# TEKST NR 145 1987

AIMS AND SCOPE
OF
APPLICATIONS AND MODELLING
IN
MATHEMATICS CURRICULA

Manuscript of a plenary lecture
delivered at ICMTA 3, Kassel, FRG
8.-11.9.1987

MOGENS NISS

TEKSTER fra

IMFUFA ROSKILDE UNIVERSITETSCENTER
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, Roskilde Universitetscenter, Postboks 260,
DK 4000 Roskilde, DENMARK

# AIMS AND SCOPE OF APPLICATIONS AND MODELLING IN MATHEMATICS CURRICULA

by Mogens Niss

IMFUFA tekst nr. 145/87, RUC. 29 sider ISSN 0106-6242

---

ABSTRACT.

The paper is the manuscript of a plenary lecture delivered by the authour at the Third International Conference on the Teaching of Mathematical Modelling and Applications (ICTMA 3), held in Kassel (FRG) 8.-11.9.1987. A shortened version of the present manuscript will be published in the conference proceedings.

The main issues in the paper are: "Why include applications and modelling in mathematics curricula?" and "What aspects of applications and modelling should be dealt with?". Besides,the issue "How should applications and modelling be included in mathematics curricula?" will be touched upon. In order to establish a background for addressing these and related questions, a discussion of the role of mathematics in curricula of different educational levels is given, and a framework for describing and analysing different types of applications of mathematics and of mathematical modelling is outlined.

# AIMS AND SCOPE OF APPLICATIONS AND MODELLING IN MATHEMATICS CURRICULA

Mogens Niss, IMFUFA, Roskilde University Centre

Among the many issues which deserve attention in discussing the aims and scopes of applications and modelling in mathematics education the following are fundamental:

(1) The question of "why?":
Should, for a given educational level, applications tions and modelling be part of the mathematics cur- curriculum? If yes, why?

Provided the answer to this question is "yes", two more questions pose themselves:

(2) The question of "what?":
What content, products, and processes related to applications and modelling should be made object of study, teaching and activity?

(3) The question of "how?":
What means (in terms of curriculum organisation, forms of learning and teaching, material and other resources) would be appropriate for applications and modelling activities, and which are actually at the disposal of teachers and students?

It is to be expected that the answering of each of these questions is dependent on which segment of the educational system is considered. Let us agree - in this but not necessarily in any contexts - to distinguish between three types of mathematics education referring to three different educational levels:

(i): The general mathematics education of the population - predominantly supplied by the school system - which aims at preparing students for their private and social lives as individuals and citizens.

(ii): The mathematical education of people studying subjects or preparing for professions which are not, in a specific sense, mathematical in themselves, but to which mathematics - by being applied in them - has important services to offer. We shall call people in such professions users of mathematics in extra-mathematical professions, or sometimes just "extra-mathematical professionals".

(iii): The mathematical education of people who are to enter mathematics professions proper, as research mathematicians, as general applied mathematicians, or as mathematics teachers for post-elementary levels (i.e. from secondary school and above).

If we combine the three questions with the three educational levels we obtain the following "question matrix" which forms the focus of attention of the present paper:

| question / educational level | Why? | What? | How? |
|---|---|---|---|
| General mathematical education provided by the school | | | |
| Education of users of mathematics in extra-mathematical professions | | | |
| Education of mathematics professionals | | | |

In the sequel I shall attempt to analyse parts of this matrix, without claiming to be exhaustive in my discussion. Along with doing so I shall give my personal answers to a few of the cell questions.

## THE QUESTION OF "WHY?"

The most important arguments - as put forward in literature, at conferences, at educational debates etc. - for including applications and modelling in a given mathematics curriculum seem to be the following five, listed according to increasing specificity with respect to mathematics:

Applications and modelling should be part of the mathematics curriculum in order to

(1) foster among students general creative and problem solving attitudes, activities and competences.

(2) generate, develop and qualify a critical potential in students towards the use (and misuse) of mathematics in extra-mathematical contexts.

(3) prepare students to being able to practice applications and modelling - in other teaching subjects; as private individuals or as citizens, at present or in the future; or in their future professions.

(4) establish a representative and balanced picture of mathematics, its character and role in the world. Such a picture must encompass all essential aspects of mathematics, and the application of mathematics and mathematical modelling in other areas do form one such aspect.

(5) assist students' acquisition and understanding of mathematical concepts, notions, methods, results and topics, either to give a fuller body to them, or to provide motivation for the study of certain mathematical disciplines.

The character and status of these five arguments are not the same. Arguments (1) and (5) relate primarily to educational tactics. Argument (1) focusses on formative aspects of the general education and personal development of students, not on matters specific to mathematics. Mathematics and application and modelling activities are to serve as a vehicle to a general end, rather than to be of independent interest. In principle, any vehicle, be it mathematical or not, serving this end might be relevant to the curriculum.

In aiming at facilitating or improving mathematics teaching, argument (5) is concerned with subject specific teaching tactics. Here applications and modelling form a vehicle to this end, in principle replaceable by any vehicle which is considered effective in this respect. On the other hand, the argument would not make sense if mathematics were dispensed with.

In pursuing the purpose of preparing students to aspects of life outside mathematics education arguments, (2) and (3) deal with general educational strategy. In both arguments students are wanted to come to grips with the actual use of mathematics in the world; in argument (2) in an analytical but possibly "passive" way, in argument (3) in an "active", constructive but possibly "non-reflective" way. To both arguments mathematics and applications and modelling are essential components, not replaceable vehicles.

Argument (4) may be viewed as the subject specific strategic counterpart of the tactical argument (5). It is concerned with students' perception of mathematics as an entity, thus addressing epistemological issues and partly sociological ones too. In contradistinction to the general strategic arguments (2) and (3) which look at the world outside mathematics and views mathematics as a factor in this world, argument (4) looks at mathematics, if also in its totality. So, in argument (4) mathematics and its aspect "applications and modelling" cannot be replaced with something else.

All these arguments can be encountered in discussions on each of the three educational levels identified in the beginning of this paper. Still, for a given level some arguments seem to carry more weight in the didactical debate than do others. A brief way to summarise this is to fill in the "why?"-column of the question-matrix presented earlier:

| | arguments for "why?" |
|---|---|
| General mathematical education provided by the school | (1), (3), (5) |
| Education of users of mathematics in extra-mathematical professions | (3), (5) |
| Education of mathematics professionals | (3), (4), (5) |

If this is a fair summary of the debate with respect to the question of "why?", it appears that arguments (2) and (4) have not gained much attention. (Although (1) occurs only once, as does also (4), it represents a larger amount of attention since it relates to the large body of school mathematics).

In my view this priority is not satisfactory. I shall try to explain why, and to establish an alternative priority. A basis for doing so is needed. The decision to include applications and modelling studies and activities in the mathematics curriculum of a given educational level should be derived from a consideration of the overall purposes of mathematics education for that particular level. Therefore, for each of the three educational levels applied in the present paper, the analysis is commenced by a brief sketch of the role of mathematic and mathematics education at that level. Needless to say, such an analysis cannot avoid involving socio-cultural values. The following analysis reflects mine.

SCHOOL mathematic education first. Several reasons for providing the general population with a mathematics education which goes beyond elementary arithmetic with known figures have been put forward through the ages. Most frequently, the utilitarian contributions of mathematics as an instrument for tackling needs relevant to society have served to justify the presence of post-elementary mathematics in the school curriculum. Sometimes mathematics education has been seen as a vehicle for forming and developing intellectual and other mental capacities (such as logical reasoning) in people, or as a means for creative, including recreational, activity among students. Sometimes mathematics has been viewed as a source for aesthetic experiences, or as a testimony of the cultural achievements of mankind.

Most often a combination of these - and other - reasons has constituted the justification of school mathematics education.

To me the ultimate reason for giving a substantial mathematics education to the general population is that mathematics is being <u>used</u> extensively and ever increasingly in society, for better and for worse, in such a way that people's professions and lives as individuals and citizens are strongly influenced by it. The main purpose of mathematics education is to help them become competent, independent individuals in all aspects of their lives, and not victims, in their association with mathematics in society. This reasoning recognises the dual interest of rising a well-educated labour force and in promoting a democratic devlopment of society.

Also the other reasons mentioned could justify a position for mathematics in the school curriculum but on a much smaller scale than the one it actually and deservedly occupies.

Now, if the mathematics education given in school is to comply with the purpose put in front here, it must provide students with prerequisites for understanding, assessing and handling aspects of the use of mathematics in other areas. The use of mathematics is brought about through the construction and application of mathematical models, even if the modelling process is not always explicit, let alone transparent. If we agree that the prerequisites mentioned are not generated automatically by a mathematics teaching focussing solely on pure mathematical concepts, results and topics, we are led to conclude that applications and modelling should be part of the school mathematics curriculum. In terms of the arguments previously considered it appears that we are invoking <u>arguments (2)</u> and <u>(3)</u>.

This does not imply that arguments (1), (4) and (5) do not have parts to play here, but they are derived arguments of the second order, whereas arguments (2) and (3) regard the importance of applications and modelling as such. In argument (1), application and modelling activities form a vehicle to a general formative end. So, only if we want to pursue this end and are convinced that application and modelling activities are the only means available for the pursuit would argument (1) make applications and modelling an indispensable part of the school mathematics curriculum. As to argument (4), its logical position in this context appears to be the following: If the ultimate reason for giving a mathematics education to all pupils and students in school is that mathematic is being used in different places in society, argument (4) - which stresses applications and modelling as an important facet in the picture of mathematics - adds nothing new and independent to the argumentation. The same is true with argument (5) which focusses on applications and modelling to assist the acqusition of mathematical concepts, results and theories.

The basic reason for providing mathematics education to future users of mathematics in a given <u>EXTRA-MATHEMATICAL PROFESSION</u> is, of course, that mathematics is applied - or applicable - in the preparation for or in the practice of that profession. This is why mathematics is often labelled a "service subject" to such professions and to studies leading up to them.

The term "profession" should be taken to carry its broadest meaning: ranging from trades and vocations in which the role of mathematics is modest (but still present), to academic occupations in areas of research and development to which the involvement of mathematics is crucial.

At first sight it seems easy to establish that applications and modelling should be included in the mathematics curricula preparing for these professions, simply because of argument (3). I would, however, like to add argument (2) to the motivation. The background for this is the following:

When considering the range of areas to which mathematics is a service subject, it is important to keep in mind that the part which mathematics plays in different areas varies very much with the area. We are faced not only with a variation in degree and extent over the disciplines, but also with a variation in character.

In subjects such as physics, astronomy, theoretical chemistry, parts of theoretical economics, and many engineering disciplines, mathematics is integrated in the very formation of the basic scientific concepts and theories, many of which cannot even be formulated without mathematics. So, to these subjects mathematics does not just provide a way, among others, of "dressing" concepts and theories, one is simply not left with the option to leave mathematics aside. In addition to this integrative involvement of mathematics comes the highly complex part which mathematics plays in these subjects in finding solutions to problems, in establishing results etc. It is a general feature that in these disciplines mathematics is demanded to provide exact answers as far as possible, and to the extent exact answers cannot be obtained, exact estimates of errors are required.

To disciplines like biology, most branches of economics etc. mathematics is, although highly relevant and useful, not essential in the same way as with the group of disciplines mentioned above. In biology and economics, theories and results often exist which make sense and can be established and formulated in qualitative terms without the use of mathematics, but which may also benefit strongly from a mathematical formulation. Another major difference between physics and, say, macro-economics is that physical theories are established through a very elaborate interplay between experimentation/observation and exact mathematical model-/theory-building, whereas in macro-economics experiments are not possible in general. So, one would only very rarely require quantitative statements in macro-economics established by means of mathematics to be exact. Rather one would go for and expect qualitative statements. The same holds in biology, even if in biology experimentation is often a possibility. Exact quantitative statements would in general be too much to hope for, mainly because living beings display much more complex behaviours than do "dead" physical systems.

If the part played by mathematics in other areas displays a large variation in character as far as academic disciplines are concerned, the variation becomes even larger when the full range of extra-mathematical professions are considered. This indicates that the application of mathematics in extra-mathematical contexts is not an unproblematic and straightforward affair. On the contrary, applications and modelling involves practical, technical, scientific, philosophical and even political, complications which deserve careful and critical attention. Therefore, the inclusion of applications and modelling in the mathematics curriculum for users in extra-mathematical professions of all kinds should rely also on argument (2). As to arguments (1), (4) and (5) their place in this context is almost identical with their place in the school context, for which reason we shall not comment on them any further at this point.

If, finally, we turn to the mathematics education of MATHEMATICS PROFESSIONALS, we encounter a somewhat different situation. To these professionals and to their functions, the role of mathematics cannot be reduced to its application in other areas, even if it might be its application that constitutes the ultimate interest of the system which employs them. They are mathematicians, whether in pure research, in general applications, or in teaching (beyond the level of elementary arithmetic). By definition, mathematics forms a crucial and indispensable component in their work. In my opinion, however, it is important that they obtain a representative and balanced picture of mathematics in all its aspects, including applications and modelling. Not only do they become more competent research mathematicians, general appliers or teachers of mathematics, if their outlook on mathematics is broader than one having only mathematics as a theoretical edifice in its field of vision. Also their social functions as expert citizens gain in wisdom and hence in quality if they are educated to become broad-minded professionals with sound judgement, rather than just specialists positioned as wheels in a huge social system whose mode of operation they are unconscious of.

---

So, I shall invoke first of all argument (4) and secondly - referring to the above reasoning concerning the varying character of the role of mathematics in different areas - argument (2) to motivate the inclusion of applications and modelling studies in the curriculum for future mathematics professionals. Again, some of the remaining arguments have their parts to play, except - perhaps - argument (1). Argument (3) is derived from arguments (4) and (2): If students are to obtain a genuine and substantial impression of the application of mathematics, they must be prepared as if they were to practice it - at least on a small scale - themselves. (This also has the pragmatic reason that it prepares students to a broader spectrum of occupational possibilities than to the narrow mathematical ones.) Finally, argument (5) is relevant in relation to the education of mathematics professionals to the extent applications and modelling do in fact assist the motivation for or the acquisition and understanding of mathematical concepts, methods, results and topics belonging to that educational level.

Let me summarise my answers to the "why?"-question by listing my main arguments in the "why?"-column:

| | Why? |
|---|---|
| General mathematical education (in the <u>school</u>) | (2), (3) |
| Mathematical education of users of mathematics in <u>extra-mathematical professions</u> | (2), (3) |
| Education of <u>mathematics professionals</u> | (2), (4) |

Although already implicitly given by the preceding discussion, it should be made explicit that these answers imply, for all educational level, the answer "yes!" to the question "Should applications and modelling be part of the mathematics curriculum for that level?"

The arguments, whatever they be, put forward for incorporating applications and modelling studies and activities in a given mathematics curriculum not only serve to justify that incorporation. If taken seriously they influence <u>the way</u> in which applications and modelling are treated in the curriculum and in the mathematics teaching. Applications and modelling work introduced only to motivate and assist students' acquisition of mathematical concepts etc. (argument (5)) is likely to differ from application and modelling activities designed to enable students to practice application and modelling in areas outside mathematics (argument (3)). Therefore the answers to the questions "what?" and "how?" must depend intimately, in principle and in practice, on the arguments given to the question of "why?". This takes us to the next section.

## THE QUESTION OF "WHAT?"

When discussing what content, products and processes related to applications and modelling should be made object of study, teaching and activity, some clarification of terms may prove appropriate. What do we mean by an "application" of mathematics, by "applying" mathematics and by "applied mathematics" and "applicable mathematics"? What do we mean by a mathematical "model", by mathematical "modelling", mathematical "model-building" and by "mathematisation"? And how is all this connected to "problem-solving" and "applied problem solving"? Quite a treatise would be needed if we were to give exhaustive answers to these questions. So, we shall have to content ourselves with giving a few working definitions.

Let us imagine an arbitrary area of extra-mathematical reality (a segment of "real life", whatever that is, or of another discipline). If the area is submitted to any kind of treatment which involves either mathematical notions or concepts, methods, results, topics or theories, we shall speak of the process of applying mathematics to that area. For the result of the process we shall use the term an application of mathematics.

It appears that the term an "application of mathematics" represents the broadest possible activation of mathematics to areas outside mathematics itself. The activation may range in a continuous spectrum, from giving mathematical <u>names</u> to objects and phenomenae (e.g. "triangle" to a pencil drawing), via attributing <u>qualitative</u> and <u>quantitative</u> mathematical <u>properties</u> to them (e.g. the shape of a certain piece of paper is an "isosceles right-angled triangle with hypothenuse 8"), on to obtaining - by means of mathematical <u>methods</u> - <u>results</u> and <u>conclusions</u> concerning the area at issue (e.g. "with an interest rate of 8% p.a. my nominal bank account deposit will have doubled in 9 years"), to establishing full mathematical <u>theories</u> of entire disciplines (e.g. "operator theory" to describe quantum mechanics).

The terms "applied mathematics" and "applicable nathematics" do not carry independent kinds of meaning in the present paper. When used here, they simply indicate those portions of mathematics (from notions to theories) which are, respectively: may be, activated in an applicational situation. Thus, in contradistinction to what is sometimes encountered elsewhere, "applied" or "applicable" mathematics do not refer to specific mathematical topics (e.g. "fluid dynamics", "linear programming", "numerical analysis") which, unlike "pure" topics, are supposed to be of particular applicational relevance.

When a segment of reality is submitted to any kind of treatment by mathematical means - or equivalently put: when mathematics is applied to that segment - a mathematical model is necessarily involved. Very briefly speaking, certain objects, relations between them, and structures, belonging to the area under consideration are selected and translated into mathematical objects, relations and structures, which are then said to <u>represent</u> the original ones. Now, the concept of model can be defined in two different ways. The first possibility is to simply identify a model with the collection, M, of mathematical objects, relations, structures etc., irrespective of what area is being represented by the model and how. The rationale of this option is that a given collection M might serve to model many different areas. In other words, the model is constant, whereas the area modelled varies. The second possibility is to define a <u>model</u> as the triple (A,M,f), where A is the segment of reality under consideration, and f is a mapping which translates certain items of A into items of M. This definition emphasises that a mathematical model is a model of something. We shall adopt it in the present paper, which will not prevent us from using more casual phrases such as "within the model", "properties of the model" and the like, to indicate matters related to M alone when it is convenient and there is no danger of ambiguity.

By <u>modelling</u>, or <u>model-building</u>, we shall understand the full process of constructing a mathematical model of a given area. At a minimum, it comprises: <u>(a)</u> identifying the features of reality which are to be modelled; <u>(b)</u> selecting the objects, relations etc. relevant to this end; and <u>(c)</u> idealising them into shapes suitable for a mathematical representation; <u>(d)</u> choosing a mathematical universe to hold the model (M); <u>(e)</u> performing a translation from reality to mathematics; <u>(f)</u> establishing mathematical relations between the translated objects, accompanied by assumptions and properties; <u>(g)</u> using mathematical methods to obtain mathematical results and conclusions; and (h) interpreting these as results and conclusions concerning the original area. In addition the process may include: (i) assessing the model by confronting it with reality (e.g. observed or predicted data), by comparing it with other models, by relating it to established theory; and finally: (j) building, if necessary, a new or modified model, thus running through the stages (a)-(j) once again.

As regards the term <u>mathematisation</u> it is often used as a synonym for "modelling"/"model-building". We could - and shall - attribute a more precise meaning to it by defining it to be the translation part of the modelling process, i.e. the stages (a)-(e) above.

As was hinted in a previous paragraph, any application of mathematics presupposes a model. From a philosophical point of view an implicit or explicit model is a necessary vehicle for an application of mathematics in some area to be carried out. The term "application of mathematics" is used to emphasise the <u>fact</u> that mathematics is activated towards a certain segment of reality, whereas the term "model" focusses on the <u>framework</u> in which mathematics is activated. There seems to be a tendency to choose "application" in contexts where not much attention is being paid to the underlying model - perhaps because it is considered trivial (as with arithmetic applied to money) or the model has no rivals, or because the model facet is simply ignored. A result of this is that no clear demarcation line is drawn between reality and mathematics. In contrast, the use of the term "model" tends to make the discrimination between reality and model explicit. It indicates that choices, assumptions and idealisations have been made, that thus the model is open to criticism, and that alternative models may exist. This does not imply, however, that any critical analysis of the model, let alone the modelling processes, is actually exercised when the term "model" is used.

Often the reason for activating mathematics towards some extra-mathematical area is that certain questions concerning that area need to be answered, certain problems to be solved. If mathematics is applied in such situations, i.e. if it is used as a means for solving problems, we shall speak of <u>applied problem solving</u>. The term <u>problem solving</u> carries a much wider meaning in not being confined to encompassing extra-mathematical problems only. By using the term "applied problem solving", the problems and the problem solving process are accentuated, whereas neither the fact that mathematics is being used nor the model(s) carrying that use are stressed.

So far in this section we have been exercising conceptual and terminological gymnastics. We shall now address the question of "what?".

There are different modes of making applications and modelling objects of study and activity in mathematics instruction:

<u>I</u> Students may acquire <u>knowledge</u> of

- (a) <u>existing models</u> and applications of mathematics. This may include knowledge of different <u>categories</u> of models, classified according to mathematical characteristics as well as to characteristics regarding the different applicational areas to which the models refer.

- (b) characteristics of the modelling process, either in general or with respect to specific categories of models.

One might say that point I concentrates on those aspects of applications and modelling which students may learn about, in teaching, by reading etc., but not "by doing", whereas the following point, II, stresses the active performing of modelling processes:

II Students may perform modelling themselves, either by

- (a) applying models known to the students to situations which are new to them,

or by

- (b) building new (to the students) models, or modifying known ones.

Finally,

III Students may critically analyse and assess models, existing ones or models they have constructed themselves, with respect to

- (a) their mathematical properties (work inside the model),

or to

- (b) the properties, qualities and bases of justification of the models as representations of given segments of reality.

If, for our present purpose, we define these points to form the aspects of applications and modelling, we may phrase the main question of "what?" as follows:

Which of these aspects of applications and modelling should be included in the mathematics curriculum of a given educational level (as identified in the beginning of the present paper)? To answer this question we will have to implicate the arguments which were put forward to answering the question "why?" in the preceding section. In that section one conclusion, common to all the educational levels considered, was that applications and modelling should be part of the mathematics curriculum in order to stimulate a critical potential in students towards the use of mathematics in extra-mathematical contexts (argument (2)). It seems pretty obvious that to this end students should acquire experiences in analysing and assessing models and modelling processes which are suitable for their educational level. In other words, aspect III should enter into the mathematics curriculum of any level.

Evidently, the capability of analysing and assessing models and modelling must rest on <u>some</u> knowledge of models and modelling. Hence, if for each level mathematics education is to deal with aspect III, it has to comprise also aspect I.

Moreover, it was concluded in the "why?"-section that a further argument for including applications and modelling in the school mathematics curriculum, and in the education of users of mathematics in extra-mathematical professions, was that students should be prepared to practice applications and modelling. Since practising applications and modelling implies performing modelling, and since, as previously stated, we hold the opinion that the ability to perform modelling needs educational training, we are led to infer that also aspect II should be addressed in the mathematics programmes of those educatonal levels.

As to the third level, the education of mathematics professionals, we shall recall a previous consideration, namely that if - in accordance with argument (4) - students are to obtain a genuine and substantial impression of the application of mathematics outside mathematics itself, they must be prepared as if they were to practice it - to some extent - themselves. So, aspect II ought to be part of the mathematics curriculum for mathematics professionals too.

We have, thus, reached the conclusion that <u>all the aspects here defined should be incorporated in the mathematics education given at each of the three levels considered</u>. This does not imply, however, that the aspects are equally important or should be equally time-consuming, or that they materialise in the same way, at different levels. In fact they do not. At school level, for instance, aspects I(a) and II will occupy a predominant position. In the education of extra-mathematical professionals, III(b) and II will be in focus, whereas the education of mathematics professionals will tend to stress aspects I(a) and III(a).

For each aspect adopted in a given mathematics programme several further questions - sub-questions of "what?"-arise. If we take, for instance, aspect I, "acquire knowledge of models and the modelling process", it lays near at hand to ask "what (kind of) knowledge?", or "what (categories of) models?". Likewise, the question "what particular phases of the modelling process should be traversed?" is relevant in relation to aspect II, "performance of modelling". An so forth.

Naturally, it goes far beyond the scope of a single paper to deal, in a comprehensive way, with all relevant issues of this kind, and to determine how the aspects materialise at each level. I shall confine myself to addressing a single point.

In the preceding section it was stressed that the part played by mathematics and mathematical models in different extra-mathematical areas varies strongly in extent and character with the area. The <u>epistemological position</u> of mathematical models varies with respect to:

The <u>purpose</u> of models.
Is the purpose of a given model to understand features, essential or phenomenological, of the area modelled, or is it to create a background for making decisions or taking actions related to that area?

The sources of model <u>construction</u>.
For a given model, what are the conceptual and other connections between the components of the model and the constituents (objects, relations, phenomena) of the area it is supposed to model? How, further, is the model related to theory (well-founded or not) that regards the extra-mathematical substance of the area modelled?

The nature of the <u>statements</u> produced.
Are the statements generated by a given model deterministic or stochastic? Quantitative or qualitative? Exact or approximate?

The <u>verificational</u> <u>status</u> of models.
To what extent is a given model reliable? Is it verified or verifiable? Justifiable? Correct within certain restrictions? A good approximation if interpreted with caution? Incorrect but useful for some purposes? Incorrect and useless? What is its explanatory value? On what foundation can we hope to tackle such questions? Does the model reproduce known data or phenomenae; or does it predict unknown but later discovered ones? If yes, within what range of cases? Is the model embedded in, corroborated by, or at least not in contradiction to established theory of the area at issue? Is the model related to other models in analogous situations? Etc. etc.

Unfortunately, we have to abstain from exploring in any detail examples which may illuminate the outlined epistemological variation of mathematical models. Let me just mention that the way in which plane triangles and their properties in Euclidean geometry serve as models of certain "earthly" physical objects, and the way in which the logistic growth model represents biological populations, are of a quite different nature. Similarly, the models from classical mechanics used to describe satellite orbits differ fundamentally from Poisson processes taken to model the number of costumer arrivals to a super market in certain time intervals. If the triangle and the satellite models were overruled, entire established theories would break down, not just the models. If, for a specific biological population, the logistic growth model failed to hold, well, that model simply has to be replaced with a different one, an event which would not shake the scientific foundation of anything. And the same is true with the costumer arrivals model.

Another kind of epistemological position is held by plane map projection models of (segments of) the surface of the globe. Due to principal mathematical reasons, such models can never reflect all geometrical features of the globe, let alone the planet "Earth". But for a given projection we are able to tell in exact terms how an area of the sphere deviates from its projected picture. Still another situation is encountered in financing. Though it is perfectly possible to model correctly and exactly the amortization of an annuity loan in nominal terms - because the substance concepts involved have already been given a definite mathematical form - the situation changes if we want to model the amortization in real terms, where inflation is involved. By using different inflation assumptions it is possible to model different scenarios to approximate the real course of events.

Numerous examples could be given in the same vein. In fact, every model , carefully analysed, would contribute to illustrate the points made here.

For all three educational levels I claim it to be important that students acquire knowledge of the described variation in epistemological status of mathematical models, over different extra-mathematical contexts. Such knowledge (which belongs to the scope of aspect I(a)) should be based on a number of characteristic model cases, different, of course, from level to level. Another difference between the educational levels may consist in a varying emphasis on the four epistemological facets. While at school level, and in the education of mathematics professionals, particular attention would probably be paid to the source of model construction and to the statements produced by models, the verificational status of models in relation to their purposes is likely to be in the focus of attention in the education of users of mathematics in extra-mathematical professions.

THE QUESTION OF "HOW?"

To deal with the questions of "why?" and "how?" in general terms, without considering the specific educational system and the position of mathematics within that system, which after all provides the framework for mathematics education and hence for applications and modelling work, is a problematic affair. Even more so with the question of "how?", where the abundance of specific boundary conditions and of organisational peculiarities in mathematics programmes tends to make it difficult to obtain a general treatment of sufficient interest. For that reason, and because of space restrictions, we shall leave the question of "how?" by just listing five basic approaches to including applications and modelling in mathematics curricula. The approaches - advanced in the report of the applications and modelling theme group of ICME 5, 1984, published in the congress proceedings (and chiefly written by this author) - are the following:

A1. The two-compartment approach divides the mathematics programme into two parts. The first parts consists of a normal course in pure mathematics establishing a body of topics, concepts, methods and results which are then activated towards applications and modelling work during the second part.

A2. The islands of applications approach is in essence a series of smaller instructional sequences each using the two-compartment approach. Thus a pure mathematics programme is interrupted by "islands" of applications and modelling work in which mathematics developed in the preceding period is activated.

A3. In the mixing approach, application and modelling activities and the formulation of new mathematical concepts and theories are woven together. Elements of applications and modelling motivate and illustrate new pieces of mathematics which, conversely, are used to treat applications and modelling issues

A4. The <u>mathematics curriculum integrated approach</u>. In the mixing approach, the mathematics content which is to be involved in applications and modelling activities is given by and large. In the present approach, the applications and modelling problems come first and mathematics is activated - sought and developed - subsequently. The only thing required is that the problems considered do in fact involve tractable mathematics of relevance to the mathematics curriculum at issue.

A5. The <u>interdisciplinary integrated approach</u> is similar to the preceding one, except that this approach integrates mathematical and extra-mathematical activities within an interdisciplinary framework not making "mathematics" a separate compartment.

The decision on which approach to adopt to installing applications and modelling in a given curriculum has to depend on a multitude of system specific circumstances. But above all, the decision also has to be closely dependent on how the questions of "why?" and "what?" are answered for that particular curriculum. We shall not go into these matters any further.

## CONCLUDING REMARKS

A predominant portion of the present paper has been spent to advance elements of an apparatus for analysing the role of applications and modelling in mathematics education. I am fully aware of the fact that this endeavour has contributed to making the exposition rather dry. However:

In recent years much attention has been devoted to introducing applications and modelling in the theory and practice of mathematics curricula. Papers and books have been written, discussions have been conducted, and conferences been held. Entire educational programmes have been devised and some carried out. Experimental courses have been tried out. Teaching materials have been developed many of which have been made publicly available. Ideas, materials and opinions have been exchanged. In spite of the fact that all this has taken place at the frontier of mathematics education in theory and in practice rather than in common day-to-day mathematics teaching, I think it is fair to speak of a successful movement of applications and modelling influencing mathematics education at all levels pretty strongly. It is not unusual with movements in progress that more attention is paid to the promotion of ideas and messages than to analytical reflection. In my opninion, the applications and modelling, movement - in which I am a participant - has reached a stage where a foundation for it in relation to mathematics education is needed. It is far from being clamied that such a foundation is provided by the present paper. Neither is it claimed that this delivers the first contribution in that direction. A sound foundation would require a comprehensive analysis of - at least: the epistemology of mathematics, applications and modelling; of mathematics education; and of education. It has been possible, here, only to sketch elements of such an analysis. One point deserves to be emphasised: The aims and taks of applications and modelling in mathematics curricula should be viewed as forming part of the aims and tasks of mathematics education, which in turn form part of the aims and tasks of education as preparing its receivers to live and act in society. This does not imply that the aims and taks of applications and modelling can be reduced to the aims and taks of education in general.

Looked upon from an appropriately distant point of view, the ultimate raison d'etre of exercising mathematics and mathematics education at a large scale in society is that mathematics is important in the pursuit of extra-mathematical purposes, brought about through its application to other areas. In no way does this mean to say that purely mathematical activities should be banned. Of course not. What it does say, however, is that no main level of mathematics education should be devoid of an applicational perspective. Therefore it was concluded in the preceding sections that basically the same arguments for including applications and modelling studies in the mathematics curriculum are valid for any educational level, from school to university mathematics, and that the same aspects of applications and modelling ought to be dealt with at all levels. What varies between the levels, and between different programmes, is the way in which these aspects are materialised, in terms - for instance - of the specific applicational areas and model(ling) cases considered, the mathematical and extra-mathematcal treatment they are submitted to, the modelling stages traversed etc.

Models and modelling may be made objects of study, teaching and activity in a multitude of ways. In conclusion one thing should be kept in mind: models are designed to model something - not to be confused with something unique. For a true perspective on applications and modelling to be generated in mathematics education, at whatever level, not only the mathematical components proper need devotion. Extra-mathematical substance has to be taken seriously too. Certainly, artificial applications and modelling situations may possess great educational value, but sometimes mathematics education must confront students with the exciting and mysterious meeting points between mathematics and reality, so difficult but so important to understand.

Mogens Niss, September 1987

References:

Mogens Niss: Applications and Modelling in the mathematics curriculum - State and trends.
Tekst nr. 1o8, 1985, Tekster fra IMFUFA, RUC
To appear in slightly modified form in Int. J. Math. Educ. Sci. Technol.

Proceedings of the Fifth International Congress on Mathematical Education (ed. Marjorie Carss), Report on Theme Group 6, pp 197-211. Birkhäuser Boston, Inc. 1986.

1/78 "TANKER OM EN PRAKSIS" - et matematikprojekt.
Projektrapport af: Anne Jensen, Lena Lindenskov, Marianne Kesselhahn og Nicolai Lomholt.
Vejleder: Anders Madsen

2/78 "OPTIMERING" - Menneskets forøgede beherskelsesmuligheder af natur og samfund.
Projektrapport af: Tom J. Andersen, Tommy R. Andersen, Gert Krenøe og Peter H. Lassen
Vejleder: Bernhelm Boss.

3/78 "OPGAVESAMLING", breddekursus i fysik.
Af: Lasse Rasmussen, Aage Bonde Kræmmer og Jens Højgaard Jensen.

4/78 "TRE ESSAYS" - om matematikundervisning, matematiklæreruddannelsen og videnskabsrindalismen.
Af: Mogens Niss
Nr. 4 er p.t. udgået.

5/78 "BIBLIOGRAFISK VEJLEDNING til studiet af DEN MODERNE FYSIKS HISTORIE".
Af: Helge Kragh.
Nr. 5 er p.t. udgået.

6/78 "NOGLE ARTIKLER OG DEBATINDLÆG OM - læreruddannelse og undervisning i fysik, og - de naturvidenskabelige fags situation efter studenteroprøret".
Af: Karin Beyer, Jens Højgaard Jensen og Bent C. Jørgensen.

7/78 "MATEMATIKKENS FORHOLD TIL SAMFUNDSØKONOMIEN".
Af: B.V. Gnedenko.
Nr. 7 er udgået.

8/78 "DYNAMIK OG DIAGRAMMER". Introduktion til energy-bond-graph formalismen.
Af: Peder Voetmann Christiansen.

9/78 "OM PRAKSIS' INDFLYDELSE PÅ MATEMATIKKENS UDVIKLING". - Motiver til Kepler's: "Nova Stereometria Doliorum Vinariom".
Projektrapport af: Lasse Rasmussen.
Vejleder: Anders Madsen.

---

10/79 "TERMODYNAMIK I GYMNASIET".
Projektrapport af: Jan Christensen og Jeanne Mortensen.
Vejledere: Karin Beyer og Peder Voetmann Christiansen.

11/79 "STATISTISKE MATERIALER".
Af: Jørgen Larsen.

12/79 "LINEÆRE DIFFERENTIALLIGNINGER OG DIFFERENTIALLIGNINGSSYSTEMER".
Af: Mogens Brun Heefelt.
Nr. 12 er udgået.

13/79 "CAVENDISH'S FORSØG I GYMNASIET".
Projektrapport af: Gert Kreinøe.
Vejleder: Albert Chr. Paulsen.

14/79 "BOOKS ABOUT MATHEMATICS: History, Philosophy, Education, Models, System Theory, and Works of".
Af: Else Høyrup.
Nr. 14 er p.t. udgået.

15/79 "STRUKTUREL STABILITET OG KATASTROFER i systemer i og udenfor termodynamisk ligevægt".
Specialeopgave af: Leif S. Striegler.
Vejleder: Peder Voetmann Ckristiansen.

16/79 "STATISTIK I KRÆFTFORSKNINGEN".
Projektrapport af: Michael Olsen og Jørn Jensen.
Vejleder: Jørgen Larsen.

17/79 "AT SPØRGE OG AT SVARE i fysikundervisningen".
Af: Albert Christian Paulsen.

18/79 "MATHEMATICS AND THE REAL WORLD", Proceedings af an International Workshop, Roskilde University Centre, Denmark, 1978. Preprint.
Af: Bernhelm Booss og Mogens Niss (eds.)

19/79 "GEOMETRI, SKOLE OG VIRKELIGHED".
Projektrapport af: Tom J. Andersen, Tommy R. Andersen og Per H.H. Larsen.
Vejleder: Mogens Niss.

20/79 "STATISTISKE MODELLER TIL BESTEMMELSE AF SIKRE DOSER FOR CARCINOGENE STOFFER".
Projektrapport af: Michael Olsen og Jørn Jensen.
Vejleder: Jørgen Larsen

21/79 "KONTROL I GYMNASIET-FORMÅL OG KONSEKVENSER".
Projektrapport af: Crilles Bacher, Per S.Jensen, Preben Jensen og Torben Nysteen.

22/79 "SEMIOTIK OG SYSTEMEGENSKABER (1)".
1-port lineært response og støj i fysikken.
Af: Peder Voetmann Christiansen.

23/79 "ON THE HISTORY AF EARLY WAVE MECHANICS - with special emphasis on the role af realitivity".
Af: Helge Kragh.

---

24/80 a+b "MATEMATIKOPFATTELSER HOS 2.G'ERE".
1. En analyse. 2. Interviewmateriale.
Projektrapport af: Jan Christensen og Knud Lindhardt Rasmussen.
Vejleder: Mogens Niss.

25/80 "EKSAMENSOPGAVER", Dybdemodulet/fysik 1974-79.

26/80 "OM MATEMATISKE MODELLER".
En projektrapport og to artikler.
Af: Jens Højgaard Jensen m.fl.

27/80 "METHODOLOGY AND PHILOSOPHY AF SCIENCE IN PAUL DIRAC's PHYSICS".
Af: Helge Kragh.

28/80 "DIELEKTRISK RELAXATION - et forslag til en ny model bygget på væskernes viscoelastiske egenskaber".
Projektrapport af: Gert Kreinøe.
Vejleder: Niels Boye Olsen.

29/80 "ODIN - undervisningsmateriale til et kursus i differentialligningsmodeller".
Projektrapport af: Tommy R. Andersen, Per H.H. Larsen og Peter H. Lassen.
Vejleder: Mogens Brun Heefelt.

30/80 "FUSIONSENERGIEN - - - ATOMSAMFUNDETS ENDESTATION".
Af: Oluf Danielsen.
Nr. 30 er udgået.

31/80 "VIDENSKABSTEORETISKE PROBLEMER VED UNDERVISNINGSSYSTEMER BASERET PÅ MÆNGDELÆRE".
Projektrapport af: Troels Lange og Jørgen Karrebæk.
Vejleder: Stig Andur Pedersen.
Nr. 31 er p.t. udgået.

32/80 "POLYMERE STOFFERS VISCOELASTISKE EGENSKABER - BELYST VED HJÆLP AF MEKANISKE IMPEDANSMÅLINGER MÖSSBAUEREFFEKTMÅLINGER".
Projektrapport af: Crilles Bacher og Preben Jensen.
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

33/80 "KONSTITUERING AF FAG INDEN FOR TEKNISK - NATURVIDENSKABELIGE UDDANNELSER. I-II".
Af: Arne Jakobsen.

34/80 "ENVIRONMENTAL IMPACT AF WIND ENERGY UTILIZATION".
ENERGY SERIES NO. I.
Af: Bent Sørensen
Nr. 34 er udgået.

35/80 "HISTORISKE STUDIER I DEN NYERE ATOMFYSIKS UDVIKLING".
Af: Helge Kragh.

36/80 "HVAD ER MENINGEN MED MATEMATIKUNDERVISNINGEN?".
Fire artikler.
Af: Mogens Niss.

37/80 "RENEWABLE ENERGY AND ENERGY STORAGE".
ENERGY SERIES NO. 2.
Af: Bent Sørensen.

---

38/81 "TIL EN HISTORIETEORI OM NATURERKENDELSE, TEKNOLOGI OG SAMFUND".
Projektrapport af: Erik Gade, Hans Hedal, Henrik Lau og Finn Physant.
Vejledere: Stig Andur Pedersen, Helge Kragh og Ib Thiersen.
Nr. 38 er p.t. udgået.

39/81 "TIL KRITIKKEN AF VÆKSTØKONOMIEN".
Af: Jens Højgaard Jensen.

40/81 "TELEKOMMUNIKATION I DANMARK - oplæg til en teknologivurdering".
Projektrapport af: Arne Jørgensen, Bruno Petersen og Jan Vedde.
Vejleder: Per Nørgaard.

41/81 "PLANNING AND POLICY CONSIDERATIONS RELATED TO THE INTRODUCTION OF RENEWABLE ENERGY SOURCES INTO ENERGY SUPPLY SYSTEMS".
ENERGY SERIES NO. 3.
Af: Bent Sørensen.

42/81 "VIDENSKAB TEORI SAMFUND - En introduktion til materialistiske videnskabsopfattelser".
Af: Helge Kragh og Stig Andur Pedersen.

43/81 1."COMPARATIVE RISK ASSESSMENT OF TOTAL ENERGY SYSTEMS".
2."ADVANTAGES AND DISADVANTAGES OF DECENTRALIZATION".
ENERGY SERIES NO. 4.
Af: Bent Sørensen.

44/81 "HISTORISKE UNDERSØGELSER AF DE EKSPERIMENTELLE FORUDSÆTNINGER FOR RUTHERFORDS ATOMMODEL".
Projektrapport af: Niels Thor Nielsen.
Vejleder: Bent C. Jørgensen.

---

45/82 Er aldrig udkommet.

46/82 "EKSEMPLARISK UNDERVISNING OG FYSISK ERKENDESE-
1+11 ILLUSTRERET VED TO EKSEMPLER".
Projektrapport af: Torben O.Olsen, Lasse Rasmussen og Niels Dreyer Sørensen.
Vejleder: Bent C. Jørgensen.

47/82 "BARSEBÄCK OG DET VÆRST OFFICIELT-TÆNKELIGE UHELD".
ENERGY SERIES NO. 5.
Af: Bent Sørensen.

48/82 "EN UNDERSØGELSE AF MATEMATIKUNDERVISNINGEN PÅ ADGANGSKURSUS TIL KØBENHAVNS TEKNIKUM".
Projektrapport af: Lis Eilertzen, Jørgen Karrebæk, Troels Lange, Preben Nørregaard, Lissi Pedesen, Laust Rishøj, Lill Røn og Isac Showiki.
Vejleder: Mogens Niss.

49/82 "ANALYSE AF MULTISPEKTRALE SATELLITBILLEDER".
Projektrapport af: Preben Nørregaard.
Vejledere: Jørgen Larsen og Rasmus Ole Rasmussen.

50/82 "HERSLEV - MULIGHEDER FOR VEDVARENDE ENERGI I EN LANDSBY".
ENERGY SERIES NO. 6.
Rapport af: Bent Christensen, Bent Hove Jensen, Dennis B. Møller, Bjarne Laursen, Bjarne Lillethorup og Jacob Mørch Pedersen.
Vejleder: Bent Sørensen.

51/82 "HVAD KAN DER GØRES FOR AT AFHJÆLPE PIGERS BLOKERING OVERFOR MATEMATIK ?"
Projektrapport af: Lis Eilertzen, Lissi Pedersen, Lill Røn og Susanne Stender.

52/82 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

53/82 "THE CONSTITUTION OF SUBJECTS IN ENGINEERING EDUCATION".
Af: Arne Jacobsen og Stig Andur Pedersen.

54/82 "FUTURES RESEARCH" - A Philosophical Analysis of Its Subject-Matter and Methods.
Af: Stig Andur Pedersen og Johannes Witt-Hansen.

55/82 "MATEMATISKE MODELLER" - Litteratur på Roskilde Universitetsbibliotek.
En biografi.
Af: Else Høyrup.

Vedr. tekst nr. 55/82 se også tekst nr. 62/83.

56/82 "EN - TO - MANGE" -
En undersøgelse af matematisk økologi.
Projektrapport af: Troels Lange.
Vejleder: Anders Madsen.

---

57/83 "ASPECT EKSPERIMENTET"-
Skjulte variable i kvantemekanikken?
Projektrapport af: Tom Juul Andersen.
Vejleder: Peder Voetmann Christiansen.
Nr. 57 er udgået.

58/83 "MATEMATISKE VANDRINGER" - Modelbetragtninger over spredning af dyr mellem småbiotoper i agerlandet.
Projektrapport af: Per Hammershøj Jensen og Lene Vagn Rasmussen.
Vejleder: Jørgen Larsen.

59/83 "THE METHODOLOGY OF ENERGY PLANNING".
ENERGY SERIES NO. 7.
Af: Bent Sørensen.

60/83 "MATEMATISK MODEKSPERTISE"- et eksempel.
Projektrapport af: Erik O. Gade, Jørgen Karrebæk og Preben Nørregaard.
Vejleder: Anders Madsen.

61/83 "FYSIKS IDEOLOGISKE FUNKTION, SOM ET EKSEMPEL PÅ EN NATURVIDENSKAB - HISTORISK SET".
Projektrapport af: Annette Post Nielsen.
Vejledere: Jens Høyrup, Jens Højgaard Jensen og Jørgen Vogelius.

62/83 "MATEMATISKE MODELLER" - Litteratur på Roskilde Universitetsbibliotek.
En biografi 2. rev. udgave.
Af: Else Høyrup.

63/83 "GREATING ENERGY FUTURES:A SHORT GUIDE TO ENERGY PLANNING".
ENERGY SERIES No. 8.
Af: David Crossley og Bent Sørensen.

64/83 "VON MATEMATIK UND KRIEG".
Af: Berhelm Booss og Jens Høyrup.

65/83 "ANVENDT MATEMATIK - TEORI ELLER PRAKSIS".
Projektrapport af: Per Hedegård Andersen, Kirsten Habekost, Carsten Holst-Jensen, Annelise von Moos, Else Marie Pedersen og Erling Møller Pedersen.
Vejledere: Bernhelm Booss og Klaus Grünbaum.

66/83 "MATEMATISKE MODELLER FOR PERIODISK SELEKTION I ESCHERICHIA COLI".
Projektrapport af: Hanne Lisbet Andersen, Ole Richard Jensen og Klavs Frisdahl.
Vejledere: Jørgen Larsen og Anders Hede Madsen.

67/83 "ELEPSOIDE METODEN - EN NY METODE TIL LINEÆR PROGRAMMERING?"
Projektrapport af: Lone Billmann og Lars Boye.
Vejleder: Mogens Brun Heefelt.

68/83 "STOKASTISKE MODELLER I POPULATIONSGENETIK"
- til kritikken af teoriladede modeller.
Projektrapport af: Lise Odgård Gade, Susanne Hansen, Michael Hviid og Frank Mølgård Olsen.
Vejleder: Jørgen Larsen.

69/83 "ELEVFORUDSÆTNINGER I FYSIK"
- en test i 1.g med kommentarer.
Af: Albert C. Paulsen.

70/83 "INDLÆRINGS - OG FORMIDLINGSPROBLEMER I MATEMATIK PÅ VOKSENUNDERVISNINGSNIVEAU".
Projektrapport af: Hanne Lisbet Andersen, Torben J. Andreasen, Svend Åge Houmann, Helle Glerup Jensen, Keld Fl. Nielsen, Lene Vagn Rasmussen.
Vejleder: Klaus Grünbaum og Anders Hede Madsen.

71/83 "PIGER OG FYSIK"
- et problem og en udfordring for skolen?
Af: Karin Beyer, Sussanne Blegaa, Birthe Olsen, Jette Reich og Mette Vedelsby.

72/83 "VERDEN IFØLGE PEIRCE" - to metafysiske essays, om og af C.S Peirce.
Af: Peder Voetmann Christiansen.

73/83 ""EN ENERGIANALYSE AF LANDBRUG"
- økologisk contra traditionelt.
ENERGY SERIES NO. 9
Specialeopgave i fysik af: Bent Hove Jensen.
Vejleder: Bent Sørensen.

---

74/84 "MINIATURISERING AF MIKROELEKTRONIK" - om videnskabeliggjort teknologi og nytten af at lære fysik.
Projektrapport af: Bodil Harder og Linda Szkotak Jensen.
Vejledere: Jens Højgaard Jensen og Bent C. Jørgensen.

75/84 "MATEMATIKUNDERVISNINGEN I FREMTIDENS GYMNASIUM"
- Case: Lineær programmering.
Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank Mølgaard Olsen.
Vejledere: Mogens Brun Heefelt og Jens Bjørneboe.

76/84 "KERNEKRAFT I DANMARK?" - Et høringssvar indkaldt af miljøministeriet, med kritik af miljøstyrelsens rapporter af 15. marts 1984.
ENERGY SERIES No. 1o
Af: Niels Boye Olsen og Bent Sørensen.

77/84 "POLITISKE INDEKS - FUP ELLER FAKTA?"
Opinionsundersøgelser belyst ved statistiske modeller.
Projektrapport af: Svend Åge Houmann, Keld Nielsen og Susanne Stender.
Vejledere: Jørgen Larsen og Jens Bjørneboe.

78/84 "JÆVNSTRØMSLEDNINGSEVNE OG GITTERSTRUKTUT I AMORFT GERMANIUM".
Specialrapport af: Hans Hedal, Frank C. Ludvigsen og Finn C. Physant.
Vejleder: Niels Boye Olsen.

79/84 "MATEMATIK OG ALMENDANNELSE".
Projektrapport af: Henrik Coster, Mikael Wennerberg Johansen, Povl Kattler, Birgitte Lydholm og Morten Overgaard Nielsen.
Vejleder: Bernhelm Booss.

80/84 "KURSUSMATERIALE TIL MATEMATIK B".
Af: Mogens Brun Heefelt.

81/84 "FREKVENSAFHÆNGIG LEDNINGSEVNE I AMORFT GERMANIUM".
Specialerapport af: Jørgen Wind Petersen og Jan Christensen.
Vejleder: Niels Boye Olsen.

82/84 "MATEMATIK - OG FYSIKUNDERVISNINGEN I DET AUTOMATISEREDE SAMFUND".
Rapport fra et seminar afholdt i Hvidovre 25-27 april 1983.
Red.: Jens Højgaard Jensen, Bent C. Jørgensen og Mogens Niss.

83/84 "ON THE QUANTIFICATION OF SECURITY":
PEACE RESEARCH SERIES NO. I
Af: Bent Sørensen
nr. 83 er p.t. udgået

84/84 "NOGLE ARTIKLER OM MATEMATIK, FYSIK OG ALMENDANNELSE".
Af: Jens Højgaard Jensen, Mogens Niss m. fl.

85/84 "CENTRIFUGALREGULATORER OG MATEMATIK".
Specialerapport af: Per Hedegård Andersen, Carsten Holst-Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Stig Andur Pedersen.

86/84 "SECURITY IMPLICATIONS OF ALTERNATIVE DEFENSE OPTIONS FOR WESTERN EUROPE".
PEACE RESEARCH SERIES NO. 2
Af: Bent Sørensen.

87/84 "A SIMPLE MODEL OF AC HOPPING CONDUCTIVITY IN DISORDERED SOLIDS".
Af: Jeppe C. Dyre.

88/84 "RISE, FALL AND RESURRECTION OF INFINITESIMALS".
Af: Detlef Laugwitz.

89/84 "FJERNVARMEOPTIMERING".
Af: Bjarne Lillethorup og Jacob Mørch Pedersen.

90/84 "ENERGI I 1.G - EN TEORI FOR TILRETTELÆGGELSE".
Af: Albert Chr. Paulsen.

---

91/85 "KVANTETEORI FOR GYMNASIET".
1. Lærervejledning
Projektrapport af: Biger Lundgren, Henning Sten Hansen og John Johansson.
Vejleder: Torsten Meyer.

92/85 "KVANTETEORI FOR GYMNASIET".
2. Materiale
Projektrapport af: Biger Lundgren, Henning Sten Hansen og John Johansson.
Vejleder: Torsten Meyer.

93/85 "THE SEMIOTICS OF QUANTUM - NON - LOCALITY".
Af: Peder Voetmann Christiansen.

94/85 "TREENIGHEDEN BOURBAKI - generalen, matematikeren og ånden".
Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank M. Olsen.
Vejleder: Mogens Niss.

95/85 "AN ALTERNATIV DEFENSE PLAN FOR WESTERN EUROPE".
PEACE RESEARCH SERIES NO. 3
Af: Bent Sørensen

96/85 "ASPEKTER VED KRAFTVARMEFORSYNING".
Af: Bjarne Lilletorup.
Vejleder: Bent Sørensen.

97/85 "ON THE PHYSICS OF A.C. HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.

98/85 "VALGMULIGHEDER I INFORMATIONSALDEREN".
Af: Bent Sørensen.

99/85 "Der er langt fra Q til R".
Projektrapport af: Niels Jørgensen og Mikael Klintorp.
Vejleder: Stig Andur Pedersen.

100/85 "TALSYSTEMETS OPBYGNING".
Af: Mogens Niss.

101/85 "EXTENDED MOMENTUM THEORY FOR WINDMILLS IN PERTURBATIVE FORM".
Af: Ganesh Sengupta.

102/85 OPSTILLING OG ANALYSE AF MATEMATISKE MODELLER, BELYST VED MODELLER OVER KØERS FODEROPTAGELSE OG - OMSÆTNING".
Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn og Susanne Stender.
Vejleder: Klaus Grünbaum.

103/85 "ØDSLE KOLDKRIGERE OG VIDENSKABENS LYSE IDEER".
Projektrapport af: Niels Ole Dam og Kurt Jensen.
Vejleder: Bent Sørensen.

104/85 "ANALOGREGNEMASKINEN OG LORENZLIGNINGER".
Af: Jens Jæger.

105/85 "THE FREQUENCY DEPENDENCE OF THE SPECIFIC HEAT AF THE GLASS REANSITION".
Af: Tage Christensen.

"A SIMPLE MODEL AF AC HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.
Contributions to the Third International Conference on the Structure of Non - Crystalline Materials held in Grenoble July 1985.

106/85 "QUANTUM THEORY OF EXTENDED PARTICLES".
Af: Bent Sørensen.

107/85 "EN MYG GØR INGEN EPIDEMI",
- flodblindhed som eksempel på matematisk modellering af et epidemiologisk problem.
Projektrapport af: Per Hedegård Andersen, Lars Boye, CarstenHolst Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Jesper Larsen.

108/85 "APPLICATIONS AND MODELLING IN THE MATEMATICS CURRICULUM" - state and trends -
Af: Mogens Niss.

109/85 "COX I STUDIETIDEN" - Cox's regressionsmodel anvendt på studenteroplysninger fra RUC.
Projektrapport af: Mikael Wennerberg Johansen, Poul Katler og Torben J. Andreasen.
Vejleder: Jørgen Larsen.

110/85 "PLANNING FOR SECURITY".
Af: Bent Sørensen

111/85 "JORDEN RUNDT PÅ FLADE KORT".
Projektrapport af: Birgit Andresen, Beatriz Quinones og Jimmy Staal.
Vejleder: Mogens Niss.

112/85 "VIDENSKABELIGGØRELSE AF DANSK TEKNOLOGISK INNOVATION FREM TIL 1950 - BELYST VED EKSEMPLER".
Projektrapport af: Erik Odgaard Gade, Hans Hedal, Frank C. Ludvigsen, Annette Post Nielsen og Finn Physant.
Vejleder: Claus Bryld og Bent C. Jørgensen.

113/85 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS 11".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

114/85 "ANVENDELSE AF GRAFISKE METODER TIL ANALYSE AF KONTIGENSTABELLER".
Projektrapport af: Lone Biilmann, Ole R. Jensen og Anne-Lise von Moos.
Vejleder: Jørgen Larsen.

115/85 "MATEMATIKKENS UDVIKLING OP TIL RENÆSSANCEN".
Af: Mogens Niss.

116/85 "A PHENOMENOLOGICAL MODEL FOR THE MEYER-NELDEL RULE".
Af: Jeppe C. Dyre.

117/85 "KRAFT & FJERNVARMEOPTIMERING"
Af: Jacob Mørch Pedersen.
Vejleder: Bent Sørensen

118/85 "TILFÆLDIGHEDEN OG NØDVENDIGHEDEN IFØLGE PEIRCE OG FYSIKKEN".
Af: Peder Voetmann Christiansen

---

119/86 "DET ER GANSKE VIST - - EUKLIDS FEMTE POSTULAT KUNNE NOK SKABE RØRE I ANDEDAMMEN".
Af: Iben Maj Christiansen
Vejleder: Mogens Niss.

120/86 "ET ANTAL STATISTISKE STANDARDMODELLER".
Af: Jørgen Larsen

121/86 "SIMULATION I KONTINUERT TID".
Af: Peder Voetmann Christiansen.

122/86 "ON THE MECHANISM OF GLASS IONIC CONDUCTIVITY".
Af: Jeppe C. Dyre.

123/86 "GYMNASIEFYSIKKEN OG DEN STORE VERDEN".
Fysiklærerforeningen, IMFUFA, RUC.

124/86 "OPGAVESAMLING I MATEMATIK".
Samtlige opgaver stillet i tiden 1974-jan. 1986.

125/86 "UVBY,β - systemet - en effektiv fotometrisk spektralklassifikation af B-,A- og F-stjerner".
Projektrapport af: Birger Lundgren.

126/86 "OM UDVIKLINGEN AF DEN SPECIELLE RELATIVITETSTEORI".
Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

127/86 "GALOIS' BIDRAG TIL UDVIKLINGEN AF DEN ABSTRAKTE ALGEBRA".
Projektrapport af: Pernille Sand, Heine Larsen & Lars Frandsen.
Vejleder: Mogens Niss.

128/86 "SMÅKRYB" - om ikke-standard analyse.
Projektrapport af: Niels Jørgensen & Mikael Klintorp.
Vejleder: Jeppe Dyre.

129/86 "PHYSICS IN SOCIETY"
Lecture Notes 1983 (1986)
Af: Bent Sørensen

130/86 "Studies in Wind Power"
Af: Bent Sørensen

131/86 "FYSIK OG SAMFUND" - Et integreret fysik/historieprojekt om naturanskuelsens historiske udvikling og dens samfundsmæssige betingethed.
Projektrapport af: Jakob Heckscher, Søren Brønd, Andy Wierød.
Vejledere: Jens Høyrup, Jørgen Vogelius, Jens Højgaard Jensen.

132/86 "FYSIK OG DANNELSE"
Projektrapport af: Søren Brønd, Andy Wierød.
Vejledere: Karin Beyer, Jørgen Vogelius.

133/86 "CHERNOBYL ACCIDENT: ASSESSING THE DATA.
ENERGY SERIES NO. 15.
AF: Bent Sørensen.

---

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen, Petr Viščör

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSESTEORETISKE FORUDSÆTNINGER"
MASTEMATIKSPECIALE: Claus Larsen
Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens første og andet møde med græsk filosofi"
Projektrapport af Frank Colding Ludvigsen
Vejledere: Historie: Ib Thiersen
Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE FASTE STOFFER" - Resume af licentiatafhandling
Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International Workshop on Teaching Nonlinear Phenomena at Universities and Schools, "Chaos in Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

139/87 "Machbarkeit nichtbeherrschbarer Technik durch Fortschritte in der Erkennbarkeit der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR - ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt