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PSEUDO-DIFFERENTIAL PROJECTIONS
AND THE TOPOLOGY OF CERTAIN SPACES
OF ELLIPTIC BOUNDARY VALUE PROBLEMS

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PSEUDO-DIFFERENTIAL PROJECTIONS
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Abstract. We calculate the homotopy groups of the space of elliptic boundary value problems for an elliptic differential operator A of first order and of the space of elliptic self-adjoint boundary value problems when A is formally self-adjoint. In particular we show that the spectral flow of an S^1 family of self-adjoint elliptic boundary value problems is well defined. This provides some information on spectral properties along the lines of the Vafa-Witten approach to spectral inequalities.

§1. The Notion of an Elliptic Boundary Value Problem

Let X be a smooth compact manifold with boundary Y. Let E and F be smooth complex vector bundles over X. For simplicity, we consider $A : C^\infty(X; E) \to C^\infty(X; F)$ an elliptic differential operator over X of first order. We discuss the results for a larger class of operators at the end of the paper. Now let us fix a Riemannian metric on X and Hermitian structures on E and F. Then in a fixed collar neighbourhood $N \cong I \times Y$ of $Y \cong 0 \times Y$ the operator A takes the form

$$(1) \qquad A(t, y) = G(t, y)\left(\partial_t + B_t(y)\right),$$

where $G(t, y) : E|_Y \to F|_Y$ is a bundle isomorphism for fixed t; ∂_t is the normal derivative; and $B_t : C^\infty(Y; E|_Y) \to C^\infty(Y; E|_Y)$ is an elliptic differential operator of first order on Y such that the principal symbol $b_t(y, \varsigma)$ of B_t has no purely imaginary eigenvalues. We will assume that $G(t, y)$ is unitary.

It is well known that the orthogonal (with respect to $L^2(Y; E|_Y)$) projection

$$P(A) : C^\infty(Y; E|_Y) \to H(A)$$

of $C^\infty(Y; E|_Y)$ onto $H(A) = \{u|_Y \mid u \in C^\infty(X; E) \text{ and } Au = 0 \text{ in } X \setminus Y\}$ is a pseudo-differential operator of order zero. It is called the *Calderón projector* of A. At each point (y, ς) of the covariant sphere bundle SY the principal symbol of $P(A)$ is the orthogonal projection $p^+(y, \varsigma) : E_y \to E_y$ onto the direct sum $E_{y,\varsigma}^+$ of the eigenspaces of the symbol $b_0(y, \varsigma)$ corresponding to the eigenvalues with positive real part.

Now let V be a Hermitian bundle over Y and let $R : C^\infty(Y; E|_Y) \to C^\infty(Y; V)$ be a pseudo-differential operator. Then we have various expressions of a *boundary value problem*, namely the pair

$$(A, R) : C^\infty(X; E) \to C^\infty(X; F) \oplus \operatorname{ran}(R),$$

the *realization* of A

$$A_R : \operatorname{dom}(A_R) \xrightarrow{A} L^2(X; F)$$

as an unbounded operator in $L^2(X; E)$ with

(2) $\qquad \mathrm{dom}(A_R) := \{u \in L^2(X; E) \mid Au \in L^2(X; F) \text{ and } R(u|_Y) = 0\}$,

and the *boundary integral*

$$RP(A) : H(A) \to \mathrm{ran}(R).$$

It has been shown in Seeley [14] that we get well defined operators with suitable regularity and finite index under the following assumptions:

(a) the range of R is closed in a suitable L^2 space and
(b) the principal symbol r of R maps the range E^+ of p^+ isomorphically onto the range of r.

REMARK. This notion includes the *local* condition (of Shapiro-Lopatinsky type) and the *global* condition (of Atiyah-Patodi-Singer type) as well. We get the local condition when $R : C^\infty(Y; E|_Y) \to C^\infty(Y; V)$ is a pseudo-differential operator such that $r : E^+ \to \pi^*(V)$ is an isomorphism where $\pi : SY \to Y$ denotes the natural projection.

Note that here we also admit operators $R : C^\infty(Y; E|_Y) \to C^\infty(Y; E|_Y)$. In this case the cokernel of R is infinite dimensional; but the point is that only the kernel of R is important. Thus, as was explained precisely in Seeley [14], we may restrict ourselves to boundary conditions which are pseudo-differential projections. This leads us to the following definition:

1 DEFINITION. We shall say that a pseudo-differential operator $R : C^\infty(Y; E|_Y) \to C^\infty(Y; E|_Y)$ defines a *generalized Atiyah-Patodi-Singer elliptic boundary condition* for A if and only if

(a) $R^2 = R$ and
(b) $r(y, \varsigma) = p^+(y, \varsigma)$ for all $y \in Y$ and $\varsigma \in SY_y$.

We denote the space of pseudo-differential projections defining generalized APS conditions for A by $Ell_Y(A)$. It carries the L^2 operator topology.

REMARK. The space $Ell_Y(A)$ does not contain all possible elliptic boundary conditions for A. Even so, it is in a certain sense generic, and one can find a close correspondence between $Ell_Y(A)$ and the Grassmannians which are used in Pressley-Segal [12], Segal-Wilson [15] and Witten [18].

The introduction of the space $Ell_Y(A)$ permits one to reduce the topological analysis of the space of elliptic boundary value problems to just the boundary as shown in the next Theorem.

2 THEOREM. *The space* $Ell_Y(A)$ *of generalized Atiyah-Patodi-Singer elliptic boundary conditions for* A *is isomorphic to the space*

$$\mathrm{Pdiff}_{p^+}(E|_Y) = \{P \mid P^2 = P \text{ is pseudo-differential of 0-th order over } Y \text{ in } E$$
$$\text{with principal symbol } p = p^+\}.$$

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§2. Spaces of Pseudo-Differential Projections With the Same Principal Symbol

The space $\mathrm{Pdiff}_{p+}(E|_Y)$, or for short, Pdiff_{p+} was introduced in [19] and its homotopy groups were determined in [21] (see also [20] for the calculations concerning the connected components of Pdiff_{p+}). In particular we know that

$$\pi_i\left(\mathrm{Pdiff}_{p+}\right) \cong \begin{cases} \mathbf{Z} & \text{for } i \text{ even} \\ 0 & \text{for } i \text{ odd} \end{cases}$$

which gives us the following result:

3 THEOREM. *Under the assumptions of Section 1 we have*

$$\pi_i\left(\mathrm{Ell}_Y(A)\right) \cong \begin{cases} \mathbf{Z} & \text{for } i \text{ even} \\ 0 & \text{for } i \text{ odd.} \end{cases}$$

We present the following example which shows how Theorem 3 works. Let us assume that X is an odd-dimensional compact manifold with boundary Y, and let us consider the Dirac operator $\displaystyle{\not{D}}: C^\infty(X; S) \to C^\infty(X; S)$ over X. It is a formally self-adjoint operator and it is of the form of equation (1), namely $\displaystyle{\not{D}} = G(\partial_t + B)$ over the collar when it is assumed that the metric is product near the boundary.

Now $G^2 = -Id$ and S decomposes over Y into $S|_Y = S_+ \oplus S_-$. With respect to this decomposition we get

$$\not{D} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \left(\partial_t + \begin{pmatrix} 0 & B_- = B_+^* \\ B_+ & 0 \end{pmatrix} \right),$$

where $B_+ : C^\infty(Y; S_+) \to C^\infty(Y; S_-)$ is the Dirac operator on Y. Let us observe that the formal self-adjointness of $\not{D}$ implies $GB = -BG$ (see below Section 3). This leads us to the following interesting observation:

Let ϕ_λ be an eigenfunction of B, $B\phi_\lambda = \lambda\phi_\lambda$ with $\lambda \neq 0$, then $G\phi_\lambda$ is an eigenfunction of B with eigenvalue $-\lambda$. This means that the spectrum of B is symmetric with respect to 0. Moreover, as shown in Palais [11; Chapter XVII], we have $\mathrm{index}\, B_+ = 0$. This means that $\dim\ker B = \dim\ker B_+ + \dim\ker B_- = 2\dim\ker B_+$.

Now let us consider the standard Atiyah-Patodi-Singer realization of $\not{D}$, i.e. $\not{D}_{\Pi_+(B)} = \not{D}$ with the domain

$$\mathrm{dom}\,\not{D}_{\Pi_+(B)} = \{u \in H^1(S) \mid \Pi_+(B)(u|_Y) = 0\}$$

where $\Pi_+(B)$ denotes the projection of $L^2(S|_Y)$ onto the subspace spanned by the eigenfunctions of B corresponding to the non-negative eigenvalues. Now, from the results of Atiyah-Patodi-Singer [2] we know that $\not{D}_{\Pi_+(B)} : \mathrm{dom}\,\not{D}_{\Pi_+(B)} \to L^2(S)$ is a Fredholm operator and that its index is given by the following formula

$$\mathrm{index}\,\not{D}_{\Pi_+(B)} = \int \alpha_0(x)\,dx - \frac{1}{2}(\eta_B(0) + \dim\ker B)$$

3

where α_0 is the index density which is equal 0 when $\dim X$ is odd, hence $\eta_B(s) = \sum_{\lambda \in \operatorname{spec} B,\ \lambda \neq 0} \operatorname{sign} \lambda |\lambda|^{-s}$ is equal 0 and we are left with the formula

$$\operatorname{index} \not{D}_{\Pi_+(B)} = -\dim \ker B_+.$$

In [20] the second author proves the following equality

$$\operatorname{index} \not{D}_{\Pi_+(B)} = \operatorname{index}(\Pi_+(B)\, P(\not{D}))$$

where he considers $\Pi_+(B)\, P(\not{D}) : H(\not{D}) \to \operatorname{ran} \Pi_+(B)$. It follows from [19] that the index of this last operator is equal 0 if and only if $\Pi_+(B)$ and $P(\not{D})$ belong to the same connected component of $\operatorname{Pdiff}_{p+}(S_- \oplus S_+)$.

Hitchin [9] showed that $\dim \ker B = 2 \dim \ker B_+$ varies when we deform the metric (the most simple non-trivial examples are all Riemannian surfaces Y with genus $g > 3$). This means that $\Pi_+(B)$ and $P(\not{D})$ change their connected components independently of each other and that under a smooth change of the metric the operator $\not{D}_{\Pi_+(B)}$ varies in a non-continuous way.

With the self-adjoint case in mind, the second author introduced the subspace $\operatorname{Pdiff}_{p+}^G$ of $\operatorname{Pdiff}_{p+}$ in his paper [21]. It is defined as follows: We fix $P_0 \in \operatorname{Pdiff}_{p+}$ and an automorphism $G : L^2(Y; E|_Y) \to L^2(Y; E|_Y)$ such that

$$(3) \qquad G^2 = -Id \qquad \text{and} \quad -GP_0G = Id - P_0.$$

Such G always exists. Moreover, the space

$$\operatorname{Pdiff}_{p+}^G := \{P \in \operatorname{Pdiff}_{p+} \mid -GPG = Id - P\}$$

is also nontrivial from a topological point of view, namely

$$\pi_i\left(\operatorname{Pdiff}_{p+}^G\right) \cong \begin{cases} 0 & \text{for } i \text{ even} \\ \mathbf{Z} & \text{for } i \text{ odd,} \end{cases}$$

as was shown in [21]. In the next Section we show how to apply this result to the theory of self-adjoint elliptic boundary value problems.

We want to close with some comments concerning applications of the main result of this Section. Theorem 3 shows us the possibility of analyzing families of elliptic boundary value problems in exactly the same way as was the case with families of elliptic operators over closed manifolds. Later, we will show precisely how - at least in the case of a simple base manifold - the index of such families arises.

§3. The Space of Self-Adjoint Elliptic Boundary Value Problems

Let us now assume that A is formally self-adjoint. In particular this implies the following identities when we consider the representation (1) over the collar

$$(4) \qquad G(t,.)B_t = -B_t^* G(t,.) \text{ and } G^*(t,.) = -G(t,.).$$

In the following we consider a slightly more special case. We assume that all metrics are of product form on the collar N of Y and that G and B do not depend on t over N. Moreover, we shall assume that

$$(5) \qquad G^2 = -Id \text{ and } B = B^*$$

which in view of (4) implies that $GB = -BG$. Note that all operators "of Dirac type" satisfy condition (5).

4 DEFINITION. We shall call the realization A_R of A a *self-adjoint elliptic boundary value problem* if and only if

$$(A_R)^* = A_R.$$

Now in view of Green's formula (and here $A = A^*$)

$$(6) \qquad \langle Au, v \rangle - \langle u, Av \rangle = \int_Y \langle G(u|_Y), v|_Y \rangle$$

we have the following fact.

5 PROPOSITION. *An element $P \in \mathrm{Pdiff}_{p+}(E|_Y)$ defines an elliptic self-adjoint boundary value problem for A if and only if $Id - P = -GPG$.*

PROOF: We have $\langle Au, v \rangle - \langle u, Av \rangle = 0$ for $u, v \in \mathrm{dom}\, A_P$, hence $\mathrm{ran}\, GP = \mathrm{ran}(Id - P)$. $\square$

The only problem left is that the space $\mathrm{Ell}_Y^*(A)$ of self-adjoint problems may be empty. To prove that this is not the case let us recall the following result which was proved in [5] and in a different way in [20], namely

$$(7) \qquad P(A^*) = GP(A)G^{-1}$$

which in our case gives exactly what we want. Hence

6 PROPOSITION. *Let A be an elliptic differential formally self-adjoint operator of first order which satisfies (5). Then the realization $A_{P(A)}$ is a self-adjoint elliptic boundary value problem.*

As a consequence we obtain the second main result of this paper:

7 THEOREM.

$$\pi_i\left(\mathrm{Ell}_Y^*(A)\right) \cong \begin{cases} 0 & \text{for } i \text{ even} \\ \mathbf{Z} & \text{for } i \text{ odd.} \end{cases}$$

In particular the fundamental group of $\mathrm{Ell}_Y^*(A)$ is $\mathbf{Z}$. This means that the notion of the spectral flow is well defined in this case. We present the simplest non-trivial examples in the next section.

§4. Examples of Non-Trivial Spectral Flow

The simplest example is of course provided by the operator $A = -i\frac{d}{dx}$ on $[0, 2\pi]$. We take the family $\{A_t\}_{t \in I}$ of realizations of A with the condition $u(1) = e^{2\pi i t} u(0)$. The eigenvalues of A_t are $\{k + t\}_{k \in \mathbf{Z}}$ and the corresponding eigenfunctions are $e^{i(k+t)x}$. Moreover, we have $A_1 = A_0$ and we see that there exists a non-trivial spectral flow in this case.

The next example is in dimension 2. Let ∂ denote the complex differentiation $\frac{\partial}{\partial z} : C^\infty(D^2) \to C^\infty(D^2)$ over the unit disc. We consider the self-adjoint operator

$$A = \begin{pmatrix} 0 & \overline{\partial} \\ \partial & 0 \end{pmatrix}$$

5

which takes over S^1 the following form

$$(8) \qquad \begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix} \left(\partial_r + \begin{pmatrix} i\partial_\phi + 1 & 0 \\ 0 & -i\partial_\phi \end{pmatrix} \right).$$

Now $H(A) = H(\partial) \oplus H(\overline{\partial}) = H_+ \oplus H_-$ where $H_+ = \{\sum_{k \geq 0} a_k z_k\}$ and $H_- = \{\sum_{k \leq 0} a_k z_k\}$. Note that $H_+ \cap H_- = \mathbb{C}$ does not violate (7) because we have a non-trivial G for ∂ (and $\overline{\partial}$). We obtain

$$P(A) = \begin{pmatrix} (Id - P_+) + p_0 & 0 \\ 0 & P_+ \end{pmatrix}$$

where P_+ is the projection onto $\mathrm{span}\{e^{ikx}\}_{k \geq 0}$ in L^2 and p_0 is the projection onto constants. Now we may choose the family A_t of realizations of A with the boundary condition $P_t = e^{it\phi} P(A) e^{-it\phi}$.

In the next dimension we have the first topologically interesting example: the restriction of the signature operator over S^3 to D^3. The precise calculations will be published separately.

§5. Spectral Estimates for Self-Adjoint Elliptic Boundary Value Problems

The main reason for our investigation of the topological structure of the space of elliptic boundary value problems is the search for non-vanishing spectral invariants. We want to transfer an idea of Vafa and Witten [17] (see also Atiyah [1]) to boundary value problems. They use the notion of spectral flow to obtain uniform inequalities for the spectrum of the extended Dirac operator (uniform with respect to the choice of the coefficients bundle) on closed manifolds. Here, we shall show how their *global reasoning* can be adapted to our context, thus providing both a short-cut alternative and a substantial supplement to the *classical (and neo-classical) spectral analysis* with its highly subtle derivations of spectral inequalities.

We consider the simplest (and probably the most interesting) case. Let $\dim X = 2k - 1$, and let A be a formally self-adjoint operator of Dirac type

$$A : C^\infty(X; S \otimes \xi) \to C^\infty(X; S \otimes \xi)$$

where ξ is a Hermitian vector bundle over X, and let Π be a suitable self-adjoint boundary condition i.e., Π is a 0'th order pseudo-differential projection such that $-G\Pi G = Id - \Pi$ (cf. Section 2 above).

Now we choose a map $g : X \to U(N)$ of the form explained in Figure 1. Since g is equal to Id near the boundary the operator $\Pi \otimes Id_{\mathbb{C}} = \Pi \oplus ... \oplus \Pi$ defines a self-adjoint boundary condition for the operator $A_1 := (Id \otimes g^{-1}) A_0 (Id \otimes g)$ with $A_0 := A \otimes Id_{\mathbb{C}}$ and in fact for all $A_t := t A_1 + (1 - t) A_0$, $0 \leq t \leq 1$. So we obtain a family of self-adjoint elliptic boundary value problems parametrized over the interval I. The family $\{(A_t)_{\Pi \otimes Id}\}_{t \in I}$ of the realizations is a continuous family of self-adjoint unbounded Fredholm operators with discrete spectra and with the periodicity $\mathrm{spec}(A_0)_{\Pi \otimes Id} = \mathrm{spec}(A_1)_{\Pi \otimes Id}$. Because of Theorem 7 there is only one homotopy invariant of such families.

So what is left is to show that the spectral flow of the family of self-adjoint boundary value problems is equal to 1.
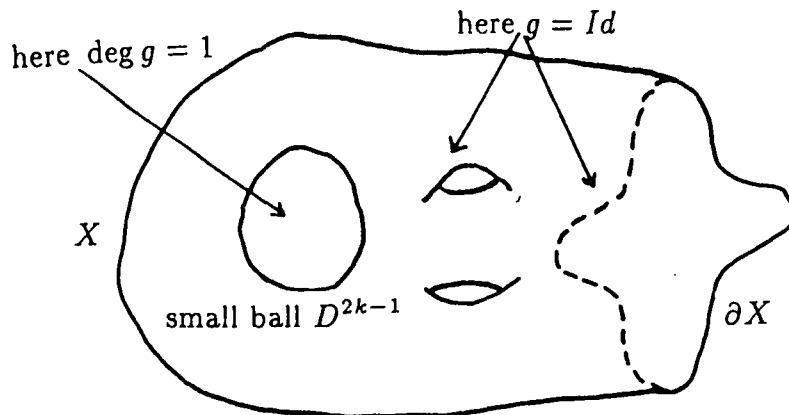
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Fig. 1. The choice of $g : X \to U(N)$

8 LEMMA. *Under the previous assumptions we have*

$$\text{sf} \left\{ (A_t)_{\Pi \otimes Id_N} \right\}_{t \in I} = 1.$$

We postpone the proof of Lemma 8. Its main consequence is the following Theorem about the spectral behaviour of the realization A_Π.

9 THEOREM. *(a) There exists a constant C such that every interval of length C contains an eigenvalue of A_Π.*

(b) There exists an upper bound $C n^{1/(2k-1)} > |\lambda_n|$ for all eigenvalues of A_Π indexed by increasing absolute value.

The constants in (a) and (b) depend only on the spin structure of the manifold X and not on the coefficients bundle ξ.

SKETCH OF PROOF: (a) follows immediately from Lemma 8. In fact, let $\{ A_t \}$ denote the family $\left\{ (A_t)_{\Pi \otimes Id_N} \right\}_{t \in I}$ and let $\{ t \mapsto j_r(t) \}_{r \in \mathbf{Z}}$ be a parametrization of the eigenvalues of the family $\{ A_t \}_{t \in I}$. In particular, this means spec $A_t = \{ j_r(t) \}_{r \in \mathbf{Z}}$. We have spec $A_1 = $ spec A_0 and by Lemma 8 we have $j_r(1) = j_{r+1}(0)$. This actually is the precise meaning of $\text{sf}\{ A_t \} = 1$.

Since $g = Id$ near the boundary of X we have $A_1 = A_0$ in a collar of the boundary. This makes the difference $A_1 - A_0$ to an L^2-bounded operator. Hence $A_1 - A_0$ is bounded in its domain in $L^2(X; (S \otimes \xi) \otimes \mathbf{C}^N)$ and by continuity we have

$$|j_r(0) - j_r(1)| \le \| A_1 - A_0 \| =: C \text{ for all } r,$$

hence $|j_r(0) - j_{r+1}(0)| \le C$. This proves that in any interval of length C there is at least one eigenvalue of A_0. The eigenvalues of A_0 are the eigenvalues of A_Π, just with N-times multiplicity. Finally, we see that C does not depend on the choice of ξ. This proves the uniformity of our estimate.

For the proof of (b) one repeats and sharpens the arguments given above. The main ingredient is the choice of a family $\{ \gamma_r \}_{r \in \mathbf{Z}}$ of twistings $g_r : X \to U(N)$ such that $\text{ch}[x \times \mathbf{C}^N, g_r] = r^{2k-1} \text{ch}[X \times \mathbf{C}^N, g_1]$ where $g_1 = g$ has "degree" 1 as above. $\square$

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REMARK. Instead of twisting the operator A over the interior of X one could choose a twisting of the boundary condition Π or of the boundary component B of the operator A over the closed manifold Y by means of a topological non-trivial map $g : Y \to U(N)$. Unfortunately, at present we are not able to give a topological formula for the spectral flow of the families defined in this way.

§6. Remarks Concerning the Proof of Lemma 8

We sketch the proof of Lemma 8. For simplicity we assume that the "tangential" component B of the operator A is invertible. The necessary generalizations to a non-invertible B may be found in [22] where the second author also discusses the relation of our results to the recent works of Bismut and Cheeger [3], [4], and of Singer [16].

Starting with an arbitrary self-adjoint condition Π, we obtain a family $\{(A_t)_\Pi\}_{t\in I}$ of self-adjoint unbounded Fredholm operators parametrized over the circle. Since B is invertible, the standard boundary condition $\Pi_+(B)$ is self-adjoint. We are going to show that $sf\{(A_t)_\Pi\}_{t\in I}$ does not depend on the special choice of Π, in fact $sf\{(A_t)_\Pi\}_{t\in I} = sf\{(A_t)_{\Pi_+(B)}\}_{t\in I}$. Afterwards, we will prove $sf\{(A_t)_{\Pi_+(B)}\}_{t\in I} = 1$.

As mentioned above, the space Pdiff_{p+}^G is path-connected. So, we may choose a path $\{\Pi_r\}_{r\in I}$ in Pdiff_{p+}^G connecting $\Pi_0 := \Pi$ with $\Pi_1 := \Pi_+(B)$. We consider the family $\{D_{t,r} = (A_t)_\Pi\}_{t,r\in I}$, (see Figure 2). Apparently, we have $sf\{D_{t,r}\} = sf\{D_{t,r'}\}$ for any r, r', hence $sf\{(A_t)_\Pi\} = sf\{(A_t)_{\Pi_+(B)}\}$.

The family $\{(A_t)_{\Pi_+(B)}\}_{t\in I}$ consists of standard Atiyah-Singer-Patodi realizations of A_t. So we can apply results of Cheeger (see [7] and especially [8, p. 209 Remark 6.11]), or we may follow the analysis from Atiyah-Patodi-Singer [2]. We obtain that $\eta_t(0) := \eta((A_t)_{\Pi_+(B)};0)$ is well-defined and that $d\eta_t(0)/dt$ is local. Because of the fixed boundary condition, the integral which defines $d\eta_t(0)/dt$ vanishes near the boundary. Hence it is given by the standard formula on the closed double $\tilde{X}$ of X out of a neighbourhood of ∂X.

The detailed calculations can be found in [22].

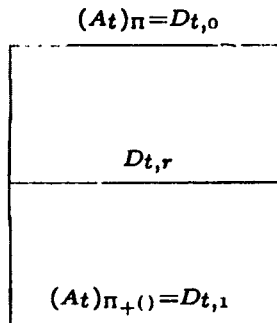$(A_t)_\Pi = D_{t,0}$

$D_{t,r}$

$(A_t)_{\Pi_+()} = D_{t,1}$

Fig. 2. The family $D_{t,r}$

§7. Generalization to Larger Classes of Operators

(1) All our results concerning the homotopy type of the spaces of elliptic boundary value problems and of self-adjoint elliptic boundary value problems (Theorem 3 and Theorem 7) easily generalise to a larger class of operators, namely to operators of the

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class $Ell^1(X, Y)$. These non-standard pseudo-differential operators with "splitting elliptic symbol" take the form (1) near the boundary where B is now pseudo-differential, see Hörmander [10] and the authors in [6].

The point is that in our global setting, the natural way of looking for spectral estimates and related results for higher order problems would be by reducing them to first order systems. This approach, however, takes us beyond the class of differential operators and we have to admit operators of the class $Ell^1(X, Y)$. This follows from the results of [6] where the authors have proven the existence of a true Calderón projector for operators of this class.

(2) Since the extended Dirac operator is fairly general from a topological point of view, one can expect that the results of Theorem 9 remain true for general formally self-adjoint differential operators A of first order with topologically non-trivial symbol class. However, the validity of Theorem 9 for operators of the class $Ell^1(X, Y)$ is less clear. In particular, it is not yet clear how we can prove Lemma 8, the essential ingredient in the proof of Theorem 9, in the context of the larger class $Ell^1(X, Y)$.

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skov, Marianne Kesselhahn og Nicolai Lomholt.
Vejleder: Anders Madsen

2/78 "OPTIMERING" - Menneskets forøgede beher-
skelsesmuligheder af natur og samfund.
Projektrapport af: Tom J. Andersen, Tommy
R. Andersen,Gert Krenøe og Peter H. Lassen
Vejleder: Bernhelm Boss.

3/78 "OPGAVESAMLING", breddekursus i fysik.
Af: Lasse Rasmussen, Aage Bonde Kræmmer
og Jens Højgaard Jensen.

4/78 "TRE ESSAYS" - om matematikundervisning,
matematiklæreruddannelsen og videnskabs-
rindalismen.
Af: Mogens Niss
Nr. 4 er p.t. udgået.

5/78 "BIBLIOGRAFISK VEJLEDNING til studiet af
DEN MODERNE FYSIKS HISTORIE".
Af: Helge Kragh.
Nr. 5 er p.t. udgået.

6/78 "NOGLE ARTIKLER OG DEBATINDLÆG OM - lærer-
uddannelse og undervisning i fysik, og - de
naturvidenskabelige fags situation efter
studenteroprøret".
Af: Karin Beyer, Jens Højgaard Jensen og
Bent C. Jørgensen.

7/78 "MATEMATIKKENS FORHOLD TIL SAMFUNDSØKONOMIEN".
Af: B.V. Gnedenko.
Nr. 7 er udgået.

8/78 "DYNAMIK OG DIAGRAMMER". Introduktion til
energy-bond-graph formalismen.
Af: Peder Voetmann Christiansen.

9/78 "OM PRAKSIS' INDFLYDELSE PÅ MATEMATIKKENS UD -
VIKLING". - Motiver til Kepler's: "Nova Stere-
ometria Doliorum Vinariom".
Projektrapport af: Lasse Rasmussen .
Vejleder: Anders Madsen.

10/79 "TERMODYNAMIK I GYMNASIET".
Projektrapport af: Jan Christensen og Jeanne
Mortensen,
Vejledere: Karin Beyer og Peder Voetmann
Christiansen.

11/79 "STATISTISKE MATERIALER".
Af: Jørgen Larsen.

12/79 "LINEÆRE DIFFERENTIALLIGNINGER OG DIFFEREN-
TIALLIGNINGSSYSTEMER".
Af: Mogens Brun Heefelt.
Nr. 12 er udgået.

13/79 "CAVENDISH'S FORSØG I GYMNASIET".
Projektrapport af: Gert Kreinøe.
Vejleder: Albert Chr. Paulsen.

14/79 "BOOKS ABOUT MATHEMATICS: History, Philosophy,
Education, Models, System Theory, and Works of".
Af: Else Høyrup.
Nr. 14 er p.t. udgået.

15/79 "STRUKTUREL STABILITET OG KATASTROFER i systemer
i og udenfor termodynamisk ligevægt".
Specialeopgave af: Leif S. Striegler.
Vejleder: Peder Voetmann Ckristiansen.

16/79 "STATISTIK I KRÆFTFORSKNINGEN".
Projektrapport af: Michael Olsen og Jørn Jensen.
Vejleder: Jørgen Larsen.

17/79 "AT SPØRGE OG AT SVARE i fysikundervisningen".
Af: Albert Christian Paulsen.

18/79 "MATHEMATICS AND THE REAL WORLD", Procee-
dings af an International Workshop, Ros-
kilde University Centre, Denmark, 1978.
Preprint.
Af: Bernhelm Booss og Mogens Niss (eds.)

19/79 "GEOMETRI, SKOLE CG VIRKELIGHED".
Projektrapport af: Tom J. Andersen,Tommy
R. Andersen og Per H.H. Larsen.
Vejleder: Mogens Niss.

20/79 "STATISTISKE MODELLER TIL BESTEMMELSE AF SIKRE
DOSER FOR CARCINOGENE STOFFER".
Projektrapport af: Michael Olsen og Jørn Jensen.
Vejleder: Jørgen Larsen

21/79 "KONTROL I GYMNASIET-FORMÅL OG KONSEKVENSER".
Projektrapport af: Crilles Bacher, Per S.Jensen,
Preben Jensen og Torben Nysteen.

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1-port lineært response og støj i fysikken.
Af: Peder Voetmann Christiansen.

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Af: Helge Kragh.

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a+b   1. En analyse. 2. Interviewmateriale.
Projektrapport af: Jan Christensen og Knud
Lindhardt Rasmussen.
Vejleder: Mogens Niss.

25/80 "EKSAMENSOPGAVER", Dybdemodulet/fysik 1974-79.

26/80 "OM MATEMATISKE MODELLER".
En projektrapport og to artikler.
Af: Jens Højgaard Jensen m.fl.

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DIRAC's PHYSICS".
Af: Helge Kragh.

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model bygget på væskernes viscoelastiske egen-
skaber".
Projektrapport af: Gert Kreinøe.
Vejleder: Niels Boye Olsen.

29/80 "ODIN - undervisningsmateriale til et kursus i
differentialligningsmodeller".
Projektrapport af: Tommy R. Andersen, Per H.H.
Larsen og Peter H. Lassen.
Vejleder: Mogens Brun Heefelt.

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ON".
Af: Oluf Danielsen.
Nr. 30 er udgået.

31/80 "VIDENSKABSTEORETISKE PROBLEMER VED UNDERVISNINGS -
SYSTEMER BASERET PÅ MÆNGDELÆRE".
Projektrapport af: Troels Lange og Jørgen Kar-
rebæk.
Vejleder: Stig Andur Pedersen.
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BELYST VED HJÆLP AF MEKANISKE IMPEDANSMÅLIN -
GER MÖSSBAUEREFFEKTMÅLINGER".
Projektrapport af: Crilles Bacher og Preben
Jensen.
Vejledere: Niels Boye Olsen og Peder Voet-
mann Christiansen.

33/80 "KONSTITUERING AF FAG INDEN FOR TEKNISK - NATUR-
VIDENSKABELIGE UDDANNELSER. I-II".
Af: Arne Jakobsen.

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TION".
ENERGY SERIES NO. I.
Af: Bent Sørensen
Nr. 34 er udgået.

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Af: Helge Kragh.

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Fire artikler.
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ENERGY SERIES NO. 2.
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38/81 "TIL EN HISTORIETEORI OM NATURERKENDELSE, TEKNOLOGI
OG SAMFUND".
Projektrapport af: Erik Gade, Hans Hedal, Henrik Lau
og Finn Physant.
Vejledere: Stig Andur Pedersen, Helge Kragh og Ib
Thiersen.
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Af: Jens Højgaard Jensen.

40/81 "TELEKOMMUNIKATION I DANMARK - oplæg til en tekno-
logivurdering".
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Jan Vedde.
Vejleder: Per Nørgaard.

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GY SUPPLY SYSTEMS".
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UDSÆTNINGER FOR RUTHERFORDS ATOMMODEL".
Projektrapport af: Niels Thor Nielsen.
Vejleder: Bent C. Jørgensen.

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45/82 Er aldrig udkommet.

46/82 "EKSEMPLARISK UNDERVISNING OG FYSISK ERKENDESE-
1+11 ILLUSTRERET VED TO EKSEMPLER".
Projektrapport af: Torben O.Olsen, Lasse Rasmussen og
Niels Dreyer Sørensen.
Vejleder: Bent C. Jørgensen.

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ENERGY SERIES NO. 5.
Af: Bent Sørensen.

48/82 "EN UNDERSØGELSE AF MATEMATIKUNDERVISNINGEN PÅ ADGANGS-
KURSUS TIL KØBENHAVNS TEKNIKUM".
Projektrapport af: Lis Eilertzen, Jørgen Karrebæk, Troels
Lange, Preben Nørregaard, Lissi Pedesen, Laust Rishøj,
Lill Røn og Isac Showiki.
Vejleder: Mogens Niss.

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Projektrapport af: Preben Nørregaard.
Vejledere: Jørgen Larsen og Rasmus Ole Rasmussen.

50/82 "HERSLEV - MULIGHEDER FOR VEDVARENDE ENERGI I EN
LANDSBY".
ENERGY SERIES NO. 6.
Rapport af: Bent Christensen, Bent Hove Jensen, Dennis
B. Møller, Bjarne Laursen, Bjarne Lillethorup og Jacob
Mørch Pedersen.
Vejleder: Bent Sørensen.

51/82 "HVAD KAN DER GØRES FOR AT AFHJÆLPE PIGERS BLOKERING
OVERFOR MATEMATIK ?"
Projektrapport af: Lis Eilertzen, Lissi Pedersen, Lill
Røn og Susanne Stender.

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Af: Arne Jacobsen og Stig Andur Pedersen.

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of Its Subject-Matter and Methods.
Af: Stig Andur Pedersen og Johannes Witt-Hansen.

55/82 "MATEMATISKE MODELLER" - Litteratur på Roskilde
Universitetsbibliotek.
En biografi.
Af: Else Høyrup.

Vedr. tekst nr. 55/82 se også tekst nr. 62/83.

56/82 "EN - TO - MANGE" -
En undersøgelse af matematisk økologi.
Projektrapport af: Troels Lange.
Vejleder: Anders Madsen.

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57/83 "ASPECT EKSPERIMENTET"-
Skjulte variable i kvantemekanikken?
Projektrapport af: Tom Juul Andersen.
Vejleder: Peder Voetmann Christiansen.
Nr. 57 er udgået.

58/83 "MATEMATISKE VANDRINGER" - Modelbetragtnin-
ger over spredning af dyr mellem småbiotoper
i agerlandet.
Projektrapport af: Per Hammershøj Jensen og
Lene Vagn Rasmussen.
Vejleder: Jørgen Larsen.

59/83 "THE METHODOLOGY OF ENERGY PLANNING".
ENERGY SERIES NO. 7.
Af: Bent Sørensen.

60/83 "MATEMATISK MODEKSPERTISE"- et eksempel.
Projektrapport af: Erik O. Gade, Jørgen Kar-
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Vejleder: Anders Madsen.

61/83 "FYSIKS IDEOLOGISKE FUNKTION, SOM ET EKSEMPEL
PÅ EN NATURVIDENSKAB - HISTORISK SET".
Projektrapport af: Annette Post Nielsen.
Vejledere: Jens Høyrup, Jens Højgaard Jensen
og Jørgen Vogelius.

62/83 "MATEMATISKE MODELLER" - Litteratur på Roskilde
Universitetsbibliotek.
En biografi 2. rev. udgave.
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Af: Berhelm Booss og Jens Høyrup.

65/83 "ANVENDT MATEMATIK - TEORI ELLER PRAKSIS".
Projektrapport af: Per Hedegård Andersen, Kir-
sten Habekost, Carsten Holst-Jensen, Annelise
von Moos, Else Marie Pedersen og Erling Møller
Pedersen.
Vejledere: Bernhelm Booss og Klaus Grünbaum.

66/83 "MATEMATISKE MODELLER FOR PERIODISK SELEKTION
I ESCHERICHIA COLI".
Projektrapport af: Hanne Lisbet Andersen, Ole
Richard Jensen og Klavs Frisdahl.
Vejledere: Jørgen Larsen og Anders Hede Madsen.

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PROGRAMMERING?"
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Vejleder: Mogens Brun Heefelt.

68/83 "STOKASTISKE MODELLER I POPULATIONSGENETIK"
- til kritikken af teoriladede modeller.
Projektrapport af: Lise Odgård Gade, Susanne
Hansen, Michael Hviid og Frank Mølgård Olsen.
Vejleder: Jørgen Larsen.

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Af: Albert C. Paulsen.

70/83 "INDLÆRINGS - OG FORMIDLINGSPROBLEMER I MATEMATIK
PÅ VOKSENUNDERVISNINGSNIVEAU".
Projektrapport af: Hanne Lisbet Andersen, Tor-
ben J. Andreasen, Svend Åge Houmann, Helle Gle-
rup Jensen, Keld Fl. Nielsen, Lene Vagn Ras-
mussen.
Vejleder: Klaus Grünbaum og Anders Hede Madsen.

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Specialeopgave i fysik af: Bent Hove Jensen.
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Projektrapport af: Bodil Harder og Linda Szko-
tak Jensen.
Vejledere: Jens Højgaard Jensen og Bent C. Jørgensen.

75/84 "MATEMATIKUNDERVISNINGEN I FREMTIDENS GYMNASIUM"
- Case: Lineær programmering.
Projektrapport af: Morten Blomhøj, Klavs Frisdahl
og Frank Mølgaard Olsen.
Vejledere: Mogens Brun Heefelt og Jens Bjørneboe.

76/84 "KERNEKRAFT I DANMARK?" - Et høringssvar indkaldt
af miljøministeriet, med kritik af miljøstyrelsens
rapporter af 15. marts 1984.
ENERGY SERIES No. 10
Af: Niels Boye Olsen og Bent Sørensen.

77/84 "POLITISKE INDEKS - FUP ELLER FAKTA?"
Opinionsundersøgelser belyst ved statistiske
modeller.
Projektrapport af: Svend Åge Houmann, Keld Nielsen
og Susanne Stender.
Vejledere: Jørgen Larsen og Jens Bjørneboe.

78/84 "JÆVNSTRØMSLEDNINGSEVNE OG GITTERSTRUKTUT I
AMORFT GERMANIUM".
Specialrapport af: Hans Hedal, Frank C. Ludvigsen
og Finn C. Physant.
Vejleder: Niels Boye Olsen.

79/84 "MATEMATIK OG ALMENDANNELSE".
Projektrapport af: Henrik Ooster, Mikael Wenner-
berg Johansen, Povl Kattler, Birgitte Lydholm
og Morten Overgaard Nielsen.
Vejleder: Bernhelm Booss.

80/84 "KURSUSMATERIALE TIL MATEMATIK B".
Af: Mogens Brun Heefelt.

81/84 "FREKVENSAFHÆNGIG LEDNINGSEVNE I AMORFT GERMANIUM".
Specialrapport af: Jørgen Wind Petersen og Jan
Christensen.
Vejleder: Niels Boye Olsen.

82/84 "MATEMATIK - OG FYSIKUNDERVISNINGEN I DET AUTO-
MATISEREDE SAMFUND".
Rapport fra et seminar afholdt i Hvidovre
25-27 april 1983.
Red.: Jens Højgaard Jensen, Bent C. Jørgensen
og Mogens Niss.

83/84 "ON THE QUANTIFICATION OF SECURITY":
PEACE RESEARCH SERIES NO. I
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Af: Jens Højgaard Jensen, Mogens Niss m. fl.

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Specialerapport af: Per Hedegård Andersen, Carsten Holst-
Jensen, Else Marie Pedersen og Erling Møller Pedersen.
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Vejleder: Torsten Meyer.

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Projektrapport af: Niels Jørgensen og Mikael Klintorp.
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Af: Ganesh Sengupta.

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VED MODELLER OVER KØERS FODEROPTAGELSE OG - OMSÆTNING".
Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn
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Vejleder: Klaus Grünbaum.

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Af: Jens Jæger.

105/85 "THE FREQUENCY DEPENDENCE OF THE SPECIFIC HEAT AF THE
GLASS REANSITION".
Af: Tage Christensen.

"A SIMPLE MODEL AF AC HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.
Contributions to the Third International Conference
on the Structure of Non - Crystalline Materials held
in Grenoble July 1985.

106/85 "QUANTUM THEORY OF EXTENDED PARTICLES".
Af: Bent Sørensen.

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ring af et epidemiologisk problem.
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FREM TIL 1950 - BELYST VED EKSEMPLER".
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Af: Jeppe C. Dyre.

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Af: Jacob Mørch Pedersen.
Vejleder: Bent Sørensen

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PEIRCE OG FYSIKKEN".
Af: Peder Voetmann Christiansen

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KUNNE NOK SKABE RØRE I ANDEDAMMEN".
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Fysiklærerforeningen, IMFUFA, RUC.

124/86 "OPGAVESAMLING I MATEMATIK".
Samtlige opgaver stillet i tiden 1974-jan. 1986.

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klassifikation af B-,A- og F-stjerner".
Projektrapport af: Birger Lundgren.

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Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

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ALGEBRA".
Projektrapport af: Pernille Sand, Heine Larsen &
Lars Frandsen.
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Projektrapport af: Niels Jørgensen & Mikael Klintorp.
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Andy Wierød.
Vejledere: Jens Høyrup, Jørgen Vogelius,
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AF: Bent Sørensen.

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen,
Petr Viscör

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSES-
TEORETISKE FORUDSÆTNINGER"
MASTEMATIKSPECIALE: Claus Larsen
Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens
første og andet møde med græsk filosofi"
Projektrapport af Frank Colding Ludvigsen
Vejledere: Historie: Ib Thiersen
Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE
FASTE STOFFER" - Resume af licentiatafhandling
Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og
Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International
Workshop on Teaching Nonlinear Phenomena
at Universities and Schools, "Chaos in
Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

13 9/87 "Machbarkeit nichtbeherrschbarer Technik
durch Fortschritte in der Erkennbarkeit
der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR -
ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With
Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircan Approach

Paper presented at the Symposium on the
Foundations of Modern Physics The Copenhagen
Interpretation 60 Years after the Como Lecture.
Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

145/87 "AIMS AND SCOPE OF APPLICATIONS AND
MODELLING IN MATHEMATICS CURRICULA"

Manuscript of a plenary lecture delivered at
ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

146/87 "BESTEMMELSE AF BULKRESISTIVITETEN I SILICIUM"

- en ny frekvensbaseret målemetode.

Fysikspeciale af Jan Vedde

Vejledere: Niels Boye Olsen & Petr Viščor

147/87 "Rapport om BIS på NAT-BAS"

redigeret af: Mogens Brun Heefelt

148/87 "Naturvidenskabsundervisning med
Samfundsperspektiv"

af: Peter Colding-Jørgensen DLH
Albert Chr. Paulsen

149/87 "In-Situ Measurements of the density of amorphous
germanium prepared in ultra high vacuum"

by: Petr Viščor

150/87 "Structure and the Existence of the first sharp
diffraction peak in amorphous germanium
prepared in UHV and measured in-situ"

by: Petr Viščor

151/87 "DYNAMISK PROGRAMMERING"

Matematikprojekt af:
Birgit Andresen, Keld Nielsen og Jimmy Staal

Vejleder: Mogens Niss