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The Dirac Equation with Light-Cone Data.

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ABSTRACT

The Dirac equation with light-cone data ;

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0.$$

$$u(-|x|,x) = f(x), x \in \mathbb{R}^3$$

is considered and explicitly solved in terms of a "Light-cone
Fourier-transform" under appropriate conditions on f .

1. Introduction.

We shall consider the characteristic Cauchy problem for the free Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0 \quad (1.1)$$

$$u(-|x|,x) = f(x) \quad (1.2)$$

under suitable conditions on f , in four space-time dimensions.

The analogous problem for the Klein-Gordon equation was considered in [1]. The wave-equation has been considered by Riesz [2] and Strichartz [3], but in a very different way. For some general results on characteristic Cauchy-problems see Hörmander [4].

We shall show that there exist a Hilbert-space \mathcal{H} of light-cone data such that (1.1) and (1.2) have a unique weak solution $u(t,x)$, with $u(t, \cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ for f in \mathcal{H} and so that all " L^2 -solutions" of (1.1) have the property that $u(t-|\cdot|, \cdot)$ is in \mathcal{H} .

The Dirac equation (1.1) will be rewritten, as an evolution

equation $i \frac{\partial u}{\partial t} = Hu$ in \mathcal{H} and the generator H will be diagonalized

by a "light-cone Fourier transform", which then allows us to give an explicit formula for $u(t,x)$ in terms of f .

2. The Dirac Equation and Associated Evolution Equations.

The Dirac equation is a hyperbolic system of linear partial differential equations which can be written as

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0 \quad (2.1)$$

where $m \neq 0$ is the mass of the particle, $u: \mathbb{R} \times \mathbb{R}^3 \rightarrow \mathbb{C}^4$, γ^0 and the three components of the vector $\gamma = (\gamma^1, \gamma^2, \gamma^3)$ are 4×4 complex matrices fulfilling the anti-commutation relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \mathbb{1} \quad (2.2)$$

where $g^{\mu\nu} = 0$ for $\mu \neq \nu$, $g^{00} = -g^{ii} = 1$, for $i = 1, 2, 3$, and $\mathbb{1}$ is the 4×4 unit matrix. An explicit representation of the γ^μ 's is given by

$$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \gamma^j = \begin{bmatrix} 0 & -\sigma^j \\ \sigma^j & 0 \end{bmatrix}, \quad \text{where } 1 \text{ in } \gamma^0 \text{ is the}$$

2×2 unit matrix and the σ^j 's are the familiar Pauli-matrices given by

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad \text{We shall use this representation}$$

in the following.

The Dirac equation (2.1) is often considered as an evolution equation with initial data on a space-like hyperplane, consider for example a $t = \text{constant}$ hyperplane and write the equation as

$$i \frac{\partial u}{\partial t} = \gamma^0 (-i \gamma \cdot \nabla + m \mathbf{1}) u, \quad \text{which can be considered as an evolution}$$

equation in the Hilbert-space $L^2(\mathbb{R}^3, \mathbb{C}^4)$. The generator

$D = \gamma^0 (-i \gamma \cdot \nabla + m \mathbf{1})$ defines a self-adjoint operator D with domain

$\mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$, the direct sum of the four identical Sobolev

spaces $H^1(\mathbb{R}^3)$. The associated spectral representation of a solution

$e^{-iDt} f$ can be written

$$u(t, x) = (2\pi)^{-3/2} \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} e^{\mp i \omega_k t + i k \cdot x} u_{\pm}(k, s) \hat{f}_{\pm}(k, s) \quad (2.3)$$

for $f(\cdot) \in \mathcal{S}(\mathbb{R}^3, \mathbb{C}^4)$, the direct sum of the four identical Schwartz-spaces $\mathcal{S}(\mathbb{R}^3)$, where $u_{\pm}(k, s)$, $s = 1, 2$, are four orthogonal eigenvectors of $\hat{D} = \gamma^0(\gamma \cdot k + m\mathbb{1})$ normalized such that $u_{\pm}^* u_{\pm} = \omega_k = (|k|^2 + m^2)^{1/2}$. The "Fourier-components" $\hat{f}_{\pm}(\cdot, s)$ is essentially given by the ordinary Fourier-Plancherel transformation,

since $L^2(\mathbb{R}^3, \mathbb{C}^4) = \bigoplus_{n=1}^4 L^2(\mathbb{R}^3)$ it follows that the ordinary Fourier-

Plancherel transform lifts in a canonical way to a unitary operator $F_0 : L^2(\mathbb{R}_x^3, \mathbb{C}^4) \rightarrow L^2(\mathbb{R}_k^3, \mathbb{C}^4)$ such that

$F_0 D F_0^* = \hat{D} = \gamma^0(\gamma \cdot k + m\mathbb{1})$ and $\hat{f} = F_0 f$. For $f(\cdot) \in \mathcal{S}(\mathbb{R}^3, \mathbb{C}^4)$ we have the following explicit formula

$$(F_0 f)_{\pm}(k, s) = \langle v_{\pm}(\cdot, k, s), f(\cdot) \rangle_{L^2(\mathbb{R}_x^3, \mathbb{C}^4)} \quad (2.4)$$

Where

$$v_{\pm}(x, k, s) = (2\pi)^{-3/2} e^{ik \cdot x} u_{\pm}(k, s) \quad (2.5)$$

The associated Parseval's formula reads

$$\int_{\mathbb{R}^3} d^3x u^*(t,x)u(t,x) = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left| \widehat{f}_{\pm}(k,s) \right|^2 \quad (2.6)$$

for every $t \in \mathbb{R}$. Consider a solution $u(t,x)$ such that $\widehat{f}_{\pm} \in \mathcal{S}(\mathbb{R}^3)$ and construct the current $J^{\mu} = \bar{u} \gamma^{\mu} u$, where $\bar{u} = u^* \gamma^0$. It follows from the Dirac equation (2.1) that J^{μ} is conserved, i.e.

$$\nabla_{\mu} J^{\mu} \equiv \frac{\partial}{\partial t} J^0 + \nabla \cdot J = 0. \text{ Intergration of this conservation equation}$$

over the region in $\mathbb{R} \times \mathbb{R}^3$ between a $t=t_0$ hyperplane, $t_0 \in \mathbb{R}$, and the lighth-cone $\mathcal{C} = \{ (t,x) : t=t' - |x|, t' < t_0 \}$ together with Gauss's theorem and a decay estimate, given in [1], gives

$$\begin{aligned} & \int_{\mathbb{R}^3} d^3x \bar{u}(t' - |x|, x) (\gamma^0 + \gamma \cdot n) u(t' - |x|, x) \\ &= \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left(\left| \widehat{f}_{+}(k,s) \right|^2 + \left| \widehat{f}_{-}(k,s) \right|^2 \right) \end{aligned} \quad (2.7)$$

where $n = x|x|^{-1}$, equation (2.7) resembles a Parseval's formula.

This indicates that there might exist a Hilbert-space \mathcal{H} of lighth-cone data $u(t-|\cdot|, \cdot)$ with the left-hand side of (2.7) as scalar product and such that the map $u(-|\cdot|, \cdot) \rightarrow u(t-|\cdot|, \cdot)$ defines a unitary strongly continuous one-parameter group $t \rightarrow U(t)$ on \mathcal{H} . This will be shown to be the case in the next section.

Let $u(0,x)$ have support in the region $|x| \leq R$ and consider $t \geq R$. It then follows from the finite propagation velocity that the support of $u(t-|x|, x)$ is in the region $|x| \leq (R+t)/2$. This means that there is a subspace $\mathcal{H}_0 \subset \mathcal{H}$ of lighth-cone data with compact support.

It follows again from the finite propagation velocity, that $U(t)\mathcal{H}_0 \subset \mathcal{H}_0$, for $t \geq 0$. This need not be the case for $t < 0$, which means that compact support of lighth-cone data, is in general, only a semi-group property.

3. The light-cone evolution equation.

Consider the Dirac equation (2.1) and make the coordinate exchange $t \rightarrow t' = t + |x|$ and $x \rightarrow x' = x$ such that a $t' = \text{constant}$ is the backward light-cone with its apex at $(t,0)$. The new coordinates will be called light-cone (LC) coordinates. In the LC coordinates the Dirac

equation becomes $i(1 + \alpha \cdot n') \frac{\partial u'}{\partial t'} = (-i\alpha \cdot \nabla' + m\gamma^0)u'$ where

$u'(t',x') = u(t' - |x'|, x')$, $\alpha = \gamma^0 \gamma$ and $n' = x' |x'|^{-1}$, note that ∇' is associated with the light-cone $\mathfrak{C} = \{ (t,x) : t = -|x| \}$ in the original coordinates and $\mathfrak{C} = \{ x' \in \mathbb{R}^3 \}$ in the new coordinates, i.e. the light-cone can be identified with \mathbb{R}^3 which will be done in the following. We shall only use the LC coordinates from now on and can therefore drop the primes. Hence the Dirac equation becomes

$$i(1 + \alpha \cdot n) \frac{\partial u}{\partial t} = (-i\alpha \cdot \nabla + m\gamma^0)u \quad (3.1)$$

where $n = x |x|^{-1}$ and $\alpha = \gamma^0 \gamma$.

Definition 3.1. Let $L^2 = L^2(\mathfrak{E}, \mathbb{C}^4)$ where the measure on \mathfrak{E} is the Lebesgue measure on \mathbb{R}^3 . Put

$$Q = \mathbf{1} + \alpha \cdot n \quad \text{and} \quad D = -i\alpha \cdot \nabla + \gamma^0 m \quad (3.2)$$

It then follows that $2^{-1}Q$ is an orthogonal projection in L^2 and that the Dirac operator D defines a self-adjoint operator in L^2 with spectrum $\sigma(D) = (-\infty, -m] \cup [m, \infty)$, $m > 0$.

The Dirac equation (3.1) can now be considered as an equation in L^2

$$iQ \frac{du}{dt}(t) = Du(t) \quad (3.3)$$

which can be written as

$$iD^{-1}Q \frac{du}{dt}(t) = u(t) \quad (3.4)$$

since $0 \notin \sigma(D)$. From equation (3.4) it follows that if $\frac{du}{dt} \in L^2$

then $u \in D^{-1}QL^2$.

Definition 3.2. Let $B = D^{-1}Q$ and $\mathcal{H}_0 = BL^2$. Furthermore put

$$\langle f, g \rangle_{\mathcal{H}_0} = \langle f, Qg \rangle_{L^2} \quad (3.5)$$

for $f, g \in \mathcal{H}_0$.

It will be proved that \mathcal{H}_0 is a pre-Hilbert space, but before doing that we have to mention two lemmas, both proved in [1].

Lemma 3.3. Let $S_0 = -i(n \cdot \nabla + \nabla \cdot n)$ with domain $\mathcal{D}(S_0) = C_0^\infty(\mathbb{R}^3)$

in $L^2(\mathbb{R}^3)$, then the following inequality holds

$$\| S_0 f \|_{L^2(\mathbb{R}^3)} \leq 4 \int_{\mathbb{R}^3} d^3x \left| \nabla f(x) \right|^2 \quad (3.6)$$

for all $f \in \mathcal{D}(S_0)$.

Lemma 3.4. The range $\mathcal{R}(S_0)$ of S_0 is dense in $L^2(\mathbb{R}^3)$.

Due to the inequality (3.6) S_0 can be extended continuously to the Sobolev space $H^1(\mathbb{R}^3)$, let this extension be denoted S , it also follows that S is symmetric in $L^2(\mathbb{R}^3)$, since S_0 is. Due to lemma 3.4. also S have dense range $\mathcal{R}(S_0)$ in $L^2(\mathbb{R}^3)$, and since S is symmetric in $L^2(\mathbb{R}^3)$ it follows that S is one-to-one, i.e. S has a densely defined inverse in $L^2(\mathbb{R}^3)$, in fact it is easy to show that

$$(S_0^{-1}g)(x) = \frac{i}{2r} \int_{-\infty}^r \rho g(\rho, x|x|^{-1}) d\rho \quad (3.7)$$

for $g \in \mathcal{R}(S_0)$.

Proposition 3.5. \mathcal{H}_0 is a pre-Hilbert space.

Proof. The non-trivial part is to prove that if $\langle f, f \rangle_{\mathcal{H}_0} = 0$ then $f = 0$

and since $\langle f, f \rangle_{\mathcal{H}_0} = \langle f, Qf \rangle_{L^2}$ this is to prove that $Qf = 0$ implies

$f = 0$ in L^2 . Let $f \in \mathcal{H}_0 \setminus \{0\}$. From the anti-commutation relation (2.2)

it follows by a direct computation that

$$(\alpha \cdot n)(\alpha \cdot \nabla) + (\alpha \cdot \nabla)(\alpha \cdot n) = (n \cdot \nabla + \nabla n) \mathbf{1} \quad (3.8)$$

on $C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. In section 2 it was shown that the Dirac operator D in

L^2 is self-adjoint with domain $\mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$. Now the above

defined operator S can be lifted in a canonical way to an operator $S \cdot \mathbf{1}$

on $\mathcal{D}(D)$, where $\mathbf{1}$ here denotes the 4×4 unit matrix. Hence

$$DQ = (2-Q)D + S \cdot \mathbf{1} \quad (3.9)$$

on $\mathcal{D}(D)$, since the left-hand side and the right-hand side agree on the

dense subspace $C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. Since $0 \notin \sigma(D)$ then D has a bounded inverse

D^{-1} defined on $\mathcal{R}(D)$, which is dense in L^2 , hence D^{-1} can be extended

to a continuous operator on L^2 also denoted by D^{-1} . Acting on (3.9) with D^{-1} from both left and right gives

$$QD^{-1} = D^{-1}(2-Q) + D^{-1}S \cdot 1D^{-1} \quad (3.10)$$

on $\mathcal{R}(D)$. By acting with Q from both left and right equation (3.10) gives

$$QB = D^{-1}S \cdot 1B \quad (3.11)$$

on $\{f \in L^2 : Qf \in \mathcal{R}(D)\}$, since $Q^2 = 2Q$. Now $f = Bg \in \mathcal{H}_0 \setminus \{0\}$ so it follows from equation (3.11) that $Qf = QBg = D^{-1}S \cdot 1Bg$ and since D^{-1} is a bounded operator and the kernel for S is trivial in $L^2(\mathbb{R}^3)$ then $D^{-1}S \cdot 1Bg \neq 0$, i.e. if $f \in \mathcal{H}_0 \setminus \{0\}$ then $Qf \in L^2 \setminus \{0\}$, or in other words if $Qf = 0$ in L^2 then $f = 0$ in \mathcal{H}_0 .

Definition 3.6. Let \mathcal{H} be the completion of \mathcal{H}_0 .

It follows from the definition that $2^{-1/2}Q$ defines an isometry from \mathcal{H} into L^2 , and that B defines a bounded self-adjoint operator on \mathcal{H} .

Proposition 3.7. The range $\mathcal{R}(B)$ of $B : \mathcal{H} \rightarrow \mathcal{H}$ is dense in \mathcal{H} .

Proof. Let $\psi \in (B\mathcal{H})^\perp$, i.e. ψ is in the orthogonal complement of $B\mathcal{H}$,

so $\langle \psi, Bf \rangle_{\mathcal{H}} = 0$ for all $f \in \mathcal{H}$. Consider $f \in \mathcal{H}_0$, then $f = Bg$ for some

$g \in L^2$. Then $\langle \psi, Bf \rangle = 0$ for all $f \in \mathcal{H}$ implies that

$\langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} = 0$ for all $g \in L^2$. Since $QD^{-1}QD^{-1}Q = (D^{-1}S \cdot 1D^{-1})^2Q$

and the commutator $[Q, D^{-1}S \cdot 1D^{-1}] = 0$ then

$$\begin{aligned} \langle (D^{-1}S \cdot 1D^{-1})^2Q\psi, g \rangle_{L^2} &= \langle QD^{-1}QD^{-1}Q\psi, g \rangle_{L^2} \\ &= \langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} \\ &= \langle \psi, D^{-1}QD^{-1}Qg \rangle_{L^2} = 0 \end{aligned} \quad (3.12)$$

for all $g \in L^2$, i.e. $(D^{-1}S \cdot 1D^{-1})^2Q\psi = 0$. Hence $Q\psi = 0$ in L^2 , since D^{-1}

is invertible and the kernel for S is trivial in $L^2(\mathbb{R}^3)$, i.e. $\psi = 0$ in \mathcal{H}

(which is proved in proposition 3.5.)

Proposition 3.7. shows that B is one-to-one since it is self-adjoint.

Definition 3.8. Put $H = B^{-1}$ with $\mathcal{D}(H) = \mathcal{R}(B)$ in \mathcal{H} .

Note that H is self-adjoint since B is. Equation (3.4) can then be written as

$$i \frac{du}{dt}(t) = Hu(t) \quad (3.13)$$

with the solution $u(t) = e^{-iHt}u(0)$, $u(0) \in \mathcal{D}(H)$, i.e. $U_t = e^{-iHt}$ is the time-evolution operator for light-cone data. From definition 3.8 it follows that

$$\langle f, Hf \rangle_{\mathcal{H}} = \langle f, Df \rangle_{L^2} \quad (3.14)$$

and

$$\langle Hf, Hf \rangle_{\mathcal{H}} = \langle Df, Df \rangle_{L^2} \quad (3.15)$$

for all $f \in \mathcal{D}(H)$, i.e. $\mathcal{D}(H) \subset \mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$.

4. The light-cone Fourier transform.

The self-adjointness of H implies, due to the spectral theorem, that

there exist a spectral family $\{ E(\lambda) \}_{\lambda \in \mathbb{R}}$ such that $H = \int_{\mathbb{R}} \lambda dE(\lambda)$.

The following well-known formula allows $E(\lambda)$ to be expressed in terms of the resolvent $R(z) = (H-z)^{-1}$, for z in the resolvent set $\rho(H)$

$$\begin{aligned} & \langle f, 1/2[(E(\beta)+E(\beta-0))-(E(\alpha)+E(\alpha-0))]f \rangle_{\mathcal{H}} \\ &= \lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f \rangle_{\mathcal{H}} \end{aligned} \quad (4.1)$$

In the following the right-hand side of equation (4.1) will be evaluated.

Let f belong to $C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ and put $g = R(z)f$ for $\text{Im}(z) \neq 0$.

It then follows that g belongs to $\mathcal{D}(H)$ and fulfills the equation

$$(H-z)g = f \quad (4.2)$$

which also can be written as $(B^{-1}-z)g = f$. Now applying B to this equation gives $(\mathbb{1}-zB)g = Bf$. Consider this equation in L^2 , then the fact that $\mathcal{D}(H) \subset \mathcal{D}(D)$ allows us to act with D , which gives

$$(D-zQ)g = Qf \tag{4.3}$$

since $B = D^{-1}Q$.

Definition 4.1. Let V_z be defined in L^2 by

$$(V_z\phi)(x) = e^{-iz|x|}\phi(x) \tag{4.4}$$

on its maximal domain.

Proposition 4.2. If ϕ is in L^2 such that $V_z^{-1}\phi$ is in $\mathcal{D}(D)$ then

$$(D-zQ)V_z^{-1}\phi = V_z^{-1}(D-z)\phi \tag{4.5}$$

Proof. Equation (4.5) follows from a straightforward computation.

Rewriting equation (4.3) as $(D-zQ)V_z^{-1}V_zg = Qf$ and using

proposition 4.2 gives

$$V_z^{-1}(D-z)V_z g = f \quad (4.6)$$

for $\text{Im}(z) < 0$, such that V_z is bounded. Equation (4.6) can be solved for g , which gives

$$g = V_z^{-1}(D-z)^{-1}V_z Qf \quad (4.7)$$

and then

$$\langle f, R(z)f \rangle_{\mathfrak{H}} = \langle V_z^{-1}Qf, (D-z)^{-1}V_z Qf \rangle_{L^2} \quad (4.8)$$

for $\text{Im}(z) < 0$. The left-hand side of (4.8) is analytic for z not in the spectra $\sigma(H)$ of H , which then shows that the right-hand side has an analytic continuation to $\text{Im}(z) > 0$.

As described in section 2 the spectral theory for the Dirac operator D in L^2 , is given by a unitary operator F_0 on L^2 , such that

$F_0 D F_0^* = \widehat{D} = \alpha \cdot k + \gamma^0 m$, $k \in \mathbb{R}^3$, with eigenvalues $\pm \omega_k$. For $f \in L^2$

put for short $\widehat{f} = F_0 f$. Equation (4.8) can then be written as

$$\langle f, R(z)f \rangle_{\mathfrak{H}} = \langle \widehat{V_{\bar{z}} Q f}, (\widehat{D} - z)^{-1} \widehat{V_z Q f} \rangle_{L^2} \quad (4.9)$$

where $(\widehat{D} - z)^{-1}$ has the following explicit representation

$$(\widehat{D} - z)^{-1} = \omega_k^{-1} \sum_{\pm} \sum_{s=1,2} (\pm \omega_k - z)^{-1} u_{\pm}(k,s) u_{\pm}^*(k,s) \quad (4.10)$$

and $u(\cdot, s)$ are four orthogonal eigenvectors of \widehat{D} normalized such

that $u_{\pm}^*(k,s) u_{\pm}(k,s) = \omega_k$ also described in section 2.

Definition 4.3. If $f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ then $f(\cdot, s)$ is defined by

$$f_{\pm}(k,s) = u_{\pm}^*(k,s) \widehat{(V_{\pm \omega_k} Q f)}(k) \quad (4.11)$$

for $s = 1, 2$.

With proper interpretation formula (4.11) also can be written as

$$\widehat{f}_{\pm}(k,s) = \langle u_{\pm}(\cdot,k,s), f(\cdot) \rangle_{\mathfrak{H}} \quad (4.12)$$

where

$$u_{\pm}(x,k,s) = (2\pi)^{-3/2} e^{\pm i\omega_k |x| + ik \cdot x} u_{\pm}(k,s) \quad (4.13)$$

Now the right-hand side of equation (4.1) can be evaluated. Let

$f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ and put

$$g_{\pm}(k,s,z) = (u_{\pm}^*(k,s) \widehat{(V_{\bar{z}} Q f)}(k))^* (u_{\pm}^*(k,s) \widehat{(V_z Q f)}(k)) \quad (4.14)$$

then formula (4.9) becomes

$$\langle f, R(z)f \rangle_{\mathfrak{H}} = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} (\pm\omega_k - z)^{-1} g_{\pm}(k,s,z) \quad (4.15)$$

Due to the fact that g_{\pm} is analytic in z and continuous in k

Fubini's theorem can be used on equation (4.15) which gives

$$\begin{aligned} & \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\epsilon) - R(\mu-i\epsilon)] f \rangle_{\mathfrak{H}} \\ &= \sum_{\pm} \sum_{s=1,2} \lim_{\epsilon \searrow 0} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu h_{\pm}(k, s, \mu, \epsilon) \end{aligned} \quad (4.16)$$

where

$$\begin{aligned} h_{\pm}(k, s, \mu, \epsilon) &= (\pm\omega_k - (\mu+i\epsilon))^{-1} g_{\pm}(k, s, \mu+i\epsilon) \\ &\quad - (\pm\omega_k - (\mu-i\epsilon))^{-1} g_{\pm}(k, s, \mu-i\epsilon) \end{aligned} \quad (4.17)$$

Since g_{\pm} is analytic in z Taylor's formula can be used and together

with Lebesgue's dominated convergence theorem equation (4.16)

becomes

$$\begin{aligned} & \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\epsilon) - R(\mu-i\epsilon)] f \rangle_{\mathfrak{H}} \\ &= \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} \pi^{-1} \lim_{\epsilon \searrow 0} \int_{\alpha}^{\beta} d\mu \Psi_{\pm}(k, s, \mu, \epsilon) \frac{\epsilon}{(\mu \mp \omega_k)^2 + \epsilon^2} \end{aligned} \quad (4.18)$$

where

$$\begin{aligned}
 \Psi_{\pm}(k,s,\mu,\epsilon) &= g_{\pm}(k,s,\mu) - g_{\pm}'(k,s,\mu)(\mu \mp \omega_k) \\
 &\quad - \epsilon(1/2[\operatorname{Re}(g_{\pm}''(\xi_{\pm})) + \operatorname{Im}(g_{\pm}''(\eta_{\pm}))](\pm\omega_k - (\mu - i\epsilon)) \\
 &\quad - 1/2[\operatorname{Re}(g_{\pm}''(\xi_{\pm}')) + \operatorname{Im}(g_{\pm}''(\eta_{\pm}'))](\pm\omega_k - (\mu - i\epsilon)))
 \end{aligned}
 \tag{4.19}$$

here g_{\pm}' denote the partial derivative of g_{\pm} , g_{\pm}'' the second partial derivative of g_{\pm} in the z -variable and $0 \leq \xi_{\pm}, \xi_{\pm}', \eta_{\pm}, \eta_{\pm}' \leq \epsilon$.

Finally since $f_{\epsilon}(y) = \pi^{-1}\epsilon(y^2 + \epsilon^2)^{-1}$ is a delta family (as $\epsilon \searrow 0$) equation (4.18) gives

$$\begin{aligned}
 \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu + i\epsilon) - R(\mu - i\epsilon)] f \rangle_{\mathcal{H}} \\
 = \sum_{\pm} \sum_{s=1,2} \int_{\alpha < \pm\omega_k < \beta} \frac{d^3k}{\omega_k} \Psi_{\pm}(k,s,\pm\omega_k,0)
 \end{aligned}
 \tag{4.20}$$

i.e.

$$\lim_{\epsilon \rightarrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\bar{\mu} \langle f, [R(\bar{\mu} + i\epsilon) - R(\bar{\mu} - i\epsilon)] f \rangle_{\mathcal{H}}$$

$$= \left\{ \begin{array}{ll} \sum_{s=1,2} \int_{\alpha < \omega_k < \beta} \frac{d^3 k}{\omega_k} | \hat{f}_+(k,s) |^2, & m < \alpha < \beta \\ \sum_{s=1,2} \int_{-\beta < \omega_k < -\alpha} \frac{d^3 k}{\omega_k} | \hat{f}_-(k,s) |^2, & \alpha < \beta < -m \\ \sum_{s=1} \int_{\omega_k < \beta} \frac{d^3 k}{\omega_k} | \hat{f}_+(k,s) |^2 \\ + \sum_{s=1,2} \int_{\omega_k < -\alpha} \frac{d^3 k}{\omega_k} | \hat{f}_-(k,s) |^2, & \alpha < -m, \beta > m \\ 0 & , \text{ otherwise} \end{array} \right. \quad (4.21)$$

duing to the fact that $\Psi_{\pm}(k,s,\pm\omega_k,0) = g_{\pm}(k,s,\pm\omega_k) = | \hat{f}_{\pm}(k,s) |^2$.
From equation (4.1) and (4.17) it then follows that

$$\langle f, [E(\beta) - E(\alpha)] f \rangle_{\mathfrak{H}}$$

$$= \left\{ \begin{array}{l} \sum_{s=1,2} \int_{\alpha < \omega_k < \beta} \frac{d^3k}{\omega_k} |\hat{f}_+(k,s)|^2, \quad m < \alpha < \beta \\ \sum_{s=1,2} \int_{-\beta < \omega_k < -\alpha} \frac{d^3k}{\omega_k} |\hat{f}_-(k,s)|^2, \quad \alpha < \beta < -m \\ \sum_{s=1,2} \int_{\omega_k < \beta} \frac{d^3k}{\omega_k} |\hat{f}_+(k,s)|^2 \\ + \sum_{s=1,2} \int_{\omega_k < -\alpha} \frac{d^3k}{\omega_k} |\hat{f}_-(k,s)|^2, \quad \alpha < -m, \beta > m \\ 0, \quad \text{otherwise} \end{array} \right. \quad (4.22)$$

Letting $\alpha \rightarrow -\infty$ and $\beta \rightarrow \infty$ formular (4.23) gives

$$\|f\|_{\mathfrak{H}}^2 = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} |\hat{f}_{\pm}(k,s)|^2 \quad (4.24)$$

for $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. Now the map defined in definition 4.3 can be extended by continuity to an isometry.

Definition 4.4. Let $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_+ \oplus \widehat{\mathcal{H}}_-$, $\widehat{\mathcal{H}}_\pm = L^2(\mathfrak{M}_\pm, \mathbb{C}^2)$ where \mathfrak{M}_\pm are the mass-hyperboloids and the measure is $\frac{d^3k}{\omega_k}$. Define

$$F : \mathcal{H} \rightarrow \widehat{\mathcal{H}} \text{ by } Ff = (\widehat{f}_+, \widehat{f}_-).$$

It follows from formula (2.7) that the map $F : \mathcal{H} \rightarrow \widehat{\mathcal{H}}$ in fact is onto, i.e. F is unitary.

Theorem 4.5. The unitary map $F : \mathfrak{H} \rightarrow \widehat{\mathfrak{H}}$, called the "light-cone Fourier transform", diagonalizes H , i.e.

$$FHf = (\omega_k \widehat{f}_+ , -\omega_k \widehat{f}_-) \quad (4.25)$$

for $f \in \mathfrak{D}(H)$.

Proof. Let $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$ then

$$\begin{aligned} \widehat{Hf}_\pm(k,s) &= \langle u_\pm(\cdot, k, s), Hf(\cdot) \rangle_{\mathfrak{H}} \\ &= \langle Du_\pm(\cdot, k, s), f(\cdot) \rangle_{L^2} \\ &= \pm \omega_k \langle u(\cdot, k, s), Qf(\cdot) \rangle_{L^2} \\ &= \pm \omega_k \widehat{f}_\pm(k, s) \end{aligned} \quad (4.26)$$

where Du_\pm is understood in weak sense. Since H is closed and F is continuous, formula (4.25) holds for all $f \in \mathfrak{D}(H)$.

The light-cone Fourier transform defines a kind of "duality" between the light-cone and the mass-hyperboloids.

Recall that formula (2.3) gives rise to a unitary map from $\widehat{\mathcal{H}}$ onto $L^2(\mathbb{R}^3, \mathbb{C}^4)$ of $t = \text{constant}$ data, i.e. there is a one-to-one correspondance between weak solutions $u(t, x)$ of the Dirac equation (2.1) with $u(t, \cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ and the solution $e^{-iHt}f$, $f \in \mathcal{H}$, of equation (3.13).

5. Conclusion.

We have shown that the characteristic Cauchy problem for the Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0$$

$$u(-|x|,x) = f(x) \quad , \quad x \in \mathbb{R}^3 \quad (5.1)$$

has a unique weak solution with $u(t,\cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ for all $f \in \mathcal{H}$ and that all "L²-solutions" are obtained in this way.

The Dirac equation was written as an evolution equation for light-cone data

$$i \frac{du}{dt} = Hu \quad (5.2)$$

and the spectral theory for H was developed including a "light-cone Fourier transform" which diagonalized H .

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Introduktion og oversættelse:

Peder Voetmann Christensen

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