{ji},$$

where E_{ij} is the (i,j)-matrix unit, and with conditional expectation

$$E(X) = \frac{1}{n}\sum_{i=1}^{n} X_{ii} \quad where \quad X = (X_{ij}) \in M_n(A).$$

Then e and E satisfy condition (III) with $\tau = \frac{1}{n^2}$.

4. Quadratic field extensions $Q(\sqrt{a})$ of the rationals where $a \in Z$ are split separable extensions. Take E to be the normalized trace $E(x + y\sqrt{a}) = x$ and $e = \frac{1}{2a}(\sqrt{a} \otimes_Q \sqrt{a} + a \otimes 1)$. Then $\tau = \frac{1}{2}$. (Then proposition 2.4 shows that the ring of integers of $Q(\sqrt{a})$ with 2a inverted is a Dedekind domain.)

   Let A be a Galois extension (of commutative rings) of S with finite group G, [1]. Then A is a split separable extension of S if $\tau = \frac{1}{|G|} \in S$. One example of such a Galois extension is a ring of G-invariant functions within the ring of continuous complex-valued functions on a compact Haussdorf space where G acts by homeomorphisms without fixed points [4].

5. The quaternion algebras $(\frac{a,b}{F})$ over a field F of characteristic $\neq 2$ are a split separable extension of F. Apply the trace E to the separability idempotent:

$$e = \frac{1}{4}(1 \otimes 1 + i \otimes ia^{-1} + j \otimes jb^{-1} - k \otimes ka^{-1}b^{-1})$$

   Note that $\tau = \frac{1}{4}$.

   More generally, a crossed product algebra $E * G$ of a Galois extension field E of F with Galois group G is a split separable extension of E (whence of F by example 3.4 and proposition 7.3). A long calculation checking that the following is a separating element over E:

$$e = \frac{1}{|G|\delta_{S,S^{-1}}}\sum_{s \in G} u_S \otimes_E u_{S^{-1}}$$

   where $\delta_{S,T}$ is the defining two-cocycle and $\{u_S\}_{S \in G}$ is the standard basis of the crossed product algebra.

4 The Basic Construction

We continue to suppose k a commutative ring, and suppose τ invertible in k. The next proposition explains our usage of the term unitality for condition (III).

Proposition 4.1 *Suppose A is a split separable extension of S with E and $e = \tau\sum_{i=1}^{n} x_i \otimes y_i$ satisfying property (III). Then $A \otimes_S A$ is a unital algebra with multiplication given by*

$$(a_0 \otimes_S a_1)(a_2 \otimes_S a_3) = a_0 E(a_1 a_2) \otimes_S a_3$$

with unity element

$$1 = \sum_{i=1}^{n} x_i \otimes y_i.$$

Proof. The multiplication is associative because

$$a_0 E(a_1 a_2) E(a_3 a_4) \otimes_S a_5 = a_0 E(a_1 a_2 E(a_3 a_4)) \otimes_S a_5$$

by S-linearity of E from both sides.

$\tau^{-1}e$ is the identity by an application of the trace lemma for separable extensions and property (III). With $E \in Hom_{S-S}(A, A)$ the expectation above we have

$$(\sum_{i=1}^{n} x_i \otimes y_i)(a \otimes b) = \sum_{i=1}^{n} x_i E(y_i a) \otimes b = Tr_{\tau^{-1}ea} E(1) \otimes b = a(\sum x_i E(y_i)) \otimes b = a \otimes b.$$

Checking that $\tau^{-1}e$ is a right identity is entirely analogous: one makes use of $\sum E(x_i) y_i = 1$. □

Proposition 4.2 *Given the hypotheses and notation of the last proposition, $A \otimes_S A$ is a split separable extension of A under the inclusion $\iota : a \longmapsto a1$ ($\forall a \in A$).*

Proof. Fix the notation $A_1 = A \otimes_S A$ and $E_1 = \tau\mu_S$ by analogy with the basic construction and the conditional expectation in von Neumann theory. Note that E_1 is a conditional expectation since

$$(i) \quad \tau\mu_S(a\tau^{-1}e) = a\mu_S(e) = a,$$

$$(ii) \quad \mu_S \text{ is an } A - A \text{ bimodule homomorphism.}$$

Now $A_1 \otimes_A A_1 \xrightarrow{\cong} A \otimes_S A \otimes_S A$ as $A_1 - A_1$ bimodules by the map

$$\Upsilon : a_1 \otimes_S a_2 \otimes_A a_3 \otimes_S a_4 \longmapsto a_1 \otimes_S a_2 a_3 \otimes_S a_4$$

where the bimodule structure on $A \otimes_S A \otimes_S A$ is given by

$$1) \quad (a_0 \otimes a_1)(a_2 \otimes a_3 \otimes a_4) = a_0 E(a_1 a_2) \otimes a_3 \otimes a_4,$$

$$2) \quad (a_0 \otimes a_1 \otimes a_2)(a_3 \otimes a_4) = a_0 \otimes a_1 \otimes E(a_2 a_3) a_4.$$

Under the identification by Υ, the multiplication map μ_A is given by

$$a_0 \otimes a_1 \otimes a_2 \longmapsto a_0 E(a_1) \otimes_S a_2.$$

We next claim that the element $f = \sum_{i=1}^{n} x_i \otimes 1 \otimes y_i$ is a separating element, which together with E_1 satisfies the unitality condition. We have:

1. $\mu_A(f) = \sum_{i=1}^n x_i \otimes y_i = 1_{A_1}$

2. $(a_0 \otimes_S a_1)f = \sum_{i=1}^n a_0 E(a_1 x_i) \otimes_S 1 \otimes_S y_i$

$$= a_0 \otimes 1 \otimes \sum_{i=1}^n E(a_1 x_i) y_i = a_0 \otimes 1 \otimes a_1$$

$$= \sum x_i E(y_i a_0) \otimes 1 \otimes a_1 = f(a_0 \otimes a_1),$$

by the trace lemma and property (III) for e and E;

3. $\mu_A(1 \otimes E_1)f = \mu_A(\tau \sum_{i=1}^n x_i \otimes 1 \otimes y_i)$

$$= \tau \sum x_i \otimes y_i = \tau 1 = \mu_A(E_1 \otimes 1)f.$$

Hence, A_1 is a split separable extension of A. □

Proposition 4.3 *If A is a split separable extension of S, then A is a finitely generated projective left or right S-module (in the natural module structures from multiplication).*

Proof. We prove that A_S is a finite projective - the left module case being entirely analogous. Let E be a conditional expectation and $e = \tau \sum_{i=1}^n x_i \otimes y_i$ a separating element satisfying the unitality condition. Then we claim that $\phi_i \in Hom_S(A, S)$ given by $\phi_i(x) = E(y_i x)$, i = 1,...,n, and $x_i \in A$, i = 1,...,n, forms a dual basis for A_S, which would complete the proof. But the trace lemma and condition (III) give

$$\sum_{i=1}^n x_i E(y_i a) = Tr_{\tau^{-1} e a} E(1) = a \sum x_i E(y_i) = a. \quad \square$$

Proposition 4.4 *If A is a split separable extension of S, then $A \otimes_S A$ with the unital algebra structure above is Morita equivalent to S.*

Proof. We prove that $A \otimes_S A$ is isomorphic to the algebra $End_S(A)$ of right S-module endomorphisms of A. But A_S is finite projective by the previous proposition, and is a generator in the category of right S-modules, since the trace ideal in S is all of S:

$$\{\sum \phi_i(x) | \phi_i \in Hom_S(A, S), x \in A\} \supseteq \{E(x) | x \in A\} = S.$$

so S and $End_S(A)$ are Morita equivalent [6].

Note the idempotent in A_1

$$e_1 = 1 \otimes_S 1$$

and note that each element of A_1 is of the form $\sum_{i=1}^{N} a_i e_1 b_i$, where $a_i, b_i \in A$. Moreover, $\forall a \in A$ the multiplication is determined by the relations,

$$e_1 a e_1 = e_1 E(a) = E(a) e_1$$

by a little computation.

Now, E is an idempotent in $End_S(A)$ since $imE = S$ and $E|_S = Id_S$. For each $a \in A$ let $\lambda(a)$ denote left multiplication by a, certainly an element of $End_S(A)$. We claim that the linear map $\theta : A_1 \longrightarrow End_S(A)$ defined by

$$\theta(a e_1 b) = \lambda(a) E \lambda(b)$$

is an isomorphism of k-algebras.

θ is a homomorphism: this follows from noting $E\lambda(b)E = \lambda(E(b)E = E\lambda(E(b))$.

θ is surjective: given $g \in End_S(A)$ and $a \in A$,

$$g(a) = g(\sum_{i=1}^{n} x_i E(y_i a)) = \sum g(x_i) E(y_i a)$$

so $\theta(\sum_{i=1}^{n} g(x_i e_1 y_i) = g$.

θ is injective: this is a three step proof using techniques of Morita theory. First, the left S-module morphism $\eta : A \longrightarrow Hom_S(A, S)$ given by $a \longmapsto E(a-)$ is injective since $E(ab) = 0 \quad \forall b \in A$ implies $a = \sum_{i=1}^{n} E(a x_j) y_j = 0$ (E is then called faithful [8]). Second, A_S is projective, whence flat by the last proposition, so $1 \otimes \eta : A \otimes_S A \longrightarrow A \otimes_S Hom_S(A, S)$ is injective. Third, θ factors as $\psi(1 \otimes \eta)$ where $\psi : A \otimes_S Hom_S(A, S) \longrightarrow End_S(A)$, defined by $\psi(a \otimes \gamma) = a\gamma(-)$, is injective by the following calculation:

$$Given \quad \psi(\sum_{i=1}^{N} a_i \otimes \eta_i) = \sum a_i \eta_i = 0,$$

$$then \quad \sum_{i=1}^{N} a_i \otimes \eta_i = \sum a_i \otimes \eta_i \sum_{j=1}^{n} x_j E(y_j -) =$$

$$\sum a_i \otimes \eta_i(x_j) E(y_j -) = \sum_{i,j} a_i \eta_i(x_j) \otimes_S E(y_j -) = 0.$$

Since θ is factored into injective maps, θ is injective. □

Remark 4.1 Note in the last proof that λ will always inject A into $End_S(A)$, but that we need A a split, finitely generated projective extension of S to ensure $End_S(A)$ Morita equivalent to S.

We call A_1 the basic construction for split separable extensions as a natural extension of terminology from von Neumann theory. Recall from section 1 that if A and S are the finite factors M and N, respectively, with [M:N] $< \infty$, then $M_1 \cong M \otimes_N M$ as C-algebras.

Let k be a field. It follows from the last proof and the observation that $S \longrightarrow A_1$ given by $s \longmapsto se_1$ is injective, that A_1 is an E-extension of the faithful conditional expectation, $E : A \rightarrow S$ [8]. However, the converse is not true: if $A \otimes_S A$ is an E-extension with respect to a split extension A of S with faithful conditional expectation $E : A \rightarrow S$, then A may not be a split separable extension. For example, take k to be a field of characteristic p, take $G = Z_p \times Z_p$, a product of two cyclic groups of prime order, and take H to be the left factor Z_p. Then k[G] has infinitely many non-isomorphic indecomposable representations, while k[H] has only finitely many by Higman's theorem (cf. Introduction). Then k[G] is not a separable extension of k[H] by Jans's theorem (cf. Introduction), although the conditional expectation defined in example 3.2 is faithful and makes $A \otimes_S A$ a unital E-extension.

An alternative definition of a split separable extension A of S is in fact the following: A is a split extension of S with conditional expectation $E : A \rightarrow S$ such that $A \otimes_S A$ is a *unital* algebra with μ_S a scalar multiple of a conditional expectation.

5 Global Dimension of Algebra and Subalgebra

The next theorem considerably sharpens the results of propositions 2.3, 2.4, and 3.2 for split separable extensions.

Theorem 5.1 *If A is a split separable extension of a subalgebra S, then (weak, right, or left) global dimension D(A) = D(S).*

Proof 1. Since A is a split extension of S, it follows from Proposition 3.2 that

$$D(S) \leq D(A) \quad + \quad \text{proj. dim. } {}_SA.$$

But A is a projective left S-module by proposition, so $D(S) \leq D(A)$. Since the basic construction A_1 is a split separable extension of A, we also have $D(A) \leq D(A_1)$. But A_1 is Morita equivalent to S, so $D(A_1) = D(S)$. Then

$$D(S) \leq D(A) \leq D(S),$$

whence D(S) = D(A). □

Proof 2. We need only assume that A is a separable, split, projective extension of S for this proof. Suppose D(S) = n, and that M is any left A-module. Then its restriction ${}_SM$ has projective dimension at most n: i.e., there exists a projective resolution of M,

$$0 \rightarrow P_n \rightarrow P_{n-1} \rightarrow \cdots \rightarrow P_0 \rightarrow_S M \rightarrow 0.$$

Some of the projectives above might be zero. Now projective resolutions will induce since A_S is projective, whence flat:

$$0 \to A \otimes_S P_n \to A \otimes_S P_{n-1} \to \cdots \to A \otimes_S P_0 \to A \otimes_S M \to 0$$

Hence, pr. dim. $A \otimes_S M \leq n$. Since A is a separable extension of S, M is a direct summand of $A \otimes_S M$ by proposition 1.2. Hence, pr. dim. $M \leq D(S)$, and taking a supremum over all A-modules we get D(A)$\leq$ D(S).

Let N be an arbitrary left S-module. If D(A) = m, then pr. dim.$(A \otimes_S N) \leq m$, i.e., there exists a projective resolution of $A \otimes_S N$,

$$0 \to P_m \to \cdots \to P_0 \to A \otimes_S N \to 0.$$

Since ${}_SA$ is projective, this projective resolution restricts to the same for ${}_S(A \otimes_S N)$. But $N|_S(A \otimes_S N)$ since A is a split extension of S. It follows that pr. dim. N $\leq$ m = D(A). Hence, D(S) $\leq$ D(A). Putting the two inequalities together we conclude D(A) = D(S).

Note that right modules may replace left modules in the proof above, since A is projective over S from both sides. Also, flat modules and resolutions may replace projective modules and resolutions, since A projective over S implies A is flat over S. □

Remark 5.1 This theorem has a non-trivial intersection with the Serre extension theorem for groups (the excluded case: k = field of characteristic p where $p|[G:H]$, c.f., [24] [29, p. 98]).

We can make use of knowing A_1 Morita equivalent to S in the next statement about "higher commutativity."

Proposition 5.1 *Suppose k is a field and suppose A is a split separable extension of a polynomial identity algebra S. Then A is a polynomial identity algebra.*

Proof. If S is a polynomial identity algebra, then so is $M_n(S)$ by a theorem of Regev [27]. Trivially, any subalgebra (not necessarily unital) of a polynomial identity algebra is itself satisfying the same polynomial identity. But A is a subalgebra of the basic construct A_1, which is Morita equivalent to S, whence of the form $gM_n(S)g$ for some integer n and idempotent g in $M_n(S)$. Hence, A satisfies a polynomial identity. □

The next proposition deals with chain conditions on one-sided ideals.

Proposition 5.2 *If A is a split separable extension of S, then S is left (respectively, right) Noetherian iff A is left (resp., right) Noetherian.*

Proof. The forward implication will use only that A is finitely generated (f.g.) over S. Suppose M is an f.g. left A-module, and N is any A-submodule of M. It will suffice to show that N is f.g. Since ${}_SA$ is f.g. by proposition, it follows that the restriction ${}_SM$ is f.g. Since S is left Noetherian, the submodule ${}_SN$ of ${}_SM$

is also f.g. Then $A \otimes_S N$ is f.g. over A. But N is the image of $A \otimes_S N$ under an A-module map, so N is f.g. Since A_S is f.g. this argument transposes from left to right.

The reverse implication depends only on a f.g. split extension. Let M be a f.g. left S-module with N a submodule. Then $A \otimes_S N$ is an A-submodule of $A \otimes_S M$, the latter being f.g. so the former is f.g. since A is left Noetherian. Then ${}_S(A \otimes_S N)$ if f.g. But $N|_S(A \otimes_S N)$ since A is a split extension of S, so N is f.g. □

Remark 5.2 Now the interested reader may easily prove that A and S are both quasi-Frobenius, left perfect, or left coherent, or both are not. In spite of being so similar homologically, it is clear from the many examples in 3.1 that A and S are not necessarily Morita equivalent. Indeed, a property that Morita equivalent rings share that is not shared by A and S is that of being a simple ring. For example, it is easy to compute that $k \times k$ is a split separable extension of k. However, in the next section we clarify the relation of split separable extension and Morita equivalence.

6 A Metatheorem for Split Separably Equivalent Rings

It is clear that behind the results of section 5 is a metatheorem, much like a metatheorem would exist for properties shared by Morita equivalent rings. In this section, we define an equivalence relation among rings that we call split separable equivalence. We then show that A and S are split separably equivalent if A is a split separable extension of S, *or* if A is Morita equivalent to S. We then prove a metatheorem that such A and S share homological properties.

Definition 6.1 Rings A and B are **split separably equivalent** if there exist bimodules ${}_AP_B$ and ${}_BQ_A$, with split surjections as A-A and B-B bimodule morphisms, respectively,

$$\nu: \quad P \otimes_B Q \longrightarrow A$$

$$\mu: \quad Q \otimes_A P \longrightarrow B$$

and **elements of adjunction** $\sum_{j=1}^n p_j \otimes q_j$ and $\sum_{i=1}^m q'_i \otimes p'_i$ ($\forall p \in P, \quad q \in Q$)

$$\sum_{i=1}^m \nu(p \otimes q'_i)p'_i = p \qquad \sum_{i=1}^m q'_i \nu(p'_i \otimes q) = q$$

$$\sum_{j=1}^n \mu(q \otimes p_j)q_j = q \qquad \sum_{j=1}^n p_j \mu(q_j \otimes p) = p$$

Remark 6.1 The last four conditions above state that the functors F $= P\otimes -$: B-Mod $\longrightarrow$ A-Mod, and G $= Q\otimes -$: A-Mod $\longrightarrow$ B-Mod, form adjunctions in either order. They also entail that P and Q are progenerators as A- and B-modules. The first two conditions above say that the counits of these adjunctions are split epis. This definition can be carried right over to categories: adjunctions and split epis are closed under composition, so the transitivity implied by the word "equivalence" does indeed hold (and may be checked directly by hand with the definition above). Symmetry and reflexivity of the relation among rings and categories is obvious.

Interestingly enough, any additive category $\mathcal{C}$ is split separably equivalent in this sense to a finite product of itself $\mathcal{C}\times\cdots\times\mathcal{C}$, since the diagonal functor has left adjoint the coproduct, right adjoint the product [21, p. 87], which coincide in the biproduct, and both counits of adjunction are split epis.

Note that a certain amount of the structure in section 4 carries over to $P\otimes_B Q$ and $Q\otimes_A P$ as well as a definition of index [A:B] and [B:A] as in section 7. We will define this more precisely in a forthcoming note.

Proposition 6.1 *If A is a split separable extension of S, then A and S are split separably equivalent.*

Proof. Take P = Q = A and B = S in definition 6.1. Note that the multiplication map $\mu_S : A\otimes_S A \longrightarrow A$ is a split surjection of A-A bimodules by proposition 2.2. Given conditional expectation $E : A \longrightarrow S$, we get the following split surjection of S-S bimodules, $A\otimes_A A \cong A \xrightarrow{E} S$. Finally, ${}_SA$ and A_S are finite projectives by proposition 4.3, while A_A and ${}_AA$ are free of rank 1.

The elements of adjunction in $P\otimes Q$ and $Q\otimes P$ are given by $\sum x_i\otimes y_i$ and $1\otimes 1$, respectively, if the former is the separating element up to τ satisfying the unitality condition. The two equations involving μ_S are trivial, the two involving E are precisely the content of the unitality condition and the trace lemma, as noted in 4.3, $\sum E(ax_i)y_i = a$ and $\sum x_iE(y_ia) = a$. □

Proposition 6.2 *If A and B are Morita equivalent rings, then A and B are split separably equivalent.*

Proof. It is well-known that one of several equivalent ways to define Morita equivalent rings A and B is to stipulate bimodules ${}_AP_B$ and ${}_BQ_A$ that satisfy

$$P\otimes_B Q \cong A$$

$$Q\otimes_A P \cong B$$

as A-A and B-B bimodules, respectively where the bimodule isomorphisms are associative (cf. [6]). The elements of adjunction are then the inverse images of 1_A and 1_B, and associativity yields the four equations of adjunction. □

Nice Properties of Modules. We shall informally say that a *property* of left modules, such as projectivity or flatness, is a choice of subclass Φ_R of all

R-modules, **R-mod**, for each ring R. A property of modules is said to *induce* (under a finite projective change of rings) if given any two rings R and S and a bimodule ${}_R P_S$, which is finite projective on either side, then $M \in \Phi_S \Rightarrow P \otimes_S M \in \Phi_R$. A property of modules is *direct sum invariant* if $M \in \Phi_R$, $N|M \Rightarrow N \in \Phi_R \ \forall$ *rings R*. A property of modules is *nice* if it is both direct sum invariant and induces under a finite projective change of rings. For example, both flatness and projectivity are nice properties.

Homological Properties of Rings. It is well-known that certain desirable properties of rings are expressible in terms of the coincidence of classes of modules. For example, "all modules are projective (flat)" or "all modules are quotients of projectives" describe important classes of rings [28]. We capture this idea as follows. Define a property of rings to be a certain binary valued function on rings, assigning to each ring a yes or no. Define a *homological* property of rings to be yes-valued precisely on rings R where two nice properties of modules coincide, $\Phi_R = \Psi_R$.

Metatheorem 6.1 *If A and B are split separably equivalent rings, then A and B share homological properties.*

Proof. Let Φ_R and Ψ_R be nice properties of modules, and suppose A is a ring such that $\Phi_A = \Psi_A$. It suffices to prove by symmetry that $\Phi_B \subseteq \Psi_B$. Suppose ${}_A P_B$ and ${}_B Q_A$ satisfy the finite projectivity and split surjectivity conditions of definition 6.1. Given $M \in \Phi_B$,$P \otimes_B M \in \Phi_A$ since nice properties induce, whence $P \otimes_B M \in \Psi_A$ by assumption. Again by inducing $Q \otimes_A P \otimes_B M \in \Psi_B$. But $M | Q \otimes_A P \otimes_B M$ by one of the split surjectivity conditions. So $M \in \Psi_B$ by direct sum invariance of nice properties. Hence, $\Phi_B \subseteq \Psi_B$. □

Note that the proof of the metatheorem requires only the weaker notion of split separable equivalence present in an earlier version of my paper where the elements of adjunction are replaced with the assumption that P and Q are finite projective as A- and B-modules. We next prove that the split surjections carry over to the Tor functors on modules in the following way.

Proposition 6.3 *Let A and B be rings where* $\mu_A : P \otimes_A Q \to B$ *and* $\mu_B : Q \otimes_B P \to A$ *are split epimorphisms of B-B and A-A bimodules, respectively, with* ${}_B P_A$ *and* ${}_A Q_B$ *bimodules finite projective on either side. Then for arbitrary A-modules* M_A *and* ${}_A N$*, there is a split epi* $\mu_n : Tor_n^B(M \otimes_A Q, P \otimes_A N) \to Tor_n^A(M, N)$ *induced from the map* $Id_M \otimes \mu_B \otimes Id_N$.

Proof. If $X. \to M$ is a projective resolution of M_A, then $X. \otimes Q \to M \otimes Q$ is a projective resolution of $M \otimes Q$ since ${}_A Q$ is flat and Q_B is projective. If μ_B is split by an A-A bimodule map σ, then $\sigma(1) = \sum q_i \otimes p_i$ is readily seen to satisfy $\sum q_i p_i = 1$ and $a \sum q_i \otimes p_i = \sum q_i \otimes p_i a$. Define two maps as follows - they are morphism of complexes by naturality:

$$X. \otimes_A N \xrightarrow{f} X. \otimes Q \otimes P \otimes N$$

$$x \otimes n \longmapsto \sum x \otimes q_i \otimes p_i \otimes n$$

$$X. \otimes Q \otimes P \otimes N \xrightarrow{g} X. \otimes_A N$$

$$x \otimes q \otimes p \otimes n \longmapsto x\mu_B(q \otimes p) \otimes_A n$$

Now $g \circ f = Id$ since $\mu_B(\sum q_i \otimes p_i) = \mu_B \sigma(1) = 1$. Then passing to the homology groups of these two complexes, g induces the split surjection μ_n as claimed. □

Not surprisingly, global dimension $\leq$ n is a homological property of rings as revealed by a close examination of proof 2 of theorem 5.1. However, the last proposition provides a convenient proof of the following.

Corollary 6.1 *If A and B are split separably equivalent rings, then $D(A) = D(B)$.*

Proof. In the notation of proposition 6.3, $Tor_n^A(M,N) | Tor_n^B(M \otimes_A Q, P \otimes_A N)$, so that $D(B) \geq D(A)$. By the symmetry in our definition of Jones equivalence, we also get $D(A) \geq D(B)$. □

7 The Jones Index and Tower of Algebras

Note that in each of the five examples given in section 3 $\tau = \mu_S(E \otimes_S 1)e$ is the inverse of the Jones index defined in [12] and more broadly in [8] and [15]. This suggests the next definition and proposition. k remains a commutative ground ring in which τ is invertible.

Definition 7.1 *Given a split separable extension A of S with conditional expectation E and separating element e satisfying unitality, $\mu_S(E \otimes 1)e = \tau 1_A = \mu_S(1 \otimes E)e$, define the index of S in A, $[A{:}S] = \tau^{-1}$.*

Note that the index is an element of k, not a positive real unless extra conditions are attached to split separable extensions. The next proposition shows that if the conditional expectation is fixed, as it often is in operator algebras, the index is well-defined for split separable extensions.

Proposition 7.1 *Suppose A is a split separable extension of S with conditional expectation E and two separating elements e and e' both satisfying the unitality condition. Suppose $\mu_S(E \otimes 1)e = \tau$ and $\mu_S(E \otimes 1)e' = \tau'$. Then $\tau = \tau'$.*

Proof. Multiplying $e = \tau\sum_{i=1}^{n} x_i \otimes_S y_i$ and $e' = \tau'\sum_{j=1}^{m} x'_j \otimes_S y'_j$ in $A \otimes_S A$ gives

$$ee' = \tau\tau'\sum_i\sum_j(x_i \otimes y_i)(x'_j \otimes y'_j) = \tau\tau'\sum_{j=1}^{m}(\sum_{i=1}^{n} x_i E(y_i x'_j) \otimes_S y'_j$$

$$= \tau\tau'\sum_{j=1}^{m} x'_j(\sum_{i=1}^{n} x_i E(y_i)) \otimes y'_j = \tau e'$$

Similarly, one checks that $ee' = \tau' e$. Then $\tau e' = \tau' e$, so applying the multiplication map μ_S we get $\tau = \tau'$. □

Proposition 7.2 *If A is a split separable extension of S with basic construction A_1, then $[A_1 : A] = [A:S]$.*

Proof. We refer to the proof and notation in Proposition 4.2 where it was shown that A_1 is a split separable extension of A. With $\tau = [\mathrm{A:S}]^{-1}$, equation 3 shows that $[A_1\mathrm{:A}] = \tau^{-1}$. □

Lemma 7.1 *If A is a split extension of S, then $1 \otimes_S a = 0$ or $a \otimes_S 1 = 0$ implies $a = 0$.*

Proof. The functors $- \otimes_S A$ and $A \otimes_S -$ are exact on split exact sequences of S-modules. Therefore, the split exact sequence $0 \to S \to A$ gives the exact sequence $0 \to S \otimes_S A \to A \otimes_S A$. Under the identification $S \otimes_S A \cong A$, $a \longmapsto 1 \otimes_S a$. Hence, $1 \otimes_S a = 0$ implies a = 0. Similarly, $a \otimes_S 1 = 0$ implies a = 0. □

Remark 7.1 From the lemma we see that S is isomorphic to the centralizer algebra of e_1 in A_1, via a map $\Phi : S \to (A_1)_{e_1} = \{z \in A_1 : e_1 z = z e_1 = z\}$ given by $x \longmapsto 1 \otimes_S x$.

Proposition 7.3 *Suppose A is a split separable extension of B, and B is a split separable extension of C. Then A is a split separable extension of C with index $[A:C] = [A:B][B:C]$.*

Proof. Let $E_1 : A \to B$ and $E_2 : B \to C$ be the conditional expectations that together with the separating elements $e_1 = \tau_1\sum_{i=1}^{m} u_i \otimes_B v_i$ and $e_2 = \tau_2\sum_{j=1}^{n} x_i \otimes_C y_i$ satisfy the unitality condition.

It is easy to check that $E = E_2 \circ E_1 : A \to C$ is a conditional expectation. We claim that $e = \tau_1\tau_2\sum_{i=1}^{n}\sum_{j=1}^{m} u_i x_j \otimes_C y_j v_i$ is a separating element. Trivially, multiplication $\mu : A \otimes_C A \to A$ sends e to 1. We obtain ae = ea $\forall a \in A$ as follows. If M is a B-B bimodule denote the B-centralized subgroup of M by $M^B = \{m \in M : bm = mb \ \forall b \in B\}$. In analogy with the trace lemma define an obvious function $\Psi : A \otimes_B A \otimes_k (B \otimes_C B)^B \to A \otimes_C A$, such that ea and ae belong to image under this map of the same point.

Finally, E and e satisfy the unitality condition in a computation that applies the trace lemma:

$$\mu(1 \otimes_S E)e = \tau_1\tau_2 \sum_i \sum_j u_i x_j E_2 \circ E_1(y_j v_i) =$$

$$\tau_1\tau_2 \sum_i u_i(\sum_j x_j E_2(y_j E_1(v_i)) = \tau_1\tau_2 \sum_i u_i E_1(v_i) \sum_j x_j E_2(y_j) = \tau_1\tau_2$$

Similarly, $\mu(E \otimes_S 1)e = \tau_1\tau_2$. Hence, A is a split separable extension of C with [A:C] = [A:B][B:C]. □

Let A be a split separable extension of S. Fix a conditional expectation $E : A \to S$. Suppose $\tau = [A : S]^{-1}$. In the next theorem we iterate the basic construction to obtain a tower of algebras above A, i.e. each unital k-algebra is a subalgebra via a canonical inclusion of the next algebra, with a countable family of idempotents satisfying braid-like relations. This is an important point since it is demonstrated in [13] that these relations permit representations of the infinite braid group.

Theorem 7.1 *There is a tower of algebras*

$$S \to A \to A_1 \to \cdots \to A_i \to A_{i+1} \to \cdots$$

where each A_i (i = 1,2,...) is the basic construction for the split separable extension A_{i-1} of A_{i-2} (where $A_0 = A$ and $A_{-1} = S$) with index τ^{-1} and conditional expectation $E_{i-1} = \tau\mu_{A_{i-2}}$ (but $E_0 = E$). The family of idempotents $\{e_i\}_{i=1}^{\infty}$ determined by $e_i = 1_{A_{i-1}} \otimes_{A_{i-2}} 1_{A_{i-1}}$ satisfy the relations:

1. $e_{i+1}e_ie_{i+1} = \tau e_{i+1}$;
2. $e_ie_{i+1}e_i = \tau e_i$;
3. $e_ie_j = e_je_i \quad i - j \geq 2$.

Proof. The properties of the basic construction are given in propositions 4.1 through 4.4 and 7.2. In the proof of 4.4 the idempotent e_i of A_i is defined and shown to be cyclic in that $A_i = A^e e_i$ is a cyclic A^e-bimodule, it is shown to satisfy, for each $a \in A$,

$$(*) \quad e_i a e_i = E_{i-1}(a)e_i.$$

In lemma 7.1 we noted that e_i is sorting in that $ae_i = 0$ or $e_i a = 0 \Rightarrow a = 0$. In remark 6.1 we have noted that e_i centralizes elements of S. Finally, one sees instantly that the conditional expectation $E_i = \tau\mu_{A_{i-2}}$ satisfies $E_i(e_i) = \tau$.

Since $A_i = A_{i-1} \otimes_{A_{i-2}} A_{i-1}$ with multiplication determined by E_{i-1} and unity element 1 by a separating element satisfying the unitality condition, A_i is an A_{i-1}-bimodule and the arrow $A_{i-1} \to A_{i-2}$ is just the map $a \longmapsto a1$ ($\forall a \in A_{i-1}$). It is clear that to complete the proof we have only to check the relations

1-3 above for the cyclic sorting idempotents e_i $(i = 1, 2, ...)$ satisfying $(*)$ and centralizing elements of A_{i-2}.

Now it follows from $(*)$ that $e_{i+1}e_ie_{i+1} = E_i(e_i)e_{i+1} = \tau e_{i+1}$. Hence, we have relation (1). Since e_{i+1} is sorting we have from (1) that

$$e_ie_{i+1}e_ie_{i+1} = \tau e_ie_{i+1} \Rightarrow e_ie_{i+1}e_i = \tau e_i$$

whence (2) follows. Since $e_j \in (A_i)_{e_i}$ if $i - j \geq 2$, then e_i and e_j commute, whence relation (3) follows. □

Corollary 7.1 *Same hypotheses and notation as in theorem 7.1. If the ground ring k has an invertible solution q of $q^2\tau = q - 1$, then each braid group on n letters, B_n, maps into A_{n-1} under a homomorphism of k-algebras $\Phi_n : k[B_n] \to A_{n-1}$.*

Proof. It is a classical fact of E. Artin that B_n has a finite presentation,

$$B_n = \{g_1, \ldots, g_{n-1} | \ g_ig_j = g_jg_i, \ \ g_{i+1}g_ig_{i+1} = g_ig_{i+1}g_i, \forall i, j \ \ |i - j| > 1\}$$

In order to define Φ_n it is sufficient to assign an invertible value $\Phi_n(g_i)$ in A_{n-1} and check the Artin relations. We define

$$\Phi_n(g_i) = qe_i - 1 \quad (i = 1, ..., n - 1).$$

$qe_i - 1$ is invertible since $(qe_i - 1)(qe_i + (1 - q)) = 1 - q$ and 1 -q is invertible in k since q and τ are invertibles.

The relation $\Phi_n(g_i)\Phi_n(g_j) = \Phi_n(g_j)\Phi_n(g_i)$ $|i - j| > 1$ is clear from relation 3 in theorem 6.1.

Using relation 1 and 2 of theorem 7.1 and the fact that the e_i's are idempotents, we get

$$\Phi_n(g_{i+1})\Phi_n(g_i)\Phi_n(g_{i+1}) = e_{i+1}(\tau q^3 - q^2 + 2q) + qe_i - q^2(e_ie_{i+1} + e_{i+1}e_i) - 1$$

$$= \Phi_n(g_i)\Phi_n(g_{i+1})\Phi_n(g_i) = e_i(\tau q^3 - q^2 + 2q) + qe_{i+1} - q^2(e_ie_{i+1} + e_{i+1}e_i) - 1,$$

since $\tau q^3 - q^2 + 2q = q$. □

Remark 7.2 Let A_∞ be the direct limit, $\lim_{\to} A_i$. It follows from the Markov relations that associate braids with links [5] that a sequence of traces $\phi_n : A_n \to k$ satisfying

$$\phi_{n+1}(x(qe_{n+1} - 1)^{\pm 1}) = \phi_n(x) \quad \forall x \in A_n.$$

gives an invariant of links in R^3 under isotopy.

The question of Pimsner and Popa has been addressed algebraically in this paper. The extent that the present paper holds for operator algebras is under investigation by the author. One can phrase several of the properties of operator algebras in terms of module, such as amenability as a cohomological dimension zero phenomenon. One can hopefully then define a relative amenability and split extension that would give a version of propositions 1.2 and 1.3 for operator algebras. The phrasing of the unitality condition is then hopefully clear, which should give an angle on several old and new theorems about shared properties of subalgebra and algebra. For example, the theorem of A. Connes that M is the hyperfinite factor iff N is hyperfinite (where M and N are II_1 factors with $N \subseteq M$ of finite index) should be reprovable in this way.

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65/83 "ANVENDT MATEMATIK - TEORI ELLER PRAKSIS".
Projektrapport af: Per Hedegård Andersen, Kirsten Habekost, Carsten Holst-Jensen, Annelise von Moos, Else Marie Pedersen og Erling Møller Pedersen.
Vejledere: Bernhelm Booss og Klaus Grünbaum.

66/83 "MATEMATISKE MODELLER FOR PERIODISK SELEKTION I ESCHERICHIA COLI".
Projektrapport af: Hanne Lisbet Andersen, Ole Richard Jensen og Klavs Frisdahl.
Vejledere: Jørgen Larsen og Anders Hede Madsen.

67/83 "ELEPSOIDE METODEN - EN NY METODE TIL LINEÆR PROGRAMMERING?"
Projektrapport af: Lone Biilmann og Lars Boye.
Vejleder: Mogens Brun Heefelt.

68/83 "STOKASTISKE MODELLER I POPULATIONSGENETIK"
- til kritikken af teoriladede modeller.
Projektrapport af: Lise Odgård Gade, Susanne Hansen, Michael Hvild og Frank Mølgård Olsen.
Vejleder: Jørgen Larsen.

69/83 "ELEVFORUDSÆTNINGER I FYSIK"
- en test i 1.g med kommentarer.
Af: Albert C. Paulsen.

70/83 "INDLÆRINGS - OG FORMIDLINGSPROBLEMER I MATEMATIK PÅ VOKSENUNDERVISNINGSNIVEAU".
Projektrapport af: Hanne Lisbet Andersen, Torben J. Andreasen, Svend Åge Houmann, Helle Glerup Jensen, Keld Fl. Nielsen, Lene Vagn Rasmussen.
Vejleder: Klaus Grünbaum og Anders Hede Madsen.

71/83 "PIGER OG FYSIK"
- et problem og en udfordring for skolen?
Af: Karin Beyer, Sussanne Blegaa, Birthe Olsen, Jette Reich og Mette Vedelsby.

72/83 "VERDEN IFØLGE PEIRCE" - to metafysiske essays, om og af C.S Peirce.
Af: Peder Voetmann Christiansen.

73/83 ""EN ENERGIANALYSE AF LANDBRUG"
- økologisk contra traditionelt.
ENERGY SERIES NO. 9
Specialeopgave i fysik af: Bent Hove Jensen.
Vejleder: Bent Sørensen.

74/84 "MINIATURISERING AF MIKROELEKTRONIK" - om videnskabeliggjort teknologi og nytten af at lære fysik.
Projektrapport af: Bodil Harder og Linda Szkotak Jensen.
Vejledere: Jens Højgaard Jensen og Bent C. Jørgensen.

75/84 "MATEMATIKUNDERVISNINGEN I FREMTIDENS GYMNASIUM"
- Case: Lineær programmering.
Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank Mølgaard Olsen.
Vejledere: Mogens Brun Heefelt og Jens Bjørneboe.

76/84 "KERNEKRAFT I DANMARK?" - Et høringssvar indkaldt af miljøministeriet, med kritik af miljøstyrelsens rapporter af 15. marts 1984.
ENERGY SERIES No. 10
Af: Niels Boye Olsen og Bent Sørensen.

77/84 "POLITISKE INDEKS - FUP ELLER FAKTA?"
Opinionsundersøgelser belyst ved statistiske modeller.
Projektrapport af: Svend Åge Houmann, Keld Nielsen og Susanne Stender.
Vejledere: Jørgen Larsen og Jens Bjørneboe.

78/84 "JÆVNSTRØMSLEDNINGSEVNE OG GITTERSTRUKTUR I AMORFT GERMANIUM".
Specialrapport af: Hans Hedal, Frank C. Ludvigsen og Finn C. Physant.
Vejleder: Niels Boye Olsen.

79/84 "MATEMATIK OG ALMENDANNELSE".
Projektrapport af: Henrik Coster, Mikael Wennerberg Johansen, Povl Kattler, Birgitte Lydholm og Morten Overgaard Nielsen.
Vejleder: Bernhelm Booss.

80/84 "KURSUSMATERIALE TIL MATEMATIK B".
Af: Mogens Brun Heefelt.

81/84 "FREKVENSAFHÆNGIG LEDNINGSEVNE I AMORFT GERMANIUM".
Specialerapport af: Jørgen Wind Petersen og Jan Christensen.
Vejleder: Niels Boye Olsen.

82/84 "MATEMATIK - OG FYSIKUNDERVISNINGEN I DET AUTOMATISEREDE SAMFUND".
Rapport fra et seminar afholdt i Hvidovre 25-27 april 1983.
Red.: Jens Højgaard Jensen, Bent C. Jørgensen og Mogens Niss.

83/84 "ON THE QUANTIFICATION OF SECURITY":
PEACE RESEARCH SERIES NO. 1
Af: Bent Sørensen
nr. 83 er p.t. udgået

84/84 "NOGLE ARTIKLER OM MATEMATIK, FYSIK OG ALMENDANNELSE".
Af: Jens Højgaard Jensen, Mogens Niss m. fl.

85/84 "CENTRIFUGALREGULATORER OG MATEMATIK".
Specialerapport af: Per Hedegård Andersen, Carsten Holst-Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Stig Andur Pedersen.

86/84 "SECURITY IMPLICATIONS OF ALTERNATIVE DEFENSE OPTIONS FOR WESTERN EUROPE".
PEACE RESEARCH SERIES NO. 2
Af: Bent Sørensen.

87/84 "A SIMPLE MODEL OF AC HOPPING CONDUCTIVITY IN DISORDERED SOLIDS".
Af: Jeppe C. Dyre.

88/84 "RISE, FALL AND RESURRECTION OF INFINITESIMALS".
Af: Detlef Laugwitz.

89/84 "FJERNVARMEOPTIMERING".
Af: Bjarne Lillethorup og Jacob Mørch Pedersen.

90/84 "ENERGI I 1.G - EN TEORI FOR TILRETTELÆGGELSE".
Af: Albert Chr. Paulsen.

91/85 "KVANTETEORI FOR GYMNASIET".
1. Lærervejledning
Projektrapport af: Biger Lundgren, Henning Sten Hansen og John Johansson.
Vejleder: Torsten Meyer.

92/85 "KVANTETEORI FOR GYMNASIET".
2. Materiale
Projektrapport af: Biger Lundgren, Henning Sten Hansen og John Johansson.
Vejleder: Torsten Meyer.

93/85 "THE SEMIOTICS OF QUANTUM - NON - LOCALITY".
Af: Peder Voetmann Christiansen.

94/85 "TREENIGHEDEN BOURBAKI - generalen, matematikeren og ånden".
Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank M. Olsen.
Vejleder: Mogens Niss.

95/85 "AN ALTERNATIV DEFENSE PLAN FOR WESTERN EUROPE".
PEACE RESEARCH SERIES NO. 3
Af: Bent Sørensen

96/85 "ASPEKTER VED KRAFTVARMEFORSYNING".
Af: Bjarne Lilletorup.
Vejleder: Bent Sørensen.

97/85 "ON THE PHYSICS OF A.C. HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.

98/85 "VALGMULIGHEDER I INFORMATIONSALDEREN".
Af: Bent Sørensen.

99/85 "Der er langt fra Q til R".
Projektrapport af: Niels Jørgensen og Mikael Klintorp.
Vejleder: Stig Andur Pedersen.

100/85 "TALSYSTEMETS OPBYGNING".
Af: Mogens Niss.

101/85 "EXTENDED MOMENTUM THEORY FOR WINDMILLS IN PERTURBATIVE FORM".
Af: Ganesh Sengupta.

102/85 OPSTILLING OG ANALYSE AF MATEMATISKE MODELLER, BELYST VED MODELLER OVER KØERS FODEROPTAGELSE OG - OMSÆTNING".
Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn og Susanne Stender.
Vejleder: Klaus Grünbaum.

103/85 "ØDSLE KOLDKRIGERE OG VIDENSKABENS LYSE IDEER".
Projektrapport af: Niels Ole Dam og Kurt Jensen.
Vejleder: Bent Sørensen.

104/85 "ANALOGREGNEMASKINEN OG LORENZLIGNINGER".
Af: Jens Jæger.

105/85 "THE FREQUENCY DEPENDENCE OF THE SPECIFIC HEAT AF THE GLASS REANSITION".
Af: Tage Christensen.

"A SIMPLE MODEL AF AC HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.
Contributions to the Third International Conference on the Structure of Non - Crystalline Materials held in Grenoble July 1985.

106/85 "QUANTUM THEORY OF EXTENDED PARTICLES".
Af: Bent Sørensen.

107/85 "EN MYG GØR INGEN EPIDEMI",
- flodblindhed som eksempel på matematisk modellering af et epidemiologisk problem.
Projektrapport af: Per Hedegård Andersen, Lars Boye, CarstenHolst Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Jesper Larsen.

108/85 "APPLICATIONS AND MODELLING IN THE MATEMATICS CURRICULUM" - state and trends -
Af: Mogens Niss.

109/85 "COX I STUDIETIDEN" - Cox's regressionsmodel anvendt på studenteroplysninger fra RUC.
Projektrapport af: Mikael Wennerberg Johansen, Poul Katler og Torben J. Andreasen.
Vejleder: Jørgen Larsen.

110/85 "PLANNING FOR SECURITY".
Af: Bent Sørensen

111/85 JORDEN RUNDT PÅ FLADE KORT".
Projektrapport af: Birgit Andresen, Beatriz Quinones og Jimmy Staal.
Vejleder: Mogens Niss.

112/85 "VIDENSKABELIGGØRELSE AF DANSK TEKNOLOGISK INNOVATION FREM TIL 1950 - BELYST VED EKSEMPLER".
Projektrapport af: Erik Odgaard Gade, Hans Hedal, Frank C. Ludvigsen, Annette Post Nielsen og Finn Physant.
Vejleder: Claus Bryld og Bent C. Jørgensen.

113/85 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS 11".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

114/85 "ANVENDELSE AF GRAFISKE METODER TIL ANALYSE AF KONTIGENSTABELLER".
Projektrapport af: Lone Biilmann, Ole R. Jensen og Anne-Lise von Moos.
Vejleder: Jørgen Larsen.

115/85 "MATEMATIKKENS UDVIKLING OP TIL RENESSANCEN".
Af: Mogens Niss.

116/85 "A PHENOMENOLOGICAL MODEL FOR THE MEYER-NELDEL RULE".
Af: Jeppe C. Dyre.

117/85 "KRAFT & FJERNVARMEOPTIMERING"
Af: Jacob Mørch Pedersen.
Vejleder: Bent Sørensen

118/85 "TILFÆLDIGHEDEN OG NØDVENDIGHEDEN IFØLGE PEIRCE OG FYSIKKEN".
Af: Peder Voetmann Christiansen

119/86 "DET ER GANSKE VIST - - EUKLIDS FEMTE POSTULAT KUNNE NOK SKABE RØRE I ANDEDAMMEN".
Af: Iben Maj Christiansen
Vejleder: Mogens Niss.

120/86 "ET ANTAL STATISTISKE STANDARDMODELLER".
Af: Jørgen Larsen

121/86 "SIMULATION I KONTINUERT TID".
Af: Peder Voetmann Christiansen.

122/86 "ON THE MECHANISM OF GLASS IONIC CONDUCTIVITY".
Af: Jeppe C. Dyre.

123/86 "GYMNASIEFYSIKKEN OG DEN STORE VERDEN".
Fysiklærerforeningen, IMFUFA, RUC.

124/86 "OPGAVESAMLING I MATEMATIK".
Samtlige opgaver stillet i tiden 1974-jan. 1986.

125/86 "UVBY,β - systemet - en effektiv fotometrisk spektralklassifikation af B-,A- og F-stjerner".
Projektrapport af: Birger Lundgren.

126/86 "OM UDVIKLINGEN AF DEN SPECIELLE RELATIVITETSTEORI".
Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

127/86 "GALOIS' BIDRAG TIL UDVIKLINGEN AF DEN ABSTRAKTE ALGEBRA".
Projektrapport af: Pernille Sand, Heine Larsen & Lars Frandsen.
Vejleder: Mogens Niss.

128/86 "SMÅKRYB" - om ikke-standard analyse.
Projektrapport af: Niels Jørgensen & Mikael Klintorp.
Vejleder: Jeppe Dyre.

129/86 "PHYSICS IN SOCIETY"
Lecture Notes 1983 (1986)
Af: Bent Sørensen

130/86 "Studies in Wind Power"
Af: Bent Sørensen

131/86 "FYSIK OG SAMFUND" - Et integreret fysik/historieprojekt om naturanskuelsens historiske udvikling og dens samfundsmæssige betingethed.
Projektrapport af: Jakob Heckscher, Søren Brønd, Andy Wierød.
Vejledere: Jens Høyrup, Jørgen Vogelius, Jens Højgaard Jensen.

132/86 "FYSIK OG DANNELSE"
Projektrapport af: Søren Brønd, Andy Wierød.
Vejledere: Karin Beyer, Jørgen Vogelius.

133/86 "CHERNOBYL ACCIDENT: ASSESSING THE DATA.
ENERGY SERIES NO. 15.
AF: Bent Sørensen.

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen, Petr Viščor

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSESTEORETISKE FORUDSÆTNINGER"
MASTEMATIKSPECIALE: Claus Larsen
Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens første og andet møde med græsk filosofi"
Projektrapport af Frank Colding Ludvigsen
Vejledere: Historie: Ib Thiersen
Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE FASTE STOFFER" - Resume af licentiatafhandling
Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International Workshop on Teaching Nonlinear Phenomena at Universities and Schools, "Chaos in Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

139/87 "Machbarkeit nichtbeherrschbarer Technik durch Fortschritte in der Erkennbarkeit der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR - ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircean Approach

Paper presented at the Symposium on the Foundations of Modern Physics The Copenhagen Interpretation 60 Years after the Como Lecture. Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

145/87 "AIMS AND SCOPE OF APPLICATIONS AND MODELLING IN MATHEMATICS CURRICULA"

Manuscript of a plenary lecture delivered at ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

146/87 "BESTEMMELSE AF BULKRESISTIVITETEN I SILICIUM" - en ny frekvensbaseret målemetode.

Fysikspeciale af Jan Vedde

Vejledere: Niels Boye Olsen & Petr Viščor

147/87 "Rapport om BIS på NAT-BAS"

redigeret af: Mogens Brun Heefelt

148/87 "Naturvidenskabsundervisning med Samfundsperspektiv"

af: Peter Colding-Jørgensen DLH
Albert Chr. Paulsen

149/87 "In-Situ Measurements of the density of amorphous germanium prepared in ultra high vacuum"

by: Petr Viščor

150/87 "Structure and the Existence of the first sharp diffraction peak in amorphous germanium prepared in UHV and measured in-situ"

by: Petr Viščor

151/87 "DYNAMISK PROGRAMMERING"

Matematikprojekt af:
Birgit Andresen, Keld Nielsen og Jimmy Staal

Vejleder: Mogens Niss

152/87 "PSEUDO-DIFFERENTIAL PROJECTIONS AND THE TOPOLOGY OF CERTAIN SPACES OF ELLIPTIC BOUNDARY VALUE PROBLEMS"

by: Bernhelm Booss-Bavnbek
Krzysztof P. Wojciechowski

153/88 "HALVLEDERTEKNOLOGIENS UDVIKLING MELLEM MILITÆRE OG CIVILE KRÆFTER"

Et eksempel på humanistisk teknologihistorie
Historiespeciale

Af: Hans Hedal

Vejleder: Ib Thiersen

154/88 "MASTER EQUATION APPROACH TO VISCOUS LIQUIDS AND THE GLASS TRANSITION"

By: Jeppe Dyre

155/88 "A NOTE ON THE ACTION OF THE POISSON SOLUTION OPERATOR TO THE DIRICHLET PROBLEM FOR A FORMALLY SELFADJOINT DIFFERENTIAL OPERATOR"

by: Michael Pedersen

156/88 "THE RANDOM FREE ENERGY BARRIER MODEL FOR AC CONDUCTION IN DISORDERED SOLIDS"

by: Jeppe C. Dyre

157/88 " STABILIZATION OF PARTIAL DIFFERENTIAL EQUATIONS BY FINITE DIMENSIONAL BOUNDARY FEEDBACK CONTROL: A pseudo-differential approach."

by: Michael Pedersen

158/88 "UNIFIED FORMALISM FOR EXCESS CURRENT NOISE IN RANDOM WALK MODELS"

by: Jeppe Dyre

159/88 "STUDIES IN SOLAR ENERGY"

by: Bent Sørensen

160/88 "LOOP GROUPS AND INSTANTONS IN DIMENSION TWO"

by: Jens Gravesen

161/88 "PSEUDO-DIFFERENTIAL PERTURBATIONS AND STABILIZATION OF DISTRIBUTED PARAMETER SYSTEMS:

Dirichlet feedback control problems"

by: Michael Pedersen

162/88 "PIGER & FYSIK - OG MEGET MERE"

AF: Karin Beyer, Sussanne Blegaa, Birthe Olsen, Jette Reich , Mette Vedelsby

163/88 "EN MATEMATISK MODEL TIL BESTEMMELSE AF PERMEABILITETEN FOR BLOD-NETHINDE-BARRIEREN"

Af: Finn Langberg, Michael Jarden, Lars Frellesen

Vejleder: Jesper Larsen

164/88 "Vurdering af matematisk teknologi
Technology Assessment
Technikfolgenabschätzung"

Af: Bernhelm Booss-Bavnbek, Glen Pate med
Martin Bohle-Carbonell og Jens Højgaard Jensen

165/88 "COMPLEX STRUCTURES IN THE NASH-MOSER CATEGORY"

by: Jens Gravesen

166/88 "Grundbegreber i Sandsynligheds-regningen"

Af: Jørgen Larsen

167a/88 "BASISSTATISTIK 1. Diskrete modeller"

Af: Jørgen Larsen

167b/88 "BASISSTATISTIK 2. Kontinuerte modeller"

Af: Jørgen Larsen

168/88 "OVERFLADEN AF PLANETEN MARS" Laboratorie-simulering og MARS-analoger undersøgt ved Mössbauerspektroskopi.

Fysikspeciale af:

Birger Lundgren

Vejleder: Jens Martin Knudsen
Fys.Lab./HCØ

169/88 "CHARLES S. PEIRCE: MURSTEN OG MØRTEL TIL EN METAFYSIK."

Fem artikler fra tidsskriftet "The Monist" 1891-93.

Introduktion og oversættelse:

Peder Voetmann Christéansen

170/88 "OPGAVESAMLING I MATEMATIK"

Samtlige opgaver stillet i tiden 1974 - juni 1988

171/88 "The Dirac Equation with Light-Cone Data"

af: Johnny Tom Ottesen

172/88 "FYSIK OG VIRKELIGHED"

Kvantemekanikkens grundlagsproblem i gymnasiet.

Fysikprojekt af:

Erik Lund og Kurt Jensen

Vejledere: Albert Chr. Paulsen og Peder Voetmann Christiansen

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173/89 "NUMERISKE ALGORITMER"

af: Mogens Brun Heefelt

174/89 " GRAFISK FREMSTILLING AF FRAKTALER OG KAOS"

af: Peder Voetmann Christiansen

175/89 " AN ELEMENTARY ANALYSIS OF THE TIME DEPENDENT SPECTRUM OF THE NON-STATONARY SOLUTION TO THE OPERATOR RICCATI EQUATION

af: Michael Pedersen

176/89 " A MAXIUM ENTROPY ANSATZ FOR NONLINEAR RESPONSE THEORY"

af : Jeppe Dyre

177/89 "HVAD SKAL ADAM STÅ MODEL TIL"

af: Morten Andersen, Ulla Engström, Thomas Gravesen, Nanna Lund, Pia Madsen, Dina Rawat, Peter Torstensen

Vejleder: Mogens Brun Heefelt

178/89 "BIOSYNTESEN AF PENICILLIN - en matematisk model"

af: Ulla Eghave Rasmussen, Hans Oxvang Mortensen, Michael Jarden

vejleder i matematik: Jesper Larsen
biologi: Erling Lauridsen

179a/89 "LÆRERVEJLEDNING M.M. til et eksperimentelt forløb om kaos"

af: Andy Wierød, Søren Brønd og Jimmy Staal

Vejledere: Peder Voetmann Christiansen
Karin Beyer

179b/89 "ELEVHÆFTE: Noter til et eksperimentelt kursus om kaos"

af: Andy Wierød, Søren Brønd og Jimmy Staal

Vejledere: Peder Voetmann Christiansen
Karin Beyer

180/89 "KAOS I FYSISKE SYSTEMER eksemplificeret ved torsions- og dobbeltpendul".

af: Andy Wierød, Søren Brønd og Jimmy Staal
Vejleder: Peder Voetmann Christiansen

181/89 "A ZERO-PARAMETER CONSTITUTIVE RELATION FOR PURE SHEAR VISCOELASTICITY"

by: Jeppe Dyre

183/89 **"MATEMATICAL PROBLEM SOLVING, MODELLING. APPLICATIONS AND LINKS TO OTHER SUBJECTS - State. trends and issues in mathematics instruction**

by: WERNER BLUM, Kassel (FRG) og MOGENS NISS, Roskilde (Denmark)

184/89 "En metode til bestemmelse af den frekvensafhængige varmefylde af en underafkølet væske ved glasovergangen"

af: Tage Emil Christensen

185/90 "EN NÆSTEN PERIODISK HISTORIE"

Et matematisk projekt

af: Steen Grode og Thomas Jessen

Vejleder: Jacob Jacobsen

186/90 "RITUAL OG RATIONALITET i videnskabers udvikling"

redigeret af Arne Jakobsen og Stig Andur Pedersen

187/90 "RSA - et kryptografisk system"

af: Annemette Sofie Olufsen, Lars Frellesen og Ole Møller Nielsen

Vejledere: Michael Pedersen og Finn Munk

188/90 "FERMICONDENSATION - AN ALMOST IDEAL GLASS TRANSITION"

by: Jeppe Dyre

189/90 "DATAMATER I MATEMATIKUNDERVISNINGEN PÅ GYMNASIET OG HØJERE LÆREANSTALTER

af: Finn Langberg

190/90 "FIVE REQUIREMENTS FOR AN APPROXIMATE NONLINEAR RESPONSE THEORY"

by: Jeppe Dyre

191/90 "MOORE COHOMOLOGY, PRINCIPAL BUNDLES AND ACTIONS OF GROUPS ON C*-ALGEBRAS"

by: Iain Raeburn and Dana P. Williams

192/90 "Age-dependent host mortality in the dynamics of endemic infectious diseases and SIR-models of the epidemiology and natural selection of co-circulating influenza virus with partial cross-immunity"

by: Viggo Andreasen

193/90 "Causal and Diagnostic Reasoning"

by: Stig Andur Pedersen

194a/90 "DETERMINISTISK KAOS"

Projektrapport af : Frank Olsen

194b/90 "DETERMINISTISK KAOS"
Kørselsrapport

Projektrapport af: Frank Olsen

195/90 "STADIER PÅ PARADIGMETS VEJ"
Et projekt om den videnskabelige udvikling der førte til dannelse af kvantemekanikken.

Projektrapport for 1. modul på fysikuddannelsen, skrevet af:

Anja Boisen, Thomas Hougård, Anders Gorm Larsen, Nicolai Ryge.

Vejleder: Peder Voetmann Christiansen

196/90 "ER KAOS NØDVENDIGT?"

- en projektrapport om kaos' paradigmatiske status i fysikken.

af: Johannes K. Nielsen, Jimmy Staal og Peter Bøggild

Vejleder: Peder Voetmann Christiansen

197/90 "Kontrafaktiske konditionaler i HOL

af: Jesper Voetmann, Hans Oxvang Mortensen og Aleksander Høst-Madsen

Vejleder: Stig Andur Pedersen

198/90 "Metal-Isolator-Metal systemer"

Speciale

af: Frank Olsen

199/90 "SPREDT FÆGTNING" Artikelsamling

af: Jens Højgaard Jensen

200/90 "LINEÆR ALGEBRA OG ANALYSE"

Noter til den naturvidenskabelige basisuddannelse.

af: Mogens Niss

201/90 "Undersøgelse af atomare korrelationer i amorfe stoffer ved røntgendiffraktion"

af: Karen Birkelund og Klaus Dahl Jensen

Vejledere: Petr Viščor, Ole Bakander

202/90 "TEGN OG KVANTER"
Foredrag og artikler, 1971-90.

af: Peder Voetmann Christiansen

203/90 "OPGAVESAMLING I MATEMATIK" 1974-1990

afløser tekst 170/88

204/91 "ERKENDELSE OG KVANTEMEKANIK"

et Breddemodul Fysik Projekt

af: Thomas Jessen

Vejleder: Petr Viščor

205/91 "PEIRCE'S LOGIC OF VAGUENESS"

by: Claudine Engel-Tiercelin
Department of Philosophy
Université de Paris-1
(Panthéon-Sorbonne)

206a+b/91 "GERMANIUMBEAMANALYSE SAMT A - GE TYNDFILMS ELEKTRISKE EGENSKABER"

Eksperimentelt Fysikspeciale

af: Jeanne Linda Mortensen

og Annette Post Nielsen

Vejleder: Petr Viščor

207/91 "SOME REMARKS ON AC CONDUCTION IN DISORDERED SOLIDS"

by: Jeppe C. Dyre

208/91 "LANGEVIN MODELS FOR SHEAR STRESS FLUCTUATIONS IN FLOWS OF VISCO-ELASTIC LIQUIDS"

by: Jeppe C. Dyre

209/91 "LORENZ GUIDE" Kompendium til den danske fysiker Ludvig Lorenz, 1829-91.

af: Helge Kragh

210/91 "Global Dimension, Tower of Algebras, and Jones Index of Split Seperable Subalgebras with Unitality Condition.

by: Lars Kadison