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PEIRCE'S LOGIC OF VAGUENESS

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Abstract:

Claudine Engel-Tiercelin, professor of philosophy at the
university of Panthéon-Sorbonne in Paris, is an expert on the
logical-semiotical inquiries of the american philosopher
C.S.Peirce (1839-1914). In the present paper she elucidates
Peirce's logic of vagueness, a thoroughgoing theme in many
scattered quotations from his Collected Papers.

The logic of vagueness is applied to general signs, symbols,
(divisible into terms, propositions and arguments), and can be
treated, according to Peirce's triadic categories under the
headings: 1 Speculative Grammar, 2. Critical Logic, and 3.
Rhetoric. The author treats the first of these divisions with
illustrative examples of vagueness vs. generality, and objective
vagueness in the so called informed breadth and depth aspects of
the sign relation. She stresses, however, in consequence of
Peirce's realistic ontology, that vagueness exists in reality, it
is not just a nuisance in the language that ought to be removed
by the logicians. Thus, application and further development of
Peirce's original ideas, as they are explained in this article,
may have a word to say about the danger of being too precise in a
mathematical modelling of the world, and about the enigmatic
status of indeterminacy in physics.

PEIRCE's LOGIC OF VAGUENESS

By logic of vagueness are usually meant today inquiries made
outside the field of classical logic in order to build multi-
valued logics. Such inquiries are most often concerned with
the vagueness surrounding ordinary language and by the ne-
cessity to appeal to some non standard logic in order to ac-
count for the paradoxes arising from vague predicates.

Therefore such a logic is intended as an analysis of the
conditions of production and determination of meaning.

Let us be clear, at the outset, that when Peirce accuses the
logicians of having neglected vagueness[1], and claims that he
has achieved a logic of vagueness almost completely (5.506),
he aims at something rather different: although he himself
makes inquiries towards a triadic logic, and is interested in
the problems related to the semantics of vagueness surroun-
ding ordinary language, his basic project deals first with
the specific semiotic as well as ontological conception he
has of logic[2]. Accordingly, Peirce's logic of vagueness
presents itself as a general theory of the vagueness affec-
ting signs. In that respect, such a logic is "everywhere"[3].
Yet it should not be confused with some general semiotic, in
so far as the only signs considered are symbols (and not
icons and indices)[4].

As a logic of symbols, the logic of vagueness will neither be
confined to a theory of vague concepts nor to a theory of
linguistic terms only. For "the rules of logic hold good of
any symbols, of those which are written or spoken as of those
which are thought" (1.559). But symbols or "indeterminate ge-
neral signs" are only signs because there is a rule, a habit,
a disposition or a convention to interpret them or to use
them as such: hence the logic of vagueness will have to elu-
cidate the rules and habits that govern the production of
symbols[5].

Finally, symbols are the only signs that allow the completion of arguments: the logic of vagueness will thus be a _formal_ logic, obeying the rules and norms that govern the sound functioning of inference, and accordingly, the principles of bivalence, contradiction and excluded middle, which are the minimal requirements for classical extensional logic.

But for Peirce, logic is also inseparable from ontology; therefore any account of vagueness will never be dealt with as such; together with the general and the individual, vagueness is indeed, but one of the three terms that are brought forth through the categorial analysis; moreover it is mereley one, together with the general, of the two figures of the indeterminate, both of which are opposed to the determinate. Thus, Peirce's project of a logic of vagueness will be more accurately defined as the project of a general theory of the various forms and relations that the indeterminate and the determinate may legitimately take.

I shall examine the general principles under which Peirce's logic of vagueness fall, such as it is conveyed to us in what constitutes the first of its divisions, namely Speculative Grammar[6], before presenting the domain of application of vagueness, namely the informed reference or, in Peirce's terms "breadth" of symbols, as well as their informed meaning or "depth". As a conclusion, I shall try to indicate what Peirce's project can bring to contemporary analyses and to wonder whether Peirce is finally favorable (or not) to the idea of giving up classical logic.

1. THE PRINCIPLES OF SPECULATIVE GRAMMAR

Speculative Grammar is that part of logic which deals with the formal conditions of symbols that have a meaning (1.559; 4.116). In modern terms, we would say that grammar has to establish the syntactical and semantical conditions which

symbols must obey, (although such modern classifications are always somewhat unfaithful, as will appear in what follows, to Peirce's own divisions). Such a normative character of grammar is decisive. Indeed its task is to establish what must be true of the <u>representamina</u> that are being used by a scientific intelligence, in order for them to embody any meaning whatsever (2.229). Hence grammar may be characterized in the following way:

First, it is a formal, "quasi-necessary" (as Peirce sometimes says) theory, (2.227; 210), which will consider the illative relation as the fundamental semiotic relation (2.444n).

Second, it will follow a rule which is in keeping with the nature of signs in general: namely a sign or symbol has only meaning within the propositional context (and as we shall see, even the assertive context), in which it is inserted. (4.583; cf. 4.56,551). In that respect, thought, being a sign, is no exception to that rule: "thought must have <u>some</u> possible interpretation for some possible interpreter", wherein lies the very being of its being (4,6) (in other words, its dialogical character) (Ms 931,5.257,5.421). Thus, Speculative Grammar is identified with some <u>Erkenntnis-theorie</u> or epistemology which considers "in what sense and how there can be any true proposition and false proposition, and what are the general conditions to which thought or signs of any kind must conform in order to assert anything" (2.206).

Thirdly, the principles put forth by Speculative Grammar will only be valuable in so far as they apply to some scientific intelligence, namely an intelligence which is incapable of intuition, in accordance with the conclusions established by the articles published in 1868 in <u>The Journal of speculative Philosophy</u> (5.213-357), an intelligence that has no other way to learn except by following the rules of inductive, abductive and deductive inference, as applied to experience, but an intelligence which has also accepted certain aims and me-

thods, among which the principle owing to which a discourse is meaningful only if its deliberate aim is to put the process of rational inquiry at the service of knowledge and truth, which can only be reached through self-control processes. Such an ethical dimension of logic is very important in order to understand Peirce's analyses. Speculative Grammar will be a theory of the conditions which any scientific discourse must be submitted to in order to have any meaning and to be capable of reaching truth. Peirce's theory of meaning is by the same token, inseparable from his theory of truth; this is why it is very difficult to talk of Peirce's theory of meaning and reference as such, in so far as such questions are intimately linked with the problems of determining truth conditions.

Fourthly, Speculative Grammar is understood as a theory of assertion. Briefly, an assertion, for Peirce, is what enables one to distinguish between the volitional content from the representative or propositional content. Assertion is the symbol which articulates the iconic elements (that is the elements which have some formal resemblance with the object) and the indexical elements (that is the elements which have some physical resemblance with the object) of the proposition. It is upon the symbol that the whole weight of assertion bears[7].

Moreover, an assertion has no meaning except through some designation that shows whether one refers to the real universe or of what universe of fiction it is about (8.368). This explains how important the indexical element of the proposition is.

An assertion is an act in which a speaker addresses a listener, formulates a propositional symbol and assumes some responsibility concerning the truth of that symbol. Then, any assertion implies, from the part of the speaker, that he believes or knows what he asserts and that he intends to convey the same belief and the same knowledge to his listener. Thus

it is first of all the speaker who has the main responsibi-
lity; it is his task to eliminate any imprecision or ambigui-
ty that might be an obstacle for communication. Which invol-
ves, on the utterer's part "a voluntary self-subjection to
penalties" in the event that the proposition turns out to be
false (Ms 517). And Peirce goes so far as to say that such
penalties are comparable to the legal penalties associated
with making a false statement under oath (Ms 517; NEM.IV,
p.249).

All this presupposes on both parts that they have a certain
competence and that they partake to a community of ideals and
aims of speech. Both want to communicate, to learn and to
know, that is to try and suppress all kinds of ambiguity and
imprecision that might creep into the rational process and
break communication.

Speculative Grammar thus involves, at least, a theory of
communication, a theory of the norms that govern communi-
cation, a theory of propositional symbols, a theory of truth,
of meaning, of belief, and of knowledge[8].

All these compounds are to be found in the way Peirce con-
ceives his logic of vagueness. As a matter of fact, the very
frame of it seems indeed unavoidable, in view of its very
definition and of the aims of logic. Indeed, how is one to
reach a correct theory of assertion, that enables one to
progress towards knowledge and truth, while taking into ac-
count the fact that our knowledge is entirely wrapped into
signs which, by nature, are utterly indeterminate?

> "Honest people, when not joking, intend to make the
> meaning of their words determinate, so that there shall
> be no latitude of interpretation at all" (5.447).

But, on the other hand, "no sign is absolutely precise".
Hence, how do we - and can we - resolve such difficulties?
Such is the challenge for a logic of vagueness.

2. THE SPECULATIVE GRAMMAR OF VAGUENESS: PEIRCE's THEORY OF INFORMED BREADTH AND DEPTH

Speculative Grammar is supposed to determine the formal conditions of the symbols that have meaning. The Speculative Grammar of vagueness deals with the formal conditions of the symbols which are determinate or indeterminate in meaning. Any symbol, as a sign, is capable of determining a further symbol which interprets it or translates it: so that it is at least potentially indeterminate.

To that leading idea of the logic of vagueness Peirce adds a twofold thesis: no term is absolutely indeterminate; no term is absolutely determinate.

First, the only way we have to know something of a term is by comparing it, or putting it into relation with another term. An absolutely indeterminate sign would designate a property which would have to be shared by <u>all</u> things. Hence we can form no conception of such a property.

> "We have no propositions whose predicate is entirely indeterminate, for it would be quite senseless to say, "A has the common characters of all things," in as much as there are no such common characters" (1.548).

Peirce writes it to William James:

> "(No term can be)... absolutely universal, since of such a term nothing could be truly asserted (,) so that it would be quite meaningless" (L.224, p.7, August 1905).

Meaningless indeed, for one the main principles of semiotic is such that any symbol must have an interpretant, hence some informed or essential depth. Otherwise, it cannot take part into communication or dialogue, for it is not capable of being interpreted.

Besides, for a term to be said absolutely indeterminate, one should be able to know all the possible predicates with respect to which it would be indeterminate. But such a list is infinite. Hence, if we stick to the level of the cognizable, any term is at least potentially determinate. As Jarrett Brock has accurately pointed out, what is here being ruled out as unintelligible, is the notion of an absolutely, simple, unanalyzable, indefinable and hence inexplicable general term- the stock in trade of traditional Cartesian Philosophy. As Brock says "We are supposed to "just understand" the meaning of such terms without being able to convey them to others. They must "just understand" them also. We might call such terms self-defining in order to show their affinity to another intuitionist's delight - the self-evident (self-justifying!) judgement[9].
We have here, in a nutshell Peirce's total rejection of "the salad of cartesianism."

Second, it is also from Peirce's anti-intuitionism and anti-reductionism that proceed the arguments in favor of the thesis according to which no term is absolutely determinate.

> "A concept determined in all respects is as fictional as
> a definite (precise) concept in all respects"(7.208).

What would such an absolutely determinate term amount to? A logical atom, "incapable of logical division" (3.93). Now, a term "however determinate, may be made more determinate still." For example, the second Phillip of Macedon is always capable of logical division into Phillip sober and Phillip drunk (3.93). An interpreter has always the right to consider that Phillip was a different man in different times and places. A term such as Phillip of Macedon may thus always be considered as a general term covering the different states of Phillip. Such an analysis is one of the most constant motives in Peirce's theory of knowledge and one of its main direct consequences is Peirce's admission of real universals or ge-

neral terms: since no percept is entirely determinate, since
no knowledge is entirely determinate, generals must have a
real existence. Such is the conclusion which is drawn by
Peirce in his 1868 articles, which are directed against car-
tesianism, and which are widely developed in his 1871 Review
of Frazer's edition of the works of George Berkeley (8.7-38).

As early as 1867, Peirce attempts to formalize those results.
In Upon Logical Extension and Comprehesion (2.391sq), a the-
ory of information is presented, which tries to accomodate
the relations of terms with a universe of discourse in De
Morgan's sense and within a given state of information. Thus,
if each term is endowed with a breadth and a depth, corres-
ponding respectively to "all the real things of which it is
predicable" (2.407) and to "all the real characters (in con-
tradistinction to mere names) which can be predicated of it"
(2.408), Peirce pays no attention to essential nor to sub-
stantial breadth and depth: indeed, ordinary information
"lies somewhere between two imaginary extremes. These are,
first, the state in which no fact would be known, but only
the meaning of terms (essential information), and second, the
state in which the information would amount to an absolute
intuition of all there is, (substantial information)"
(2.409).

Nevertheless, if the distinction which is thus established
between breadth and depth partly covers the distinction be-
tween extension and comprehension, it cannot be reduced to
it. The analysis which is only limited to terms in 1867 is
later generalized to propositions and arguments (2.407n
1;5.477), in virtue of the fact that Peirce holds such an
identification to be not only possible, but desirable
(2.407n1). As a consequence of this, in $S \prec P$, "one can
indifferently speak of S as subject, antecedent or premiss,
and of P as predicate, consequent or conclusion" (3.175n1).
The theory of informed breadth and depth now satisfies one
requirement of the logic of vagueness: its necessary appli-
cation to all symbols, terms as well as propositions and

arguments.

In so generalizing to arguments one can now associate, on the one hand, interpretant, predicate, consequent, conclusion and depth, and on the other, subject, object, antecedent, premiss, argument and breadth. Breadth will not only be defined by whatever a symbol applies to, but also by whatever implies it (its implicans) whereas depth will be defined not only by whatever is said of the symbol, but by whatever it implies (its implicate or implicates).

As you may have noticed, the category of breadth covers a wide domain ranging from questions of naming, denotation, reference, extension, application to questions of predication, and implication (since whatever is truly predicated of a given sign is in its breadth, but also whatever implies a given sign); the category of depth ranges over questions of meaning, connotation, definition, intension, intention, and also of course of predication and implication, since the predicates and/or implicates of a symbol constitute its depth.

Thus, if a symbol S implies a symbol P, not only is P a part of the depth of S, but S is also part of the breadth of P.

3. THE FORMS OF THE INDETERMINATE AND THE DETERMINATE. VAGUE, GENERAL, INDIVIDUAL

The principles that are proper to Speculative Grammar together with the relations between the indeterminate and the determinate impose a certain treatment of vagueness, according as it appears at the level of breadth or at the level of depth, but also with regard to the general on the one hand, and to the possible forms of the determinate, on the other hand; and finally according to the situation which is involved in the act of assertion.

1) Vague and general in breadth

In so far as they are both figures of the indeterminate, and despite what opposes them, vagueness and general are, from a formal point of view, "on a par" (5.506). Vagueness is "the antithetical analogue of generality" (5.505). Let us have a look at the texts:

> "A sign (under which designation I place every kind of thought, and not alone external signs), that is in any respect objectively indeterminate (i.e. whose object is undetermined by the sign itself) is objectively <u>general</u> in so far as it extends to the interpreter the privilege of carrying its determination further. Example: "Man is mortal." To the question, "What man?" the reply is that the proposition explicitly leaves it to you its assertion to what man or men you will. (cf. 2.357). A sign that is objectively indeterminate in any respect is objectively <u>vague</u> in so far as it reserves further determination to be made in some other conceivable sign, or at least does not appoint the interpreter as its deputy in this office. Example: "A man whom I could mention seems to be a little conceited." The <u>suggestion</u> here is that the man in view is the person addressed; but the utterer does not authorize such an interpretation or <u>any</u> other application of what she says. She can still say, if she likes, that she does <u>not</u> mean the person addressed. Every utterance naturally leaves the right of further exposition in the utterer; and therefore, in so far as a sign is indeterminate, it is vague, unless it is expressly or by a well-understood convention rendered general." (5.447).

Second text where the definitions are to be found:

> "A sign is objectively <u>general</u>, in so far as, leaving its effective interpretation indeterminate, it surrenders to the interpreter the right of completing the

determination for himself. "Man is mortal". "What man?"
"Any man you like". A sign is objectively <u>vague</u> in so
far as, leaving its interpretation more or less inde-
terminate, it reserves for some other possible sign or
experience the function of completing the determination.
"This month", says the almanach-oracle, "a great event
is to happen". "What event?" "Oh, we shall see. The al-
manac does not say that." (5.505)

But Peirce also gives the formal conditions that vagueness
and generality obey respectively:

"Perhaps a more scientific pair of definitions would be
that anything is <u>general</u> in so far as the principle of
excluded middle does not apply to it and is <u>vague</u>, in so
far as the principle of contradiction does not apply to
it. Thus, although it is true that "Any proposition you
please, <u>once you have determined its identity</u>, is either
true or false," yet, <u>so long as it remains indeterminate
and so without identity</u>, it need neither be true that any
proposition you please is true, nor that any proposition
you please is false. So likewise, while it is false that
"a proposition <u>whose identity I have determined</u> is both
true and false," yet, until it is determinate, it may
be true that a proposition is true and that a proposition
is false" (5.448).

Thus, it is the context or the situation of assertion that
gives the rules of the right functioning of vagueness and
generality. But the situation is far from being edenic: the
question is not so much to describe a situation of communi-
cation or of dialogue between speakers that care about one
another than to provide the <u>rules</u> of a game. As Risto Hil-
pinen has pointed out rightly, such analyses have much in
common with the strategy adopted by Hintikka in his Game-
Theoretical semantics[10].

What is indeed at stake here, is not any kind of communi-

cation: it is the communication of truth. Now, for Peirce,
truth goes hand in hand with the adoption of beliefs:
therefore, the speaker must, one way or another have his
belief adopted by his listener. For the aim of communication
is nothing, but "an endeavour to make the person addressed
(i.e. the interpreter) think in a certain way," that is be-
lieve something (Ms 284):

> "The assertion consists in furnishing of evidence by the
> speaker to the listener that the speaker believes some-
> thing, that is, finds a certain idea to be definitively
> compulsory on a certain occasion" (Ms 787; 2.335).

No wonder then if the assertion situation finds its expres-
sion in descriptions which have more in common with conflict
than dialogue. The speaker who asserts a proposition accepts
to be held responsible for it, and subjects himself to pos-
sible penalties, in case the proposition turned out to be
false:

> To assert a proposition means to accept responsbility
> for it, so that if it turns out ill, or as Mr. Schiller
> says (by implication) unsatisfactory, in a certain way
> which we need not define, but which is called proving to
> be false, he who asserted it regrets having done so,"(Ms
> 280).

The speaker is a defender of his own position; as for the li-
stener, it is his interest to try and detect a possible fal-
sehood committed by the speaker, since "the affirmation of a
proposition may determine a judgement to the same effect in
the mind of the interpreter to his cost" (Ms 517; NEM 4,
p.249). Hence the utterer and the interpreter have opposite
interests and attitudes with regard to the truth of any pro-
position asserted by the former '1. So difficult it is, as
everybody knows to give up one's beliefs:

> "The utterer is essentially a defender of his own posi-

tion and whishes to interpret it so that it will be defensible. The interpreter, not being so interested, and being unable to interpret it fully without considering to what extreme it may reach, is <u>relatively</u> in a hostile attitude, and looks for the interpretation least defensible" (Ms 9, pp.3-4).

This is why Peirce occasionally calls the interpreter of a proposition its "oppponent" (e.g,. in Ms 515).

Thus, the language-game occurring between the speaker and the interpreter with respect to an indeterminate proposition is very close to what Hintikka calls a zero-sum game[12].

Indeed, all communication implies the mutual respect of a certain number of tacit assumptions, some agreement upon the aim of communication on both parts. This is why honest people when not joking, intend to make the meaning of their words determinate, and want to avoid any "latitude of interpretation" (5.447). Which means that "the character of their meaning consists in the implications and non-implications of their words; and they intend to fix what is implied and what is not implied" (5.447).

Nevertheless, the necessarily asymetrical situation which prevails between both speakers does not make the elimination of indetermination in all cases desirable. The speaker may have some interest in remaining in a certain fuzziness.

Indeed, vagueness, is on the side of the speaker; it is not linked with the very nature of the matter. It is due to the deliberate intention of the speaker:

"The phrase "a <u>certain</u> man" means that the determination which is left <u>uncertain</u> to the reader or auditor is nevertheless, or once was, <u>certain</u> either to the utterer or to some other person" (5.505, n.1, cf. 3.94).

If, however, the speaker wants to convince, or to communicate an information, it is up to him to qualify vagueness. To qualify it does not mean: to suppress it. The speaker can use also indexical signs which would be relatively indeterminate, or indefinite, such that "the sign is not sufficiently expressing itself to allow of an indubitable determinate interpretation" (5.448, n.1).

The removal of vagueness does not mean, then, the suppression of every indetermination. Nonetheless communication requires that indetermination is not complete (otherwise it would be meaningless).

What kind of means are at our disposal if we want to communicate some information after all, and even in certain cases to "eliminate every attempt at interpretation?" Peirce here has several ways of answering this question.

1) First, vagueness can be removed if the speaker accepts to extend "to the interpreter the privilege of carrying its determination further" (5.447). The assertion which was vague becomes general. The fact that a sign ceases to be vague by becoming general does not mean that we push the problem further, first, because "no sign can be at once vague and general in the same respect, since insofar as the right of determination is not distinctly extended to the interpreter it remains the right of the interpreter." (5.506) second, because generality escapes an absolute indetermination, in so far as "a sign which should make its interpreter its deputy to determine its signification at his pleasure would signify anything, unless nothing be its significate." (5.448, n.1)

Finally, and most importantly, the general does not introduce any absolute indetermination: for in that case we would have to accept that generality can effect the depth or the meaning of a term. Strictly speaking, this is impossible. A general term is in fact the product of an hypostatic abstraction, the operation by which we introduce abstract entities the iden-

tification of which is done through the relationship which they have with something which is familiar to us. For instance, if we conceive heat as something which explains the occurence of certain phenomena such as our sensations of warmth, or expansion, of contraction etc., we shall say that heat is a concept which is formed through hypostatic abstraction. The same is true of the famous "dormitive virtue" invoked as an explanation of the tendency that some people have of sleeping after having taken opium: what was meant by this hypostatic abstraction, says Peirce, was simply that there is "some particularity in the opium to which the sleep must be due" (5.534). A concept which has been introduced thus by hypostatic abstraction can have a possible application only if it actually corresponds to something true that we have thus assumed. The idea of dormitive virtue of opium has no application if there is nothing in opium which accounts for the state of sleepiness of the opium smokers[13].

However, if what we have thus assumed is true, then the idea which is introduced by hypostatic abstraction must apply to something. And everything to which it can be applied will be of the same nature; for it is part of what is assumed. If, consequently, there is dormitive virtue in opium, it will be fundamentally the same in every instance of opium, although the concept formed by hypostatic abstraction does not specify the nature of its presumed object.

This is why the speakers leaves it to the interpreter to complete the determination of the implication, as when he says: "This being is filthy, in every sense of that term", there is no reason to fear that indeterminacy be lost in every sense: in fact, the interpreter puts under the term "filthy" the general and customary properties which are conventionally attached to this term. His freedom will not be a licence. In Peircian terms, generality always bears on breadth, and never on depth.

We need here to stress the importance of the collateral in-

formation which helps the system of conventions to facilitate interpretation: the hearer will always be able to interpret correctly a general sign in breadth, because he is familiar with the signs, or because he remembers them and knows the names of the individuals which populate the universe of discourse[14].

2) Once it is admitted that we have avoided the risk of an absolute indetermination by transforming vagueness into generality, how can one be sure, if one wants to satisfy the requirements of communication, that generality actually involves a reference to an object or to reality?

a) Here the roles of indexes is outstanding: Peirce says explicitly that "every proposition actually asserted must refer to a non-general subject." Now there is no individual which is absolutely determinate. Even if we pass from the *individuum vagum* to the *individuum signatum*, Philip remains always devisible into Philip drunk and Philip sober. How can we solve this difficulty?

It can be solved in two ways, either by adopting a more or less determinate index (that is by a proper name or a _singular_) or by contenting oneself of an indeterminate or indefinite index, that is an indeterminate indexical symbol which represents more than one singular object (Ms 283). Common nouns perform, most of the time, this function. Whereas a proper name "denotes a unique individual the existence of which is well known both by the speaker and by the interpreter" (Ms 517, NEM, IV, p.243), a common name, although it functions as the subject of the proposition (and therefore as an index), remains indefinite:

> The function of a common noun is the same as that of a proper name. That is, it merely draws attention to an object and so puts its interpreter into condition to learn whatever there may be to be learned from such attention. Now attention can only be drawn to what is al-

ready in experience. A proper name can only function as such if the utterer and interpreter are already more or less familiar with the object it names. But the peculiarity of a common noun is that it undertakes to draw attention to an object with which the interpreter may have no acquaintance. For this purpose it calls up to his mind such an image as a verb calls up, appeals to his memory that he has seen different objects (as) the subjects of that image... and then of those which might be so recollected or imagined, the noun indefinitely names one (Ms 516).

In what sense does an indeterminate index (which Peirce sometimes calls a "precept") direct a speaker and an interpreter towards the proper way of finding a singular object or the index of a singular object, so that this object can function as the subject of an assertion?

Peirce's analysis here can be compared to Hintikka's interpretation of quantifiers in game-theoretical semantics[15].

A determinate index is indefinite only if the speaker is free to choose (through the selectors and the quantifiers) the object which the index is supposed to represent; that is if he is free to give his interpretation of the assertion. An existential quantifier signals the choice of the speaker in the language game[16]. A definite proposition can be considered as the choice made by the speaker of leaving no latitude of interpretation:

A definite proposition is the one the assertor (i.e. the utterer) of which leaves himself no loop-hole for escape against attack by saying that he did not mean so and so, but something else. (Ms 515, p.25).

The utterer, however, can do even more: He can go so far as letting "his opponent choose" which singular object is going to refute the proposition, like in "Any man you please is

mortal" (Ms 515).

In other terms, when the index of an assertion is a general index, the interpreter can choose the singular object which will be instantiated by it. The universal quantifier thus transfers the choice of the singular to the interpreter. Peirce says

> It seems an odd thing, when one comes to ponder it, that there should be such a mode of signification as the latter (i.e. generality), in which the utterer of a sign transfers to its interpreter the office of determining what that sign is to be apprehended as meaning. Its familiarity blinds the wonder of it. (Ms 283).

Since the choice of the object cannot be done simultaneously by the two interlocutors, every indeterminate index must be either individual (non general) or definite (non indefinite) (Ms 9, § 2); on the other hand, "whichever of the two makes his choice of the object he is to choose, after the other has made his choice, is supposed to know what that choice was. This is an advantage to the defense or attack, as the case may be." (Ms 9,3).

b) The context of an assertion is the second means invoked by Peirce to explain how an object of experience can be identified, besides its being singularized. It is well known that "every sign has a unique object, even if this unique object can be a unique set or a unique continuum of objects. No general description can identify an object." How can this problem be solved? Here context, or if one prefers, the pragmatic dimension of Peirce's theory of reference is all-important:

> The common sense of the interpreter of the sign will assure him that the object must be one of a limited collection of objects. Suppose, for example, two Englishmen to meet in a continental railway carriage. The

total number of subjects of which there is any appreciable probability that one will speak to the other perhaps does not exceed a million, and each will have perhaps half that million not far below the surface of consciousness, so that each unit is ready to suggest itself. If one mentions Charles the Second, the other need not consider what Charles the Second is meant. It is no doubt the English Charles the Second. Charles the Second of England was a quite different man on different days; and it might be said that without further specification the subject is not identified. But the two Englishmen have no purpose of splitting hairs in their talk; and the latitude of interpretation which constitutes the indeterminacy of a sign must be understood as a latitude which might affect the achievement of a purpose. For two signs whose meanings are for all pos-sible purposes equivalent are absolutely equivalent. This, to be sure, is rank pragmaticism; for a purpose is an affection of action. (5.448, n.1)."

This is rank pragmatism indeed if we bring this together with the famous Peircian maxim:

"Consider what effects that might conceivably have practical bearings, we conceive the object of our conception to have. Then our conception of these effects is the whole of our conception of the object."

What does this mean? That the meaning of a concept does not consist in the abstract image or in the mental image to which it corresponds, but rather in the use, the method or the technique of action in what can be expected from it, or its aim. We have here much more than an introduction of pragmatic elements in the elucidation of meaning: in fact the whole pragmatist theory of meaning is at stake, and together with it, the idea that any definition must go through the description and the exhibition of the rules, the conventions and the uses which the term implies.

2) Vagueness in depth

There are principally two headings under which Peirce deals with the kind of vagueness which has to do with the category of depth, and in each case he intends to claim, on the one hand that this kind of vagueness is irreducible, and on the other hand that we have no reason here to believe that such a vagueness can be an obstacle on the way to knowledge and truth.

1) The first form of vagueness comes from the real indeterminacy which is constitutively tied to our beliefs and habits. It is illustrated by the definition given in he Baldwin Dictionary:

> Vague (in logic). Indeterminate in intention. A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. By intrisically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker's habits of language were indeterminate; so that one day he would regard the proposition as excluding, another as admitting, those states of things. Yet this must be understood to have reference to what might be deduced from a perfect knowledge of his state of mind; for it is precisely because these questions never did, or did not frequently, present themselves, that his habit remained indeterminate. (Vol.II. p.748).

In this sense, vagueness can be removed, because it is entrenched in the beliefs that we entertain on the world, our common sense, almost instinctive beliefs, which, for this very reason are undubitable:

The question whether a certain newly found skeleton was the skeleton of a man rather than an anthropoid ape, the reply "Yes and no" might, in a certain sense, be justifiable. Namely, owing to our conception of what a man is having been formed without thinking of the possibility of such a creature as to which this skeleton belongs, the question really has no definite meaning. (Ms 596, p.16).

Had I said his hair was red, that would not be quite "precise", but a little "vague", since there are shades of hair between sandy and red which I might one day call red while on another day I might say "No, that is reddish, but not red" (Ms 48, p.8 n.d).

There are here two sources of vagueness. First the vagueness which is tied to the way we usually refer to colours, that is through reference to colour charts or to paradigmatic case (we do not learn any definition af colour), but through an habit or an usage contracted through those apprenticeships and which function as a rule or as a definition, and which, consequently do not allow us to give an invariant answer to questions about colours. Second the vagueness which is attached to the impossibility of a strict definition of the domains of application of the concept.

2) For Peirce, this second form of vagueness is directly tied to ontology and to the hypothesis of the existence of the continuum. For if colour is a real continuum, then the possibility of borderline cases is ineliminable. Now this hypothesis is the one towards which Peirce constantly moves[17]. If the colours do form a continuum, for instance in the sense of Kant's principle of affinity or continuity, which prescribes "that we move from one species to another through a constant increasing of diversity," then the Kantian-Peircian principle according to which there is a potentially infinite number of intermediaries is satisfied. It seems therefore that one can-

not reach a vocabulary of colours which would be so precisely
defined that it would rend border-line cases impossible:

> Suppose the chat of our Englishmen had fallen upon the
> colour of Charles II's hair. Now that colors are seen
> quite differently by different retinas is known. That
> the chromatic sense is much more varied than it is po-
> sitively known to be is quite likely. It is very unli-
> kely that either of the travelers is trained to observe
> colors or is a master of their nomenclature. But if one
> says that Charles II had dark auburn hair, the other
> will understand him quite precisely enough for all their
> possible purposes, and it will be a determinate predica-
> tion. (5.448, n.1).

Vagueness is here tied to the fact that reality presents it-
self to us as a continuum: this is what explains, according
to Peirce, the sorites paradoxes raised by border-line cases,
limit cases, or fuzzy sets:

> Indefiniteness...consists in a sign leaving it doubt-
> ful...what its intended interpretation was...as to a
> great multitude or even a continuum of possible inter-
> pretations no two of which differ without the doubt ex-
> tending to intermediate interpretations (Ms 283, rejec-
> ted page 139, cf. 4.171-2)

All this confirms, according to Peirce, that vagueness is a
universal real principle, and not "a defect of our knowledge
or of our thought" (4.344). As Peirce notes:

> Whereever degree of or any other possibility of conti-
> nuous variation subsists, absolute precision is impos-
> sible. (5.506).

Not only it would be vain to think of removing it, but one
must, as far as possible, take it into accout in our know-
ledge. To assert that "vagueness is no more to be done away

with in the world of logic than friction in mechanics"
(4.513) is the same thing as to assert that our ideal of
knowledge is inseparable from the examination and elucidation
of the methods and principles of logic, of science, and of
ontology.

> Get rid, thoughtful Reader, of the Ockhamistic prejudice
> of political partizenship that in thought, in being, and
> in development the indefinite is due to a degeneration
> from a primal state of perfect definiteness. The truth
> is rather on the side of the scholastic realists that
> the unsettled is the primal state, and that definiteness
> and determinateness, the two poles of settledness, are,
> in the large, approximations, developmentally, episte-
> mologically, and metaphysically (6.348;4.344).

4. THE MEANING AND SCOPE OF THE PEIRCIAN PROJECT OF A LOGIC OF VAGUENESS

It is no doubt difficult to judge correctly the project of a
logic of vagueness, not only because of its scope, but also
because of its originality. Must we nevertheless consider
that it is enough to relegate it among the curiosities? This
is, to a certain extent. Susan Haack's conclusion at the be-
ginning of the chapter that she devotes to vagueness in her
book Deviant Logic. The Peircian conception of vagueness is
so "excentric", in particular because Peirce is almost alone
in thinking about vagueness by comparing and contrasting it
with generality, because his conception is so remote from the
usual way of posing the problem of vagueness that it contri-
butes more to the obscurity of the subject than it sheds
lights on it. Now it seems that on this point "a definiton of
vagueness which would not be too remote from ordinary and
predominant usage is desirable."[18]

It is possible to avoid this difficulty by saying that
Peirce's position must be envisaged in its specificity, and

that we cannot earn much by looking for answers or solutions to problems which a writer did not really have. This position, however, is not satisfactory, at least when we consider this project, for although it can appear as baroque or disconcerting, Peirce would have liked to see it discussed and submitted to the verdict of the scientific community. One must therefore, if one wants to be faithfull to Peirce's ethics of research, ask what can be the credentials of this project, and pose at least the following questions: does his analysis shed any light about the problems of a semantics of vagueness, for instance the distinction between vagueness, ambiguity, generality, imprecision, etc.? Does a reflexion on objective vagueness allow us to make any progress on the problems posed by the sorites paradoxes? And what is in the end Peirce's position about classical logic? Does be believe that it is necessary to do away with it or not?

1) A first thing for which apparently, Peirce should be praised, is to have seen where vagueness should be localised and in respect to what it should be thought about: vagueness is objective. Thus vagueness affects depth (the indefinite in depth) or meaning, but to a certain extent also, it affects breadth, reference. More exactly, as far as this second aspect is concerned, even if it be eliminated, either because the speaker introduces indices, or because he transfers his right of determining to the interpreter, so that vagueness then becomes generality, in both cases, a certain form of irreducible indeterminacy remains. First because indices are never totally determined, second because generality is indeed a new form of the indeterminate. Therefore, in either case, the irreducibility of the third category - which is Peirce's main thesis - shows the strength of its reality.

In so doing, Peirce draws a first important distinction, a distinction which may seem obvious and natural today, especially since Max Black gave some explanations about it in his famous 1937 article[19], but such authors as Russell or Waismann do not always seem to draw it: I mean the distinction

between vagueness and ambiguity. Peirce clearly indicates that such irreducible vagueness, which concerns meaning as well as reference must not be interpreted as some form or cause of ambiguity:

> "...A term which is indefinite between distinct alternative interpretations has a sort of indeterminacy. it is equivocal or ambiguous. But it is very different from genuine indefiniteness in which there is an indistinguishable, or indistingushed mass of cases. The former is the indefiniteness of the expression "bright color" (disregading the indefiniteness in degree); the latter is that of saying of a history that it is written in a somewhat French style... Indefiniteness...consists in the sign's leaving it doubtful just what its intended interpretion was, not between two or more separate interpretations, which would be ambiguity, but would not be indefiniteness..." (Ms 283).

Therefore ambiguity arises from the equivocacy that is made possible through its lack of index or object, or as Mark Sainsbury says, it comes from the fact that one single sentence may be used to convey or ask more then one thing. Before communication proceeds, one must therefore determine what the thing is that is being conveyed or asked[20].

On the opposite, one may wonder whether Peirce draws a clear enough distinction between vagueness and relativity. Contrary to vagueness, relativity can very well cope with precise expressions. Take for example the phase "taller than the average". If, as Sainsbury points out, it is not difficult to assign numbers to individuals, in order to measure their heigth, this has nothing to do with vague properties. A person is taller than the average on the condition that the number used to measure her size be larger than the number used to measure the average size; and this is a completely precise condition. If many vague predicates are also relative (such as the predicate "somewhat French", which Peirce alludes to,

and which may be regarded as giving more precise conditions of application, to a certain extent, by numbering a certain amount of characteristics of the French style), their relativity must not be confused with their vagueness[21].

At all events, as far as ambiguity is concerned, not only can it be eliminated, but is _must_ be so. Peirce's attitude towards ordinary language is, in that respect, highly significant. One of the obvious sources of ambiguity comes indeed from language; but not only because it has objective vagueness. Peirce's position is very subtle here. On the one hand, he adopts a position which in some ways might be labeled "wittgensteinian," according to which he is inclined to saying that ordinary language is perfect as it stands, and that there is no need to substitute some ideal language to it. More exactly, it is necesary that philosophy should adopt in its language, a corpus of words with vague meanings, with which it may identify those vague ideas of ordinary life which it is its duty to analyze; and to that end, the vernacular expressions, although they do not lend themselves to this in a perfect way, will non the less be the best (Ms 280, c.1905). Indeed,

> No words are so well understood as vernacular words, in one way; yet they are invariably vague. (6.494).

In that area, Peirce has never gone so far as Frege, who thought, as Crispin Wright has pointed out, that "a language containing vague predicates was essentially defective" and that it was "philosohically intolerable that predicates should occur for which it was not always determinate whether or not they could truly be ascribed to an object."[22]

But Peirce is also very critical about the illusion consisting in believing that one should or could put an end to the vagueness of ordinary language through precision. To say that I have 123000 blond hair is undoubtedly precise and remains none the less vague. Hence, it is a bit useless to be over-

precise (1.112;8.244).

> It is easy to speak with precision upon a general theme.
> Only, one must commonly surrender all ambition to be
> certain. It is equally easy to be certain. One has only
> to be sufficiently vague (4.237)[23].

On the other hand, Peirce also says very clearly that "the
case of philosophy is very peculiar in that it has positive
need of popular words in popular senses - not as its own
language (as it has usually used those words), but as objects
of its study. It thus has a peculiar need of a language di-
stinct and detached from common speech, such a language as
Aristotle, the scholastics and Kant endeavored to supply,
while Hegel endeavored to destroy it" (2.223; cf. 8.169).

In keeping with this, one may understand why he refused any
compromise between logic and ordinary language or grammar,
why he insisted upon the necessity of some strict ethics of
terminology, of creating, when necessary, neologisms, why fi-
nally, he cared so much about scolastic distinctions.

Hence, ordinary language is perfect as it stands: moreover,
it can help the phenomenologist-philosopher to identify some
indeterminate characters of reality. But for Peirce, obvious-
ly there are levels in perfection. Indeed, the objective va-
gueness of language has not to be condemned. However, it is
useless to add to it other sources of ambiguity than those it
is already endowed with. In that respect, Peirce cannot be re-
garded as an ordinary-language philosopher. He remains, first
and foremost, a logician. But a logician, who by tradition
would refuse the Frege-Russellian syntactical approach to
logic, according to which a language approaches fulfilment of
its function only in proportion as it approaches logical per-
fection, i.e. in inverse proportion to its vagueness.
Peirce's semiotical and semantical approach to logic[24] would
rather incline him to favor the idea, developed under several
forms in formal semantics, of the possibility of a precise

semantical description of a given natural language, namely the framing of theoretical models - the semiotical classifications are a first attempt in that direction -, for trying to account for, at least, the obvious regularity which is manifested by language. In other words, as far as Peirce's attitude towards ordinary language is concerned, it could be summarized in the following way: "even if "bald", say, is imprecise, this does not require any inexactitude in an <u>account</u> of its sense[26]."

2) A second important distinction which is brought forth by Peirce's analogies has to do with vagueness on the one hand, and generality on the other hand. Again, such a distinction may now seem obvious. The finite domain or field of application of a word is its generality, whereas the vagueness of that word is indicated by the lack of specification of its boundaries[26]. However it is not certain that this is always perfectly mastered in the discussions. Thus Russell may be found writing that "a vague word is not to be identified with a general word[27]," proceeding to say that nevertheless "in practice the distinction is apt to be blurred," and blurring it himself in declaring that "a memory is vague when it is appropriate to <u>many</u> occurrences"[28] and that "the fact that meaning is a one-many relation is the precise statement of the fact that all language is more or less vague."[29]

In any case, it is not sure that all the authors are always careful enough indistiguishing possible types of indeterminacy.

As far as he is concerned, Peirce thinks that he has good reasons to be careful, and not only for reasons which have to do, as Susan Haack believes, with a special taste for logical symmetries[30].

3) Most often, philosophers or logicians analyze the problem of vagueness from the standpoint of a certain ideal of knowledge or formalization, in virtue of which what one should

tend to reach is something <u>precise</u>. As Frege put it, the law of excluded middle is really nothing, but another form of the requirement that concepts should have a sharp boundary[31]. The various processes of indexicalisation used by Peirce show in what respect such an idea was not utterly foreign to him. But at the same time, he thought that there is some illusion in believing that one can eliminate indeterminacy simply by replacing it with determinacy, precision or individuality. By stressing that a concept ceases to be semantically vague in becoming general, Peirce points out that the other character of indeterminacy, namely the reality of the third category is both an irreducible and decisive element of our knowledge.

In defining generality as the antithetical analogue of vagueness, Peirce has proposed the following insight: indeterminacy must not be, as such, considered as an obstacle to communication, knowledge or truth. On the contrary, it is an essential component of it. Generality is the indefinite series of interpretants (1.339), the idea that knowledge is for ever open. Of course, in so far as it has to do with potentiality, quality, in short, Firstness, generality is indeed of a "negative sort" (1.427); but in so far as it belongs to the conditional necessity of law, to thirdness, it is of that "positive sort" which "with infiniteness, continuity, growth and intelligence" (1.340), are part of those ideas of third category to which philosophy and the sciences should pay most attention. Generalisation, "the most important operation of the mind in mathematics as everywhere else" (1.82) is indeed first and foremost an operation of "specification" (Ms 283).

It means the discovery by reflection upon a number of cases of a general description applicable to them all. Thus, generality is as such the very manifestation of intelligence; since it is less the extension of an idea one already had than a progress, in the definiteness of conceptions that one applies to known things (2.422). To eliminate generality would thus amount to dreaming that one can reduce the meaning or the intelligence which is in things, which is illusory

(1.344-5).

Consequently, to define, to specify is not necessarily to precise; in other words, to suppress any kind of indeterminacy; it is exactly the opposite. There is food for meditation here about what Peirce has to say on the temptation of precision or overprecision. If a term is completely precise, completely specified, it does not enable to raise interesting questions any longer, i.e. to make information progress. If it is too precise, either it runs the risk of raising pointless questions (how interesting would it be, for ex. to know, concerning such statements as "Peirce wrote that paper", what was the colour of the ink he used or the name of the father of the man who made the ink, or the movement of the planets at the time of his birth, etc.? (5.448n1)), or it runs the risk of inducing us into error. As the French historian and philosopher of science Pierre Duhem wrote, one can be sure of the truth of "John is tall", but less of the truth of "John is one meter ninety centimeters and three millimeters high." In that respect, the vagueness which the speaker can, if he wishes, choose to give to his statements is not necessarily on his part a proof of machiavelism or unfairness: it may be the only way he has sometimes to convey the right information. In other words, "between precision and certitude, there is a kind of compensation; the one cannot grow, but in opposition to the other."[32]

Therefore, Peirce's attitude towards indeterminacy is not only a position of comfort - which is after all not so bad, when one is so willing (as he is) to respect the principles of an economy of research -. It is also related to an ethical or simply pragmatistic attitude, namely to the idea that the meaning of a concept lies in the future, in its conceivable effects, i..e. in the idea of some reality of the possible.

Finally, it is clear that his caring so much more for the reality of indeterminacy than for precision is linked with the fact that his antilogicism naturally carries him towards

regarding logic less in view of its possible utility in mathematics than in view of what it can bring at the level of method and inquiry in the natural sciences. Now, as one knows, imprecision, asymptotic values, errors of measurement must be taken into account in physical sciences to such an extent that a fallibilistic epistemology, capable of capturing such elements of uncertainty will be estimated by Peirce as the best epistemology to handle such problems.

4) Another important insight in Peirce's analysis of vagueness bears upon the question whether one should consider vagueness as something purely subjective or due to some defect in our knowledge.

According to him, vagueness is objective. But by centering the problem of the objectivity of vagueness on interpretation and the realiy of the third category, Peirce has underlined that vagueness is less a matter of semantics or as Kit Fine would put it of "deficiency in meaning"[33] than it is irreducibly linked with pragmatical considerations: first because all interpretation takes place in a context of communication: in that sense, it is finally not important that certain predicates should suffer from logico-semantical indeterminacy, so long as the context is there to help us in specifying the context of assertion[34]. But also because assertion involves beliefs, habits, which manifest themselves in our use of signs (and to that extent in our conceptual as well as linguistic use). This point is interesting because it means that one should not perhaps confine the discussions on vagueness to a linguistic investigation, i.e. to a theory of the production and determination of meaning, nor to the question whether some logics should be created which would enable one to account for the objective vagueness of ordinary language. Perhaps one should also try to analyze the mode of production and the role played by our beliefs in the formation of our concepts and in their uses.

5) But Peirce not only intended to show that there is objective vagueness, i.e. states of affairs whose application it is objectively impossible to determine. He also intended to hold that vagueness is real because reality itself is vague. His project of a logic of vagueness is thus inseparable from his ontological account of vagueness. Now, this may help to have a better grasp at what Peirce means and what his position is towards classical logic. Indeed, Peirce made several attempts in direction of a multivalued logic, admitting the indeterminacy value (as a limit)[36]. Turquette has shown how the formal developments conducted by Peirce had anticipated some discoveries by Lukasiewicz and Post. Now the correspondence from that time indicates what was the context in which such inquiries were made, and show that they were parallel to the investigations into modalities[36]. Thus, while analyzing the triad "Potentiality", "Actuality", and "Necessity", Peirce defines them in the following way:

> Potentiality is the absence of Determination (in the usual broad sense) not of a mere negative kind, but a positive capacity to be Yes and to be Nay; not ignorance, but a state of being... Actuality is the Act which determines the merely possible... Necessitation is the support of Actuality by reason...(Ms 277,1908).

In another text, written somewhat later (1910), "The Art of Reasoning elucidated", one can see even better how Peirce viewed the link between his inquiries:

> ...A simply assertory Proposition differs just half as much as the assertion of a Possibility, or that of a Necessity, as these two differ from each other. For as we have seen above, that which characterizes and defines an assertion of Possibility is its emancipation from the Principle of Contradiction, while it remains subject to the Principle of Excluded Third; while that which characterizes and defines an assertion of Necessity is that it remains subject to the Principle of Contradiction,

but throws off the yoke of the Principle of Excluded Third; and what characterizes and defines an assertion of Actuality or simple existence, is that it acknowledges allegiance to both formulae, and is just midway between the two rational "Modals" as the modified forms are called by all the old logicians.

Does this mean a denial of classical logic? A negative answer is the only acceptable one.

In a letter to William James, dated February 26, 1906, Peirce clearly states his position:

> I have long felt that it is a serious defect in existing logic that it takes no heed of the limit between two realms. I do not say that the Principle of Excluded Middle is downright false; but I do say that in every field of thought whatsoever there is an intermediate ground between positive assertion and positive negation which is just as Real as they. Mathematicians always recognize this, and seek for the limit as the presumable lair of powerful concepts, while metaphysicians and old fashioned logicians, the sheep and goat separators- never recognize this. The recognition does not involve any denial of existing logic, but it involves a great addition to it.

Such a text finally throws light upon the deep meaning of Peirce's project of a logic of vagueness.

The logic of vagueness is, but the continuing of the project of constituting a semiotics; now, not only is logic part of it (logic as critic of arguments), but it is saturated through and through with semiotics. So to dream of some possible exclusion of formal logic, in its classical sense, from it would be perfectly contradictory with such a semiotical definition of logic itself.

This is why Peirce's works on triadic logic or on the formal theory of assertion, on modal logic, on doxastic logic, must be viewed as <u>additions</u> to the existing logic, but not as its denial.

Wherever possible, one has to suppress the indefiniteness so as to reach the definite, namely symbols which obey bivalence, excluded middle and non contradiction. Far from him the idea, for example, that it would suffice to develop a three valued logic to have a better grasp of the vagueness surrounding ordinary language. Even less would he find it valuable - when one considers his ethical views on logic - to try to use such logics in order to account for meaningless sentences[37]. Finally, one may wonder whether Peirce would not have agreed with Russell's views, developed in his classical article on vagueness, but without accepting Russell's consequences: what vagueness shows is that logic is inapplicable to ordinary language, which does not mean that it is false, but simply inappropriate[38]. Right or wrong, Peirce thinks that, without abandoning the minimal requirements of classical logic, but through some necessary "addings", and especially, through getting semiotical and ontological categories into formal logic, one can also account for a number of properties of natural language, not because of its having particular categories, but because of its being also part of the real: now Peirce is convinced that "the logical principles are not only regulatively valid, but as truths of being" (1.487; cf. 7.480, 8.113).

Finally, one may wonder whether Peirce's attitude is not quite close, ontological committment put aside, to the attitude Kit Fine recommends lately toward formal logic. According to Fine, the question of adopting non classical logic or not, to account for vague predicates is useless. One needs just to decide conventionally to treat vagueness while sticking to the strict framework of classical logic, that is to say, by choosing for instance to specify the conditions allowing to speak of borderline-cases[39]. Let us suppose for a

while that one should acopt such a view: then it is easy to reason in the following way: it may be the case that our powers of perceptual discrimination be limited, and that we cannot decide whether such and such object belongs to the category "red", or not. Never mind. Our inability to learn does not concern our capacity to signify. Never mind the objectivity of vagueness: it is always possible to make a predicate precise. But as Fine points out, it always remains possible that the whole of the admissible specifications should itself be vague, i.e. that there be a narrow link between the vagueness of language and reality.

If that were the case, then what language would mean would be an intrinsically vague fact[40]. Now, such is precisely Peirce's position, who adopts here a realistic standpoint. If the logical categories are the mirror of the categories of the real, then our logic has to take it into account. However, this does not imply, that some of its principles and rules should be abandoned. As Peirce writes:

> It may be that the world does not conform precisely to the rigidity of our idea of something sufficiently stable to be represented by a sign. The reader knows several cases of insolubilia of that kind, as the logicians call them, namely cases in which any attempt to reason leads to an absurdity (4.78).

Even if "all the propositions in the world should turn out to be so elusive", it would not "affect the truth of the Nota Notae, which only amounts to say that to the extent that things conform to our idea of successive inclusion, to that extent, (unless we have committed almost unconceivable blunders), the Nota Notae remains valid" (4.78).

This is why, while opposing logic, as a science of facts to mathematics as the domain par excellence of abstractions, Peirce can also write that:

The logician asserts nothing, contrary to the geometer; but there are certain assumed truths which he hopes, which he trusts, on which he relies, in a way which is totally different from the mathematician. Logic teaches us to expect some residue of dreaminess in the world and even contradictions, but we do not expect to be confronted to such phenomenon, or at any rate, we are forced to run the risk of it. The assumptions of logic differ from those of geometry, not only by the fact that they are not held in an assertoric way, but also because they are much less defined (4.79).

Of course, one may be tempted to conclude, with Michael Dummett that "the notion acording to which things could really be vague, together with being vaguely described is not properly intelligible[41]."

But such is precisely Peirce's challenge: to try not only to think the intelligibility of vagueness through and through, but also to insert that logic into a much more ambitious project of his: the constitution of a coherent realism of indeterminacy[42].

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References

All references are taken from <u>The Collected Papers of C.S.Peirce</u>, Harstshorne, Weiss and Burks, eds. Harvard 1931-1958, 8 vols. and from the Manuscripts annotated by R.Robin, Amherst, Massachussetts, 1967.

1. (5.505; cf. 5.446. "Logicians have been at fault in giving Vagueness the go-by, so far as not even to analyze it. The present writer has done his best to work out the Stecheiology (or Stoicheiology), Critic, and Methodeutic of the subject...").

2. I have developed such themes in "Logique, psychologie et métaphysique: les fondements du pragmatisme selon C.S.Peirce", in <u>Zeitschrift für allgemeine Wissenschaftstheorie</u>, 1985, pp. 229-250).

3. cf. Jarrett Brock, "Principal themes in Peirce's Logic of Vagueness", <u>Peirce Studies</u>, Lubbock, Texas, 1979, pp. 41-50. For a fuller development of Peirce's Logic of vagueness, one should consult J.Brock's pioneering and excellent (yet unpublished!) doctoral dissertation: <u>C.S.Peirce's Logic of Vagueness</u>, Urbana, Illinois, 1969, to which this paper is much indebted. See also Mihai Nadin: "Peirce's Logic of Vagueness and the category of synechism", in <u>The Monist</u>, juil. 1980, vol.63,n'3, pp.351-363.

4. "I extend logic to embrace all the necessary principles of semiotic, and I recognize a logic of icons, and a logic of indices, as well as a logic of symbols" 4.9;1905).

5. "A symbol is a sign, which is constituted a sign merely or mainly by the fact that it is used and understood as such, whether the habit is natural or conventional, and without

regard to the motives which originally governed its selection" (2.307, 1901;cf. 4.531)

6. I shall only consider one of the three branches of the logic of vagueness which Peirce claims he has worked through, namely Speculative Grammar (a term borrowed from the Pseudo-Scot), because it is the first one, in order, and "each division depend on the one which precedes it" (1.191). But obviously, any complete account should have to consider the two other branches of the logic of vagueness, namely, critical logic, i.e. "the theory of the general conditions of the reference of symbols to their assigned objects" (2.93), which classes reasonings and "determines the validity and degree of strength of any species" (1.191), as well as speculative methodecutic or rhetoric, which is "the general theory of the proper way to conduct inquiry" (2.106), or again the "method of the discovery of methods" (2.108).

7. "Any symbol involves an assertion, at least rudimentary" (2.34; cf. C.Chauviré, "Peirce, le langage et l'action", Les Etudes Philosophiques, 1979, n'1, pp. 3-17 et J. Brock "An Introduction to Peirce's Theory of Speech-Acts", Transactions of the C.S.Peirce Society, 1981, vol.17, pp. 319-326.

8. J. Brock, "Peirce's Conception of Semiotic", Semiotica, 14:2, 1975, pp. 124-141, p.129.

9. Brock. "Principal themes in Peirce's LOV", art. cit. p.43.

10. Risto Hilpinen "On Peirce's Theory of the Proposition: Peirce as a precursor of Game-Theoretical Semantics", in The Monist, 65, 1982, pp. 182-188. On the same topic, see J.Brock, "Peirce's anticipation of Game-theoretical Logic and semantics", in M.Herzfeld and M.Lehnart edits. Semiotics 1980, Plenum Press, New York and London, 1980, pp. 55-64. For a general exposition of Hintikka's semantics, see "Quantifiers in Logic and Quantifiers in Natural Language", in Saarinen ed. Game-Theoretical Semantics, Reidel, Dordrecht,

1979, pp. 27-47 and other articles by Hintikka in the same volume.

11. Hilpinen, art.cit. p.185.

12. Ibid. p.185. On that topic, see Jaakko Hintikka, "Quantifiers vs. Quantification Theory", in Saarinen, op. cit. pp. 49-79, p.51.

13. cr. T.L.Short "Peirce and the Incommensurability of theories." The Monist, 1980, vol.63, n'3, pp. 316-328, p.322.

14. On collateral experience, see Ms 318 and 8.178. "Collateral" observation is distinguished from "acquaintance" with the system of signs (8.179, which involves a "previous acquaintance which involves a "previous acquaintance with what the sign denotes" (8.179).

15. cf. Hilpinen, art. cit. p.185.

16. Ibid.

17. In keeping with the synechistic hypothesis according to which "all that exists is continuous" (1.172; cf. 6.116; 6.164;4.121).

18. S.Haack Deviant Logic Cambridge University Press, 1974, chap.6, pp.108-9.

19. Max Black; "Vagueness: an exercise in logical analysis." Philosophy of Science, VI.1937, pp. 427-455.

20. M.Sainsbury, Paradoxes, Cambridge University Press, 1988, chap.2. "Vagueness: the paradox of the heap", p.27.

21. Sainsbury, op.cit. p.27.

22. C.Wright, "On the coherence of vague predicates', Syn-

these, 30, 1975, pp. 325-365, p.325.

23. The idea according to which there would be perhaps some illusion to want to give up ordinary laguage (Quine) in order to adopt some ideal language has often been taken up again nowadays (by Putnam or Davidson, for ex.).

24. cf "Peirce ou la version sémiotique du courant sémantique en logique formelle" Actes du Colloque "La Sémantique formelle" Grenoble, sept.1987, Cahiers du Groupe de recherches sur la Philosophie et le Langage, n'10, 1989, pp.39-71.

25. C.Wright, art.cit. p.326.

26. M. Black, art.cit. p.432.

27. Russell, Analysis of Mind, p.184, cit par Black, rt.cit. P432.

28. Ibid. p.182.

29,. Russell, "Vagueness", Australasian Journal of Philosophy, vol. 1, 1923, p.89.

30. Vagueness being defined as that to which the principle of non contradiction cannot apply and general becoming that to which the excluded middle cannot apply. S.Haack, op.cit. p.109.

31. Frege, Foundations of Arithmetics, Austin edit. 1952, p.159.

32. P.Duhem. La théorie physique, son objet, sa structure, Paris, 1906, 2e édit. 1914,chap. 5, la loi physique, p.271, cf. Swinburne, "Vagueness inexactness and imprecision", British Journal for the Philosophy of Science", 19, 1969.

33. Kit Fine, "Vagueness, truth and Logic", Synthese, 1975,

pp. 265-300.

34. In that respect Peirce may be said to have anticipated the trend of the so-called "conceptualistic" theoricists. cf. Peter Bosch "Vagueness is Context-Dependence: a solution to the Sorites paradox", in Approaching Vagueness, Th.T.Ballmer & M.Pinkal edits. North Holland Linguistic series, Amsterdam, 1983, pp.189-210.

35. On Peirce's triadic Logic, see Ms 277 and M.Fisch & A. Turquette, "Peirce's triadic Logic", Transactions of the C.S.Peirce Society, 1966, p.78 and P.Thibaud, La Logique de C.S.Peirce: de l'algébre aux Graphes, 1975, Université de Provence, p.46.

36. Turquette, art.cit. pp.78-9.

37. For ex. Hallden The Logic of Nonsense, Uppsala Universitets Årsskrift, 1949, Goddard, "Predicates, relations and categories", Australasian Journal of Philosophy, 44n, 1966 or Routley "The Need for Nonsense", Australasian Journal of Philosophy, 47, 1969.

38. Russell, art.cit. pp. 88-9." All traditional logic habitually assumes that precise symbols are being employed. It is therefore inapplicable to this terrestrial life, but only to an imagined celestial existence."

39. Kit Fine, art.cit. 297.

40. Ibid. p.298.

41. M.Dummett, "Wang's Paradox', Synthese, 430, 1975, pp. 301-24, repr. in Truth and Other Enigmas, Duckworth, London, 1978, ppp. 248-68, p. 260.

42. On Peirce's realism of vagueness and his position on metaphysics, see C.Engel-Tiercelin, "Le vague est-il réel. Sur

le réalisme de Peirce", in <u>Philosophie</u>, 1966, n'10, pp. 69-96
and C.Engel-Tiercelin, <u>Le Probléme des Universaux</u>, thése de
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Projektrapport af: Biger Lundgren, Henning Sten Hansen
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Projektrapport af: Morten Blomhøj, Klavs Frisdahl
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102/85 OPSTILLING OG ANALYSE AF MATEMATISKE MODELLER, BELYST
VED MODELLER OVER KØERS FODEROPTAGELSE OG - OMSÆTNING".
Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn
og Susanne Stender.
Vejleder: Klaus Grünbaum.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International
Workshop on Teaching Nonlinear Phenomena
at Universities and Schools, "Chaos in
Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

13 9/87 "Machbarkeit nichtbeherrschbarer Technik
durch Fortschritte in der Erkennbarkeit
der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

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Projektrapport af Finn C. Physant
Vejleder: Ib Thiersen

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Splitting Elliptic Symbols"

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Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircean Approach

Paper presented at the Symposium on the
Foundations of Modern Physics The Copenhagen
Interpretation 60 Years after the Como Lecture.
Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

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Manuscript of a plenary lecture delivered at
ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

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Fysikspeciale af Jan Vedde
Vejledere: Niels Boye Olsen & Petr Viščor

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redigeret af: Mogens Brun Heefelt

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Albert Chr. Paulsen

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germanium prepared in ultra high vacuum"
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Matematikprojekt af:
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