TEKST NR 101 1985

EXTENDED MOMENTUM THEORY FOR WINDMILLS

IN PERTURBATIVE FORM

GANESH SENGUPTA

TEKSTER fra

IMFUFA ROSKILDE UNIVERSITETSCENTER
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES
FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, Roskilde Universitetscenter, Postboks 260, 4000 Roskilde

Extended Momentum Theory for Windmills in perturbative form

Ganesh Sengupta
Niels Bohr Institute, Blegdamsvej 17, DK 2100 Copenhagen Ø

IMFUFU tekst nr. 101/1985 20 sider ISSN 0106 - 6242

Abstract:

Momentum theory, in this paper, is presented in a more consistent form. Besides considering axial and rotational velocity of the slip stream (general momentum theory), the present treatment also includes drag. The different results belonging to the different theories (approximations) are reproduced from the general expression of the present paper, when the corresponding approximations are applied (perturbative formalism). The implications of the results, deduced above from analytical treatment, are narrated geometrically.

Uddrag af fysikspeciale

Vejleder: Peder Voetmann Christiansen

Introduction:

The present interest of search of alternative energy resources to supplement or replace the fossil energy, has increased the effort to utilise the wind energy. This has motivated a very wide activity for the windmill theory in terms of more minute and detailed study, application and further development through research.

In the present work, we study a very important theory for windmill called Momentum Theory or actuator disc theory. Momentum theory makes it possible to assess the average power output of a windmill or windturbine or often termed as rotor, without the knowledge of its geometry. The wind turbine, in momentum theory, is assumed to be a fictive mechanism, the actuator disc, which absorbes energy from the wind resulting in its slowing down. This slowing down of the wind velocity (axial) produces a divergence of the streamlines in the flow directions and causing air to spil over the tip of the turbine, see fig. 1. We also see that there is a discontinuity in pressure across the disc plane; this is a necessary condition for energy transfer to the wind turbine.

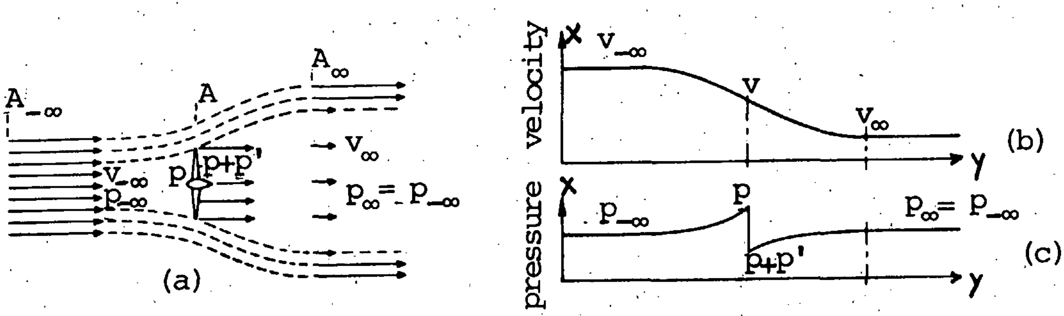

Fig.1. (a): idealised (laminar) flow lines, (b):axial velocity curve along the rotational axis of the turbine and (c): pressure curve along the rotational axis.

In the windmill problem, the determination of the induced axial and rotational velocity is essential; the induced velocity affects the mass flow through the disc plane, and the rotational velocity of the wind behind the disc plane, termed as slipstream, determines the torque acting on the turbine. These induced velocities are expressed by interference factors a and a', defined by

$$v=(1-a)v_{-\infty} \quad \text{and} \quad \omega=2a'\Omega$$

The rotational velocity imparted to the slip-stream is given by $a'\Omega$. The rotational velocity of the slip-stream is termed as ω. Since one half of ω is induced to the slip-stream by the rotor, $\omega/2=a'\Omega$. This implication comes from the demand, that the airflow upstream from the rotor plane has no rotational component.

One important result of the momentum theory is that the axial velocity on the disc plane is the average of the axial velocities far up and down stream. In other words half of the total slowing down of the axial velocity is reached at the disc plane.

Besides the loss of axial velocity, the wind behind the turbine acquires rotation also due to conservation of angular momentum.

The momentum theory exist at different level of approximations. The level of approximation, where only slowing down of the axial wind velocity is taken into consideration, is called AXIAL MOMENTUM THEORY. In the next level, where rotation of the slipstream is also considered it is termed as GENERAL MOMENTUM THEORY. In textbooks (1,2,3), the two theories are treated seperately and independently as two different theories.

But we know that we have only one theory. The present paper deduces expressions for the momentum theory in general form, and then reproduces the results, belonging to the different theories, axial, general momentum theory, by insertion of the respective approximations (perturbative presentation). Besides this generalisation, the present paper makes an extension by including drag. The present paper presents the results in a more comprehensive manner, making it possible to study the characteristic of the important variables and parameters through analysis.

In contradistinction to standard litterature (1,2,3), the present paper includes the equation of the conservation of energy. This enables us to express the unknown variable, rotation of the slipstream, through other known and unknown variables, like $v_{-\infty}$, v_{∞}, Ω etc.

After deduction of the results and discussion of their behaviour analytically, the present paper further treats the various results geometrically to make the implication of the inclusion of the different factors (rotation of the slipstream, drag) more comprehensive. We know that as we include factors, like rotation, drag, it results in a decrease in the power potential of the turbine. Geometrically, we observe this by looking at the force component in the disc plane, which is directly proportional to the power. As we include the respective factors (rotation, drag), the force component in the disc plane is reduced.

The geometrical treatment concludes with a drawing of a series of curves for the thrust coefficient C_T as a function of λ, parametrised by efficiency η, where λ, termed as tip-speed ratio, is given by $r\Omega/v_{-\infty}$. From this graph, it is possible, for example, to read how much pressure thrust is required to get a definite efficiency for a given λ for a windturbine.

Notation:

$v_{-\infty}$, v_{∞}, v	: velocity far in front, behind and on the disc plane.
$v_{rot\infty}$, v_{rot}	: rotational velocity far behind and on the rotor plane.
Ω, ω	: rotational velocity of the windturbine and slipstream.
$p_{-\infty}$, p_{∞}, p, $p+p'$	: pressure far infront, behind, just infront and just behind the disc plane.
T, C_T	: axial thrust and coefficient
P, C_p, η	: Power and power coefficient and efficiency.
dL, dD, ε, ϕ	: lift, drag, $\tan\varepsilon = dD/dL$ and ϕ is the angle of the relative incoming wind velocity with the disc plane.
H_{up}, H_{down}	: total pressure head in infront and behind the disc plane.
λ , b	: $= r\Omega/v_{-\infty}$ and $b = 1 - \tan\varepsilon\tan\phi$
a, a'	: axial and rotational interference factors
dA	: differential area
C_x	: force component (tangential) along the rotor plane

As mentioned before, in momentum theory, the windturbine is replaced by an imaginary energy absorbing machine, called the actuator disc. The windturbine is smeared equally all over the disc area. For such a representation, the geometry of the windturbine plays no part in the theory (2).

In order to deduce the desired expressions, the following theorems are utilised:

1. Bernoulli's equation
2. Conservation of momentum
3. Conservation of angular momentum
4. Conservation of energy

1. <u>Bernoulli's equation</u>:

We may write Bernoulli's equation as

$$\rho/2\, v^2 + p = H \qquad (1)$$

where ρ is the fluid density, v is the velocity and p is the pressure of the fluid; H is the total pressure head, which is constant along a flow line.

Since the actuator disc slows the wind, Bernoulli's equation implies that pressure will increase. Calling H_{up} and H_{down} being the total pressure heads in front and behind the disc plane (abbreviated as H_u and H_d), the power P is given by :

$$P = \int (H_u - H_d)\, v\, dA \qquad (2)$$

Equation (2) implies that in order to have a non zero power ($H_u \neq H_d$) a discontinuity either in v or in p is required across the disc plane.

From the demand on validity of the equation of continuity

$$v dA_1 = v_2 dA_2 = v dA \qquad (3)$$

the axial velocity is required to be continuous across the disc plane (ρ is assumed to be constant). Thus a discontinuity in pressure is a neccessary criterion (see fig. 2)

Bernoulli's equation in front and behind the disc plane is written as:

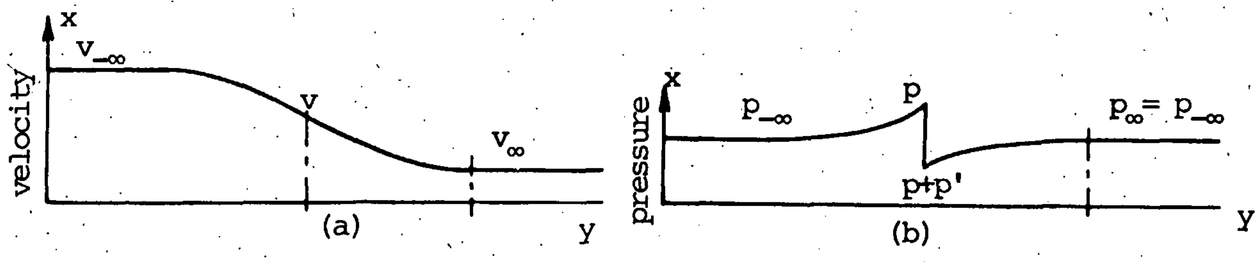

fig. 2.

<u>infront</u>: $$\rho/2\, V_{-\infty}^2 + p_{-\infty} = \rho/2\, V^2 + p = H_{up} \tag{4}$$

<u>behind</u>: $$\rho/2\,(V^2 + V_{rot}^2) + p + p' = \rho/2\,(V_\infty^2 + V_{rot\,\infty}^2) + p_\infty = H_{down} \tag{5}$$

From equation (4) and (5), we get

$$p' = \rho/2\,(V_\infty^2 - V_{-\infty}^2) + \rho/2\,(V_{rot\infty}^2 - V_{rot}^2) + (p_\infty - p_{-\infty}) \tag{6}$$

v_{rot} is normally small compared to $v_{-\infty}$ and v_∞ , therefore v_{rot} may be considered of first order. Justification of this statement is shown experimentally (2). Thus $v_{rot}^2 - v_{rot}^2$ ($\approx O(3)$) may be neglected. Since $p_{-\infty}$ and p_∞ are the pressure far infront and behind the disc plane, they are of the same magnitude. It may also be shown (3) that, $p_{-\infty} - p_\infty \approx 0$. Thus

$$\left.\begin{aligned} v_{rot\infty}^2 - v_{rot}^2 &\cong 0 \quad \text{and} \\ p_{-\infty} - p_\infty &\cong 0 \end{aligned}\right\} \tag{7}$$

Thus

$$p' = \rho/2\,(V_\infty^2 - V_{-\infty}^2) \tag{8}$$

and the thrust dT on a differential area dA on the disc plane, due to the change in velocity, is given by

$$dT = \rho/2\,(V_\infty^2 - V_{-\infty}^2)\,dA$$

Thus from Bernoulli's theorem, we have deduced one important equation:

$$dT_B = \rho/2\,(V_\infty^2 - V_{-\infty}^2)\,dA \quad \text{(axial)} \tag{9}$$

2. Momentum theorem:

The pressure thrust (axial) due to the change in velocity from $v_{-\infty}$ far infront to v_{∞} far behind the disc with a mass flux $\rho v dA$, on a small area dA on the disc plane is given by

$$dT = p' dA = \rho V (V_{\infty} - V_{-\infty}) dA \qquad (10)$$

3. Angular momentum:

Conservation of angular momentum gives

$$r v_{rot} = r_{\infty} v_{rot\infty} \qquad (11)$$

The torque is given by $d\tau = \rho r v_{rot} v dA$, and

the power is given by $dP_I = \Omega d\tau = \rho r v_{rot} v dA\Omega$, or

$$dP_I = \rho r . r\omega V \Omega 2\pi r dr \qquad (12)$$

The subscript I indicates, that the expression is based upon the torque.

4. Drag

From hydrodynamical analysis it may be shown, that when an airfoil is exposed to a fluid flow, causing a change in the flow lines, the airfoil is acted upon by a force, termed as lift dL, ⊥ to the direction of the undisturbed flow. The presence of the drag, dD, tries to draw the airfoil along with the flow; therefore the drag is defined along the incoming flow line, although there are components in other directions, but they are small. Thus the total force the airfoil experiences is the vector sum of $d\vec{L}$ and $d\vec{D}$, see fig 3.

Thus $dD/dL = \tan\varepsilon$, where ε is small

Without drag (dD=0):

$dT = dL.\cos\phi$

$dQ = dL.\sin\phi$

dT is perpendicular and dQ is parallel to the disc plane.

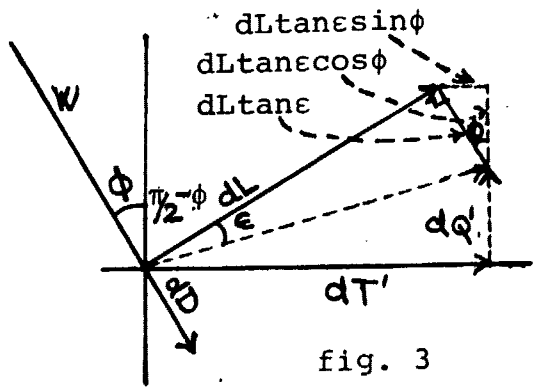

fig. 3

With drag:

$$dT' = dT + dL\tan\varepsilon\sin\phi = dT(1 + \tan\varepsilon\tan\phi) \qquad (13)$$

$$dQ' = dQ - dL\tan\epsilon\cos\phi = dQ(1-\tan\epsilon\cot\phi) \qquad (14)$$

$$dP_I = \Omega\, d\Upsilon = \Omega r\, dQ'$$

$$\Rightarrow \quad dP_I = \rho V \Omega r^2 \omega\, dA(1-\tan\epsilon\cot\phi) \qquad (15)$$

5. Energy

From the theorem of conservation of energy, we find that the energy absorbed by the rotor, through an area dA, on the disc plane, during a time dt, is given by (assuming $p_{-\infty} \simeq p_{\infty}$)

$$\Delta E = \rho/2\,(V_{-\infty}^2 - V_{\infty}^2 - V_{rot\infty}^2)V\,dt\,dA - d\vec{D}\cdot\vec{W}\,dt\,dA$$

Thus the power, which is the time rate of change of ΔE is given by:

$$dP_E = \rho/2\,(V_{-\infty}^2 - V_{\infty}^2 - V_{rot\infty}^2)V\,dA - d\vec{D}\cdot\vec{W}\,dA$$

see fig. 3. The subscript E indicates, the expression dP_E is based on energy. Since $v_{rot\infty}^2 - v_{rot}^2 \simeq 0$ (see equation 7), we may write:

$$v_{rot\infty}^2 \simeq v_{rot}^2 \quad , \text{ thus}$$

$$dP_E = \rho/2\,(V_{-\infty}^2 - V_{\infty}^2 - V_{rot}^2)V\,dA - d\vec{D}\cdot\vec{W}\,dA \qquad (16)$$

$$= \rho/2\,(V_{-\infty}^2 - V_{\infty}^2 - r^2\omega^2)V\,dA - V\,dL\tan\epsilon\cos\phi(\tan\phi + \cot\phi)\,dA$$

$$\Rightarrow \quad dP_E = \rho/2\,(V_{-\infty}^2 - V_{\infty}^2 - r^2\omega^2)V\,dA - V\,dL\sec\phi\tan\epsilon\cot\phi\,dA \qquad (17)$$

The above equations (9-12) belonging to sections 1-3, without the energy equation, belonging to section 5, makes basis for the so called General Momentum theory (3). The evolution of momentum theory began with the axial momentum theory, in which rotation was not taken into account.

Setting $dP_E = dP_I$ (eq. (15)=(17)) and

Calling $b = 1 - \tan\epsilon \cot\phi$, we get

$$\rho V r^2 \Omega \omega b = \frac{\rho}{2}\left(V_{-\infty}^2 - V_{\infty}^2 - r^2\omega^2\right) - dL \sec\phi (1-b)$$

$$\Rightarrow r^2\omega^2 + 2r^2 b\Omega\omega - \left[V_{-\infty}^2 - V_{\infty}^2 - {}^2/_\rho\, dL \sec\phi(1-b)\right] = 0$$

$$\Rightarrow \omega = \Omega b\left[-1 \pm \sqrt{1 + \frac{V_{-\infty}^2}{r^2 b^2 \Omega^2}\left[1 - \left(V_\infty/V_{-\infty}\right)^2 - \frac{2dL}{\rho V_{-\infty}^2}\sec\phi(1-b)\right]}\right]$$

$$\Rightarrow \omega = \Omega b\left[-1 + \sqrt{1 + \lambda^{-2} b^{-2}\left[C_T - A\right]}\right]$$

$$\Rightarrow \omega = \Omega b\left[-1 + \sqrt{1 + \lambda^{-2} b^{-2} B}\right] \qquad (18)$$

where $\lambda = r\Omega/V_{-\infty}$, $b = 1 - \tan\epsilon \cot\phi$

$$C_T = 1 - \left(V_\infty/V_{-\infty}\right)^2, \quad A = \frac{2dL}{\rho V_{-\infty}^2}\sec\phi(1-b)$$

and $$B = C_T - A$$

Observe the minus sign is omitted.

The thrust, power and efficiency coefficients are given as follows:

$$-dT = \rho/2 v_{-\infty}^2 C_T dA$$

$$dP = \rho/2 v_{-\infty}^3 dA C_P \qquad (19)$$

$$dP = \rho/2 v_{-\infty}^3 dA_{-\infty} \eta$$

Since we are interested in studying the geometrical implications, we have taken the absolute value of dT (note dT is negative).

The difference between the power coefficient C_P and the efficiency η is given in the following way: for C_P an area dA on the disc plane is prolonged linearly to far in front of the disc plane; whereas for efficiency, the area dA on the disc plane is prolonged to far in front along the stream lines, see fig. 4 below.

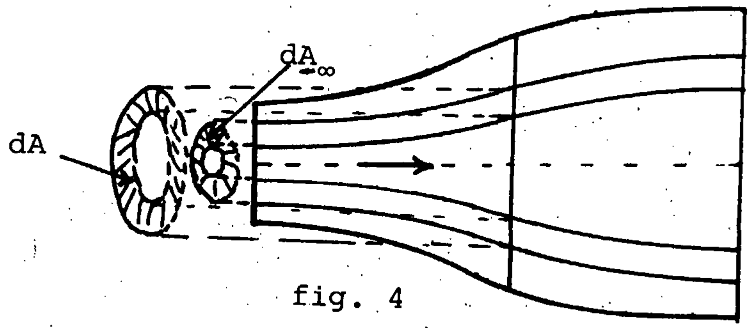

fig. 4

Inserting the expressions of dT and dP in equation (19), we get

$$C_T = 1 - (v_\infty/v_{-\infty})^2$$

$$C_P = 2(v/v_{-\infty})(r\Omega/v_{-\infty})(r\omega/v_{-\infty})(1-\tan\varepsilon\tan\phi) \qquad (20)$$

$$\eta = 2(r\Omega/v_{-\infty})(r\omega/v_{-\infty})(1-\tan\varepsilon\tan\phi)$$

Inserting eq. (18) in eq. (20), we get

$$C_P = 2(v/v_{-\infty})\lambda^2 b^2 \left[-1+\sqrt{1+\lambda^{-2} b^{-2} B}\right] \qquad (21)$$

For analysis and approximation, series (taylor) expansion is done. Assuming $\lambda^{-2}b^{-2}B<1$, we have

$$C_p = 2\left(\frac{V}{V_{-\infty}}\right)\lambda^2 b^2\left[\frac{1}{2}\lambda^{-2}b^{-2}B - \frac{1}{2}\frac{1}{4}\lambda^{-4}b^{-4}B^2 + \frac{1}{2}\frac{1}{4}\frac{3}{6}\lambda^{-6}b^{-6}B^3 - \cdots\right] \quad (22)$$

$$\Rightarrow C_p = (V/V_{-\infty})B - 2(V/V_{-\infty})B\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2(n!)-1}{2(n+1)!}\lambda^{-2n}b^{-2n}B^n \quad (23)$$

The set of equations, belonging to sections 1-5, makes the basis for a theory, we call EXTENDED MOMENTUM THEORY. Let us recollect the different expressions:

6. <u>Set of equations in Momentum theory deduced so far including drag</u>

<u>Bernoulli</u>:

$$dT_B(\text{axial}) = \frac{\rho}{2}\left(V_\infty^2 - V_{-\infty}^2\right)dA \quad (A)$$

<u>Momentum</u>:

$$dT_M = \rho\left(V_\infty - V_{-\infty}\right)V\,dA \quad (B)$$

<u>Angular Momentum</u>:

$$dP_I = \rho V\Omega r^2\omega\,dA\,(1-\tan\epsilon\tan\phi) \quad (C) \qquad (24)$$

<u>Energy</u>:

$$dP_E = \frac{\rho}{2}\left(V_{-\infty}^2 - V_\infty^2 - r^2\omega^2\right)V\,dA - dL\sec\phi(1-b) \quad (D)$$

$$\omega = \Omega b\left[-1+\sqrt{1+\lambda^{-2}b^{-2}B}\right] \quad (E)$$

$$C_p = 2(V/V_{-\infty})\lambda^2 b^2\left[-1+\sqrt{1+\lambda^{-2}b^{-2}B}\right] \quad (F)$$

$$C_p = (V/V_{-\infty})B\left[1-2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2(n!)-1}{2(n+1)!}\lambda^{-2n}b^{-2n}B^n\right] \quad (G)$$

Equation (24 A-G) are the most general expressions possible, with in the domain of our work containing information of axial, rotational velocity and drag. From this set of unified expressions, we are able to reproduce results, belonging to the theories, known as axial momentum theory and general momentum theory. The required results, belonging to the respective theories, are received by successive introduction of the corresponding approximations; this mode of presentation is termed as perturbative representation.

7. Axial momentum theory:

No rotation and no drag or $\omega=0, \lambda$ is large and $b=1$, $B = C_T$

Bernoulli:

$$dT_B = \rho/2\,(V_\infty^2 - V_{-\infty}^2)\,dA \qquad \text{(A)}$$

Momentum:

$$dT_M = \rho\,(V_\infty - V_{-\infty})\,V\,dA \qquad \text{(B)} \qquad (25)$$

Energy:

$$dP_E = \rho/2\,(V_{-\infty}^2 - V_\infty^2)\,V\,dA \qquad \text{(C)}$$

Power coefficient:

$$C_P = (V/V_{-\infty})\,C_T \qquad \text{(D)}$$

From equation (25 A-B), we find

$$v = (v_{-\infty} + v_\infty)/2$$

which means that one half of the total slowing down of the wind, in the axial direction has been reached on the rotor plane.

From eq. (25 C), we see that energy lost by the wind $\frac{\rho}{2}(v_{-\infty}^2 - v_\infty^2)$ is absorbed by the turbine at a rate $\rho/2v(v_{-\infty}^2 - v_\infty^2)$. This must be the upper limit or the ideal turbine.

8. General momentum theory:

Rotation but no drag; or $\omega \neq 0$ and $b=1$, $B=C_T$.

Bernoulli:

$$dT_B(\text{axial}) = \rho\,(V_\infty^2 - V_{-\infty}^2)\,dA \qquad \text{(A)}$$

Momentum: (26)

$$dT_M = \rho\,(V_\infty - V_{-\infty})\,V\,dA \qquad \text{(B)}$$

Angular momentum:

$$dP_I = \rho \Omega V r^2 \omega \, dA \qquad \text{(C)} \qquad (26)$$

Energy:

$$dP_E = \rho/2 \left(V_{-\infty}^2 - V_{\infty}^2 - r^2\omega^2\right) V dA \qquad \text{(D)}$$

$$\omega = \Omega\left[-1 + \sqrt{1 + \lambda^{-2} C_T}\,\right] \qquad \text{(E)}$$

Power coefficient:

$$C_p = 2\left(V/V_{-\infty}\right)\lambda^2\left[-1 + \sqrt{1+\lambda^{-2}C_T}\,\right] \qquad \text{(F)}$$

$$= \left(V/V_{-\infty}\right)C_T - 2\left(V/V_{-\infty}\right)C_T \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2(n!)-1}{2(n+1)!} \lambda^{-2n} C_T^{\,n} \qquad \text{(G)}$$

from equation (26 A-B), we find again

$$V = \frac{V_{-\infty} + V_{\infty}}{2}$$

From equation (26 D), we see that energy absorption by the turbine is diminished due to rotation in the slip stream. From eq. (26 E), we have

$$\omega = \Omega\left[-1 + \sqrt{1 + V_{-\infty}^2/r^2\Omega^2 \, C_T}\,\right]$$

For a given C_T, we see that as Ω gets large or λ^{-1} gets small, ω is small. Thus as

$$\Omega \rightarrow \infty : \omega \rightarrow 0 .$$

Qualitatively this phenomenon may be understood by appreciating the fact, that the faster the turbine rotates, the less effectively the wind feels the presence of the blade at a certain place. In other words the faster the the turbine rotates, the more effectively the blades move away from the wind, as it approaches the disc plane, causing the wind to pass the rotor (turbine) plane undisturbed. Thus from designing point of view, we should strive for attaining as high a Ω as possible.

From eq. (26 F), we also see that as $\lambda \rightarrow 0$: $C_p \rightarrow 0$.

From eq. (26 G), we have

$$C_p = (V/V_{-\infty}) C_T \left[1 - 2\sum_{n=1}^{\infty}(-1)^{n-1} \frac{2(n!)-1}{2(n+1)!} \lambda^{-2n} C_T^{\,n}\right.$$

$$= C_p(\text{axial})\left[1 - 2\sum_{n=1}^{\infty}(-1)^{n-1} \frac{2(n!)-1}{2(n+1)!} \lambda^{-2n} C_T^{\,n}\right] \tag{26}$$

Upto 2nd order

$$C_p \approx C_p(\text{axial})\left[1 - \frac{1}{2}\lambda^{-2} C_T\right] \tag{H}$$

We see that power absorption is reduced due to rotation of the slip stream. As λ gets large, which is equivalent to rotation of the slip stream gets small, $C_P \to C_P(\text{axial})$.

9. Extended momentum theory:

with rotation and drag; or $\omega \neq 0$ and $b<1$

See equation (24 A-G)

From equation (24E) $\omega = \Omega b\left[-1 + \sqrt{1 + \lambda^{-2} b^{-2} B}\right]$

we find that ω(without drag, b=1) < ω(with drag, b<1). Assuming v and v_∞ are unchanged (this is not a bad assumption), we see thus from eq. (24D), that dP_E(without drag) > dP_E(with drag).

From eq. (24 G), taking upto the Ist order, we find

$$C_p \simeq C_p(\text{axial})\left[1 - 2\cdot\frac{1}{2}\cdot\frac{1}{4}\lambda^{-2} b^{-2} B\right]$$

which shows, that when drag is included (b<1), C_P is additionally reduced.

10. Geometrical analysis of the three situaions: axial, axial + rotation and axial + rotation + drag, discussed above

Axial: ($\omega=0$, $b=1$)

When we consider a model, where only the axial component of the wind is slowed down, then the turbine observes the wind to approach the rotor plane with an axial component v and a tangential component $r\Omega$, see fig 5.

The only force, we have, is dL, which makes an angle γ with the rotor axis. The force parallel to the rotor plane, which we are most interested in, is

$$dQ = dL\sin\gamma$$

In this case $\gamma=\phi$ and

$$\tan\gamma = \tan\phi = \frac{v}{r\Omega} \qquad (27)$$

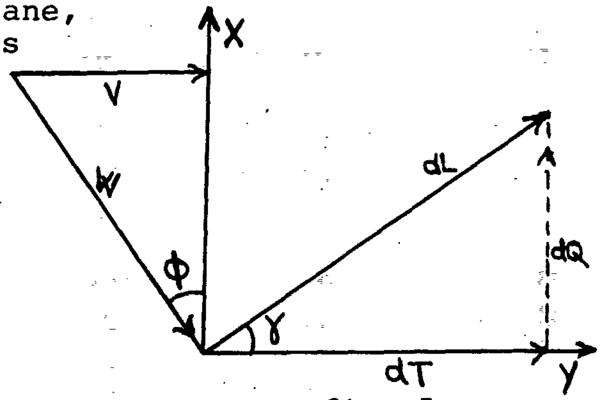

fig. 5

Axial + rotation: $(\omega\neq 0,\ b=1)$

When rotation in the slip stream is considered, but otherwise the rest is unchanged, then only the tangential component gets an additional component $r\omega/2$ see fig. 6. This results in the force dL, having an angle γ with the y axis, is rotated to a smaller angle γ', such that the new component dQ', parallel to the rotor (disc) plane, gets reduced.

$$dQ' = dL'\sin\gamma'$$

From eq. (26 B–C) we find

$$dQ' = \rho r\omega v dA \quad \text{and}$$

$$dT' = \rho v(v_{-\infty} - v_{\infty})dA$$

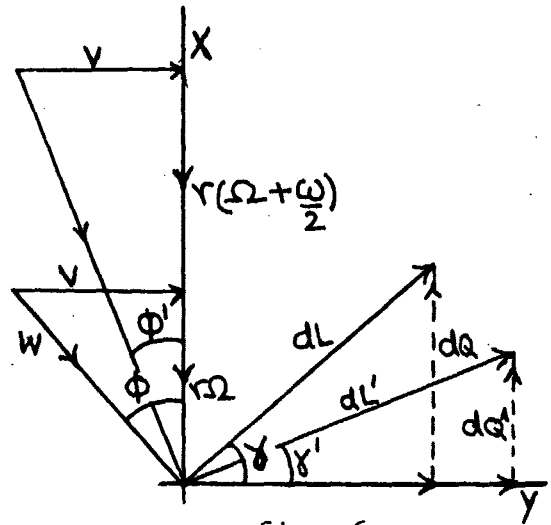

fig. 6

$$\tan\gamma' = \frac{dQ'}{dT'} = \frac{\rho r\omega v dA}{\rho v(V_{-\infty} - V_{\infty})dA} = \frac{r\omega}{V_{-\infty} - V_{\infty}}$$

Inserting the value of ω, from equation (26 E), we get

$$\tan\gamma' = \frac{r\Omega}{V_{-\infty} - V_{\infty}}\left[-1 + \sqrt{1 + \lambda^{-2}C_T}\right] \qquad (28)$$

Considering $\lambda^{-2}C_T < 1$, we may write

$$\tan\gamma' = \frac{r\Omega/V_{-\infty}}{1 - V_{\infty}/V_{-\infty}}\left[-1 + 1 + \lambda^{-2}C_T - \frac{1}{2}\frac{1}{4}\lambda^{-4}C_T^{2} + \cdots\right]$$

$$= \frac{V_{-\infty}+V_{\infty}}{2V_{-\infty}} \frac{1}{\lambda}\left[1 - \frac{1}{4} C_T/\lambda^2 + \cdots\cdots \right]$$

$$= V/r\Omega \left[1 - \frac{1}{4} C_T/\lambda^2 + \cdots\cdots \right]$$

up to 2nd. order in $1/\lambda$

$$\tan\gamma' = V/r\Omega\left[1 - \frac{1}{4}\lambda^{-2}C_T\right]$$

$$\Rightarrow \tan\gamma' = \tan\gamma\left[1 - \frac{1}{4}\lambda^{-2}C_T\right] \qquad (29)$$

Equation (29), shows approximately how γ is reduced to γ'. In other words we see, that in order to get maximum power, we should be interested in making γ or γ' as large as possible.

Axial + rotation + drag: ($\omega \neq 0$, $b<1$)

From fig. 7 we see, that

$$d\tilde{Q} = dQ'(1 - \tan\epsilon \cot\phi')$$

where $dQ' = dL'\sin\phi'$

$$d\tilde{T} = dT'(1 + \tan\epsilon \tan\phi')$$

where $dT' = dL'\cos\phi'$

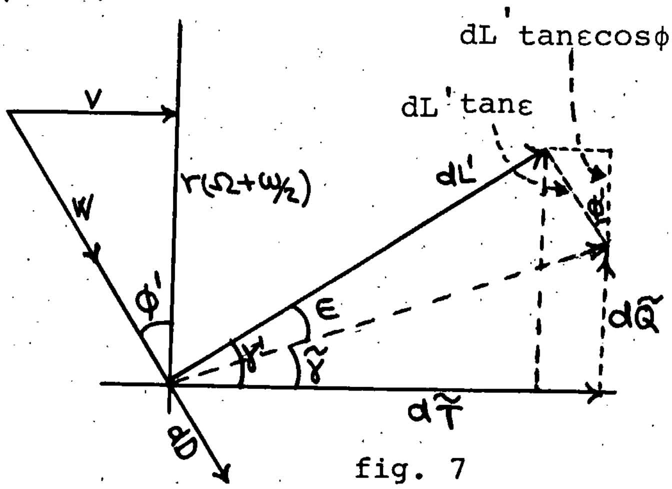

fig. 7

Thus,

$$\tan\tilde{\gamma} = \frac{d\tilde{Q}}{d\tilde{T}} = \frac{dQ'}{dT'}\left[\frac{1 - \tan\epsilon\cot\phi'}{1 + \tan\epsilon\tan\phi'}\right]$$

$$\tan\tilde{\gamma} = \tan\gamma' \frac{1 - \tan\epsilon\cot\phi'}{1 + \tan\tan\phi'} \qquad (30)$$

Thus we see that due to drag γ' is additionally reduced to $\tilde{\gamma}$ through the equation (30), which according to the above discussion, expresses additional power reduction. Thus from equation (28) taking up to 2nd order, we find

$$\tan\tilde{\gamma} \approx \underbrace{\tan\gamma}_{\text{axial}}\underbrace{\left(1-\tfrac{1}{4}\lambda^{-2}C_T\right)}_{\text{rotation}}\underbrace{\frac{1-\tan\epsilon\cot\phi'}{1+\tan\epsilon\tan\phi'}}_{\text{drag}} \qquad (31)$$

<u>Discussion:</u>

The equation set (24), deduced in the present work, makes the basis for what we have termed as extended momentum theory. The equation set contains slowing down of the axial velocity, rotation of the slipstream and drag.

In the original calculation, all the variables are given positive values, so that at the end of the calculation the direction or the sign of the variables (dT, v, v_∞ etc.) automatically falls out. Such calculations shows that the pressure force dT is negative, which is obvious. In text books (1), the calculations are started with assuming that dT is negative or that there is a pressure fall across the disc plane, so that in the real calculation the numerical value of dT is considered.

From eq. (24F), we find

$$C_p = 2(V/V_{-\infty})\lambda^2 b^2\left[-1+\sqrt{1+\lambda^{-2}b^{-2}B}\right];\quad B = C_T - A$$

$$A = \frac{2dL}{\rho V_{-\infty}^2}\sec\phi(1-b)$$

or for $\lambda^{-2}b^{-2}B<1$

$$C_p = \left(\frac{V}{V_{-\infty}}\right)B - 2\left(\frac{V}{V_{-\infty}}\right)B\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2(n!)-1}{2(n+1)!}\lambda^{-2n}b^{-2n}B^n$$

Expressing v and C_T through interference factor a (see introduction), we get

$$C_p = 2(1-a)\lambda^2 b^2\left[-1+\sqrt{1+\lambda^{-2}b^{-2}[4a(1-a)-A]}\right] \qquad (32)$$

or

$$C_p = (1-a)[4a(1-a)-A]\left[1-2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2(n!)-1}{2(n+1)!}\lambda^{-2n}b^{-2n}(4a(1-a)-A)^n\right] \qquad (33)$$

Neglecting drag or setting b=1, we get

$$C_p = 2(1-a)\lambda^2\left[-1+\sqrt{1+\lambda^{-2}4a(1-a)}\right] \tag{34}$$

or

$$C_p = 4a(1-a)^2\left[1-2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2(n!)-1}{2(n+1)!}\lambda^{-2n}\{4a(1-a)\}^n\right] \tag{35}$$

Drawing C_P as a function of λ neglecting drag, we find that for $\lambda=0$ $C_P=0$, which is seen from eq. (34); as λ gets large, C_P goes asymptotically to $(v/v_\infty)C_T = 4a(1-a)^2$ (see eq. (25C)). It may be shown that $C_P=C_{Pmax}$ for a=1/3, thus $C_{Pmax}=16/27$, which is well-known result. Thus for $0<\lambda<\lambda_{ideal}$, we have a situation, where rotation of the slip stream is not negligible, see fig. 8

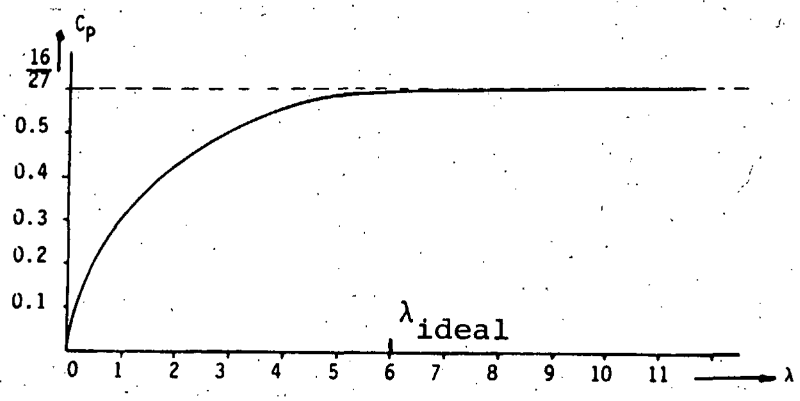

fig. 8

Similarly we find that when drag, expressed through $b\in[0,1]$ is included and gets more dominating, $b\to0$, then $C_P\to0$. Thus some important results, well-known and seen in the text books, (1,2,3), are reproduced analytically from one general equation.

The same phenomena are also shown geometrically. We see that the most important parameter in this context is the force component along the rotor plane. The basic objective is to make this force component C_x as large as possible. Neglecting drag, we see that the rotation of the rotor and that of the slip stream pulls the incoming wind velocity (relative) more and more parallel to the disc plane (ϕ becomes smaller), resulting in diminishing the force component C_x. Thus the slower the turbine rotates, the larger the force component C_x becomes. But from the output point of view, the slower the turbine rotates, the less the power output gets. Thus there is a definite value of C_x and λ, for which the power is maximum.

Inclusion of the drag draws the force even more in the direction of y axis resulting in additional reduction of C_x. Therefore we see, there is direct relation between C_x and the power potential of the wind turbine.

Looking the equation (26E), we find

$$\omega = \Omega\left[-1+\sqrt{1+\left[1-(v_\infty/v_{-\infty})^2\right]\lambda^{-2}}\,\right]$$

Inserting a and a', defined as $v=(1-a)v_{-\infty}$, $v_\infty=(1-2a)v_{-\infty}$ and $\omega=2a'\Omega$, we find

$$a' = \frac{1}{2}\left[-1+\sqrt{1+4a(1-a)\lambda^{-2}}\,\right] \qquad (36)$$

This is the solution of a well-known equation,

$$\lambda^2 a'(1+a') = a(1-a)$$

obtained geometrically.

From eq. (20), with the application of equation of continuity and neglecting drag, we get

$$\eta = 2.\lambda^2[-1+\sqrt{1+\lambda^{-2}C_T}] \text{ or } C_T = \lambda^2[(\eta/2\lambda^2+1)^2-1] \qquad (37)$$

This is a useful equation, and the relation amongst λ, η and C_T is seen by drawing C_T as a function of λ parametrised by η, see fig. 9.

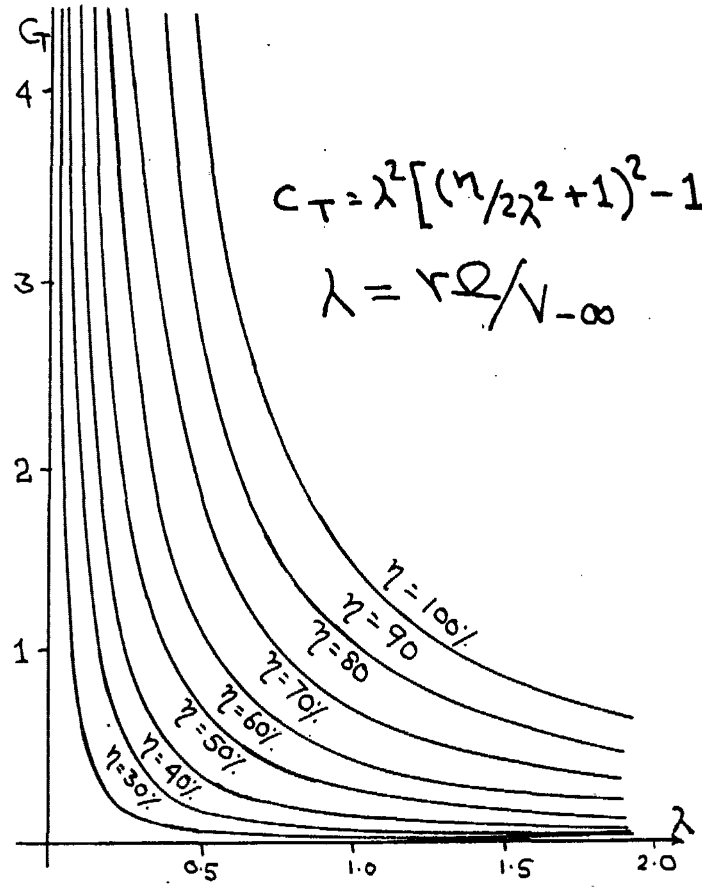

fig. 9

Conclusion:

Momentum theory is deduced with the help of Bernoulli's, momentum, angular momentum and energy theorem. The energy loss factor, drag, is also included in the momentum theory to make an extension of it.

The application of the additional energy conservation theorem makes it possible to express the unknown rotational velocity of the slipstream with the help of the known and other unknown variables. The different equations deduced, involving ω, v and v_∞, may also be expressed with the help of interference factors a and a' to identify them with the text books, see discussion.

The theory is presented in its most general form; then the different momentum theories: axial, general momentum theories etc. are reproduced by insertion of the corresponding approximations. This mode of presentation is known as perturbative presentation. This mode of presentation is the reversal of that, given in standard textbooks (1) and is considered to be more consistent.

With a general expression, eq. (24E-G), it is possible to study behaviour of the different variables under varied conditions. Physically well known results are reproduced through analysis: for example we saw analytically, how ω varies asymptotically and for different values of Ω.

It is always sound to visualise physical phenomena and their implications geometrically, although it is not always easy. In the present work, it has been possible to show a direct relation between the magnitude of the force C_x along the rotor (xz) plane and the power. Introduction of the different power absorbing factors (rotation, drag) implies a reduction of C_x.

Graphical presentation of the efficiency equation makes it possible to see, how the three variables C_T, λ and η are related to each other. This information may be utilised in connection with construction of a horizontal axis wind turbine.

References:

1. Basismateriale for beregning af propelvindmøller, Peter S. Andersen et. al., Risø, 1981.

2. Applied aerodynamics of windpower mechanics, Wilson & Lissaman, Oregon State University, 1974

3. Aerodynamic theory, volume IV, W.F.Durand, Peter Smith, 1976.

1/78 "TANKER OM EN PRAKSIS" - et matematikprojekt.
Projektrapport af Anne Jensen, Lena Lindenskov, Marianne Kesselhahn og Nicolai Lomholt.
Vejleder: Anders Madsen.

2/78 "OPTIMERING" - Menneskets forøgede beherskelsesmuligheder af natur og samfund.
Projektrapport af Tom J. Andersen, Tommy R. Andersen, Gert Kreinøe og Peter H. Lassen.
Vejleder: Bernhelm Booss.

3/78 "OPGAVESAMLING", breddekursus i fysik. — Nr. 3 er a jour ført i marts 1984
Lasse Rasmussen, Aage Bonde Kræmmer, Jens Højgaard Jensen.

4/78 "TRE ESSAYS" - om matematikundervisning, matematiklæreruddannelsen og videnskabsrindalismen. — Nr. 4 er p.t. udgået.
Mogens Niss.

5/78 "BIBLIOGRAFISK VEJLEDNING til studiet af DEN MODERNE FYSIKS HISTORIE". — Nr. 5 er p.t. udgået.
Helge Kragh.

6/78 "NOGLE ARTIKLER OG DEBATINDLÆG OM - læreruddannelse og undervisning i fysik, og - de naturvidenskabelige fags situation efter studenteroprøret".
Karin Beyer, Jens Højgaard Jensen og Bent C. Jørgensen.

7/78 "MATEMATIKKENS FORHOLD TIL SAMFUNDSØKONOMIEN". — Nr. 7 er udgået.
B.V. Gnedenko.

8/78 "DYNAMIK OG DIAGRAMMER". Introduktion til energy-bond-graph formalismen.
Peder Voetmann Christiansen.

9/78 "OM PRAKSIS' INDFLYDELSE PA MATEMATIKKENS UDVIKLING". - Motiver til Kepler's:"Nova Stereometria Doliorum Vinarioum".
Projektrapport af Lasse Rasmussen.
Vejleder: Anders Madsen.

10/79 "TERMODYNAMIK I GYMNASIET".
Projektrapport af Jan Christensen og Jeanne Mortensen.
Vejledere: Karin Beyer og Peder Voetmann Christiansen.

11/79 "STATISTISKE MATERIALER"
red. Jørgen Larsen

12/79 "LINEÆRE DIFFERENTIALLIGNINGER OG DIFFERENTIALLIGNINGSSYSTEMER". — Nr. 12 er udgået
Mogens Brun Heefelt

13/79 "CAVENDISH'S FORSØG I GYMNASIET".
Projektrapport af Gert Kreinøe.
Vejleder: Albert Chr. Paulsen

14/79 "BOOKS ABOUT MATHEMATICS: History, Philosophy, Education, Models, System Theory, and Works of Reference etc. A Bibliography".

Else Høyrup.

Nr. 14 er p.t. udgået.

15/79 "STRUKTUREL STABILITET OG KATASTROFER i systemer i og udenfor termodynamisk ligevægt".

Specialeopgave af Leif S. Striegler.

Vejleder: Peder Voetmann Christiansen.

16/79 "STATISTIK I KRÆFTFORSKNINGEN".

Projektrapport af Michael Olsen og Jørn Jensen.

Vejleder: Jørgen Larsen.

17/79 "AT SPØRGE OG AT SVARE i fysikundervisningen".

Albert Christian Paulsen.

18/79 "MATHEMATICS AND THE REAL WORLD", Proceedings of an International Workshop, Roskilde University Centre, Denmark, 1978. Preprint.

Bernhelm Booss & Mogens Niss (eds.).

19/79 "GEOMETRI, SKOLE OG VIRKELIGHED".

Projektrapport af Tom J. Andersen, Tommy R. Andersen og Per H.H. Larsen.

Vejleder: Mogens Niss.

20/79 "STATISTISKE MODELLER TIL BESTEMMELSE AF SIKRE DOSER FOR CARCINOGENE STOFFER".

Projektrapport af Michael Olsen og Jørn Jensen.

Vejleder: Jørgen Larsen.

21/79 "KONTROL I GYMNASIET - FORMÅL OG KONSEKVENSER".

Projektrapport af Crilles Bacher, Per S. Jensen, Preben Jensen og Torben Nysteen.

22/79 "SEMIOTIK OG SYSTEMEGENSKABER (1)".
1-port lineært response og støj i fysikken.

Peder Voetmann Christiansen.

23/79 "ON THE HISTORY OF EARLY WAVE MECHANICS - with special emphasis on the role of realitivity".

24/80 a+b "MATEMATIKOPFATTELSER HOS 2.G'ERE".
1. En analyse. 2. Interviewmateriale.

Projektrapport af Jan Christensen og Knud Lindhardt Rasmussen.

Vejleder: Mogens Niss.

Nr. 24 a+b er p.t. udgået.

25/80 "EKSAMENSOPGAVER", Dybdemodulet/fysik 1974-79.

26/80 "OM MATEMATISKE MODELLER".
En projektrapport og to artikler.

Jens Højgaard Jensen m.fl.

27/80 "METHODOLOGY AND PHILOSOPHY OF SCIENCE IN PAUL DIRAC's PHYSICS".

Helge Kragh.

28/80 "DIELEKTRISK RELAXATION - et forslag til en ny model bygget på væskernes viscoelastiske egenskaber".

Projektrapport, speciale i fysik, af Gert Kreinøe.

Vejleder: Niels Boye Olsen.

29/80 "ODIN - undervisningsmateriale til et kursus i differentialligningsmodeller".

Projektrapport af Tommy R. Andersen, Per H.H. Larsen og Peter H. Lassen.

Vejleder: Mogens Brun Heefelt

30/80 "FUSIONSENERGIEN - - - ATOMSAMFUNDETS ENDESTATION".

Oluf Danielsen.

Nr. 30 er udgået.

Udkommer medio 1982 på Fysik-, Matematik- og Kemilærernes forlag.

31/80 "VIDENSKABSTEORETISKE PROBLEMER VED UNDERVISNINGSSYSTEMER BASERET PÅ MÆNGDELÆRE".

Projektrapport af Troels Lange og Jørgen Karrebæk.

Vejleder: Stig Andur Pedersen.

Nr. 31 er p.t. udgået

32/80 "POLYMERE STOFFERS VISCOELASTISKE EGENSKABER - BELYST VED HJÆLP AF MEKANISKE IMPEDANSMÅLINGER OG MÖSSBAUEREFFEKTMÅLINGER".

Projektrapport, speciale i fysik, af Crilles Bacher og Preben Jensen.

Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

33/80 "KONSTITUERING AF FAG INDEN FOR TEKNISK-NATURVIDENSKABELIGE UDDANNELSER. I-II".

Arne Jakobsen.

34/80 "ENVIRONMENTAL IMPACT OF WIND ENERGY UTILIZATION".

ENERGY SERIES NO.1.

Bent Sørensen.

Nr. 34 er udgået.

Publ. i "Renewable Sources of Energy and the Environment", Tycooli International Press, Dublin, 1981.

35/80 "HISTORISKE STUDIER I DEN NYERE ATOMFYSIKS UDVIKLING".

Helge Kragh.

36/80 "HVAD ER MENINGEN MED MATEMATIKUNDERVISNINGEN ?".

Fire artikler.

Mogens Niss.

37/80 "RENEWABLE ENERGY AND ENERGY STORAGE".

ENERGY SERIES NO.2.

Bent Sørensen.

38/81 "TIL EN HISTORIETEORI OM NATURERKENDELSE, TEKNOLOGI OG SAMFUND".

Projektrapport af Erik Gade, Hans Hedal, Henrik Lau og Finn Physant.

Vejledere: Stig Andur Pedersen, Helge Kragh og Ib Thiersen.

Nr. 38 er p.t. udgået

39/81 "TIL KRITIKKEN AF VÆKSTØKONOMIEN".

Jens Højgaard Jensen.

40/81 "TELEKOMMUNIKATION I DANMARK - oplæg til en teknologivurdering".

Projektrapport af Arne Jørgensen, Bruno Petersen og Jan Vedde.

Vejleder: Per Nørgaard.

Nr. 40 er p.t. udgået

41/81 "PLANNING AND POLICY CONSIDERATIONS RELATED TO THE INTRODUCTION OF RENEWABLE ENERGY SOURCES INTO ENERGY SUPPLY SYSTEMS".

ENERGY SERIES NO.3.

Bent Sørensen.

42/81 "VIDENSKAB TEORI SAMFUND - En introduktion til materialistiske videnskabsopfattelser".

Helge Kragh og Stig Andur Pedersen.

43/81 1. "COMPARATIVE RISK ASSESSMENT OF TOTAL ENERGY SYSTEMS".

2. "ADVANTAGES AND DISADVANTAGES OF DECENTRALIZATION".

ENERGY SERIES NO.4.

Bent Sørensen.

44/81 "HISTORISK UNDERSØGELSE AF DE EKSPERIMENTELLE FORUDSÆTNINGER FOR RUTHERFORDS ATOMMODEL".

Projektrapport af Niels Thor Nielsen.

Vejleder: Bent C. Jørgensen.

45/82

46/82 I+II "EKSEMPLARISK UNDERVISNING OG FYSISK ERKENDELSE - ILLUSTRERET VED TO EKSEMPLER".

Projektrapport af Torben O. Olsen, Lasse Rasmussen og Niels Dreyer Sørensen.

Vejleder: Bent C. Jørgensen.

47/82 "BARSEBÄCK OG DET VÆRST OFFICIELT-TÆNKELIGE UHELD".

ENERGY SERIES NO.5.

Bent Sørensen.

48/82 "EN UNDERSØGELSE AF MATEMATIKUNDERVISNINGEN PÅ ADGANGSKURSUS TIL KØBENHAVNS TEKNIKUM".

Projektrapport af Lis Eilertzen, Jørgen Karrebæk, Troels Lange, Preben Nørregaard, Lissi Pedersen, Laust Rishøj, Lill Røn, Isac Showiki.

Vejleder: Mogens Niss.

49/82 "ANALYSE AF MULTISPEKTRALE SATELLITBILLEDER".

Projektrapport af Preben Nørregaard.

Vejledere: Jørgen Larsen & Rasmus Ole Rasmussen.

50/82 "HERSLEV - MULIGHEDER FOR VEDVARENDE ENERGI I EN LANDSBY". ENERGY SERIES NO.6.

Rapport af Bent Christensen, Bent Hove Jensen, Dennis B. Møller, Bjarne Laursen, Bjarne Lillethorup og Jacob Mørch Pedersen.

Vejleder: Bent Sørensen.

51/82 "HVAD KAN DER GØRES FOR AT AFHJÆLPE PIGERS BLOKERING OVERFOR MATEMATIK?"

Projektrapport af Lis Eilertzen, Lissi Pedersen, Lill Røn og Susanne Stender.

52/82 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS"

Bernhelm Booss & Krzysztof Wojciechowski.

53/82 "THE CONSTITUTION OF SUBJECTS IN ENGINEERING EDUCATION".

Arne Jakobsen & Stig Andur Pedersen.

54/82 "FUTURES RESEARCH" - A Philosophical Analysis of Its Subject-Matter and Methods.

Stig Andur Pedersen & Johannes Witt-Hansen.

55/82 "MATEMATISKE MODELLER" - Litteratur på Roskilde Universitetsbibliotek.
En bibliografi.
Else Høyrup.

Vedr. tekst nr. 55/82: Se også tekst 62/83.

56/82 "ÉN - TO - MANGE" - En undersøgelse af matematisk økologi.
Projektrapport af Troels Lange.
Vejleder: Anders Madsen.

57/83 "ASPECT EKSPERIMENTET" - Skjulte variable i kvantemekanikken?
Projektrapport af Tom Juul Andersen.
Vejleder: Peder Voetmann Christiansen.

Nr. 57 er udgået.

58/83 "MATEMATISKE VANDRINGER" - Modelbetragtninger over spredning af dyr mellem småbiotoper i agerlandet.
Projektrapport af Per Hammershøj Jensen & Lene Vagn Rasmussen.
Vejleder: Jørgen Larsen.

59/83 "THE METHODOLOGY OF ENERGY PLANNING".
ENERGY SERIES NO. 7.
Bent Sørensen.

60/83 "MATEMATISK MODEKSPERTISE" - et eksempel.
Projektrapport af Erik O. Gade, Jørgen Karrebæk og Preben Nørregaard.
Vejleder: Anders Madsen.

61/83 "FYSIKS IDEOLOGISKE FUNKTION", som et eksempel på en naturvidenskab - historisk set.
Projektrapport af Annette Post Nielsen.
Vejledere: Jens Høyrup, Jens Højgaard Jensen og Jørgen Vogelius.

62/83 "MATEMATISKE MODELLER" - Litteratur på Roskilde Universitetsbibliotek.
En bibliografi. 2. rev. udgave
Else Høyrup

63/83 "CREATING ENERGY FUTURES: A SHORT GUIDE TO ENERGY PLANNING".
ENERGY SERIES No. 8
David Crossley & Bent Sørensen

64/83 "VON MATHEMATIK UND KRIEG".
Bernhelm Booss og Jens Høyrup

65/83 "ANVENDT MATEMATIK - TEORI ELLER PRAKSIS".
Projektrapport af Per Hedegård Andersen, Kirsten Habekost, Carsten Holst-Jensen, Annelise von Moos, Else Marie Pedersen, Erling Møller Pedersen.
Vejledere: Bernhelm Booss & Klaus Grünbaum

66/83 "MATEMATISKE MODELLER FOR PERIODISK SELEKTION I ESCHERICHIA COLI".
Projektrapport af Hanne Lisbet Andersen, Ole Richard Jensen og Klavs Frisdahl.
Vejledere: Jørgen Larsen og Anders Hede Madsen

67/83 "ELIPSOIDE METODEN - EN NY METODE TIL LINEÆR PROGRAMMERING?"

Projektrapport af Lone Biilmann og Lars Boye

Vejleder: Mogens Brun Heefelt

68/83 "STOKASTISKE MODELLER I POPULATIONSGENETIK" - til kritikken af teoriladede modeller.

Projektrapport af Lise Odgård Gade, Susanne Hansen, Michael Hviid, Frank Mølgård Olsen.

Vejleder: Jørgen Larsen.

69/83 "ELEVFORUDSÆTNINGER I FYSIK" - en test i 1.g med kommentarer

Albert Chr. Paulsen

70/83 "INDLÆRINGS- OG FORMIDLINGSPROBLEMER I MATEMATIK PÅ VOKSENUNDERVISNINGSNIVEAU"

Projektrapport af Hanne Lisbet Andersen, Torben J. Andreasen, Svend Åge Houmann, Helle Glerup Jensen, Keld Fl. Nielsen, Lene Vagn Rasmussen.

Vejleder: Klaus Grünbaum & Anders H. Madsen

71/83 "PIGER OG FYSIK" - et problem og en udfordring for skolen?

Karin Beyer, Sussanne Blegaa, Birthe Olsen, Jette Reich & Mette Vedelsby

72/83 "VERDEN IFØLGE PEIRCE" - to metafysiske essays, om og af C.S. Peirce.

Peder Voetmann Christiansen

73/83 "EN ENERGIANALYSE AF LANDBRUG" - økologisk contra traditionelt

ENERGY SERIES No. 9

Specialeopgave i fysik af Bent Hove Jensen

Vejleder: Bent Sørensen

74/84 "MINIATURISERING AF MIKROELEKTRONIK" - om videnskabeliggjort teknologi og nytten af at lære fysik

Projektrapport af Bodil Harder og Linda Szkotak Jensen.

Vejledere: Jens Højgaard Jensen og Bent C. Jørgensen

75/84 "MATEMATIKUNDERVISNINGEN I FREMTIDENS GYMNASIUM" - Case: Lineær programmering

Projektrapport af Morten Blomhøj, Klavs Frisdahl, Frank Mølgaard Olsen

Vejledere: Mogens Brun Heefelt & Jens Bjørneboe

76/84 "KERNEKRAFT I DANMARK?" - Et høringssvar indkaldt af miljøministeriet, med kritik af miljøstyrelsens rapporter af 15. marts 1984.

ENERGY SERIES No. 10

Af Niels Boye Olsen og Bent Sørensen

77/84 "POLITISKE INDEKS - FUP ELLER FAKTA?" Opinionsundersøgelser belyst ved statistiske modeller

Projektrapport af Svend Åge Houmann, Keld Nielsen, Susanne Stender

Vejledere: Jørgen Larsen & Jens Bjørneboe

78/84 "JÆVNSTRØMSLEDNINGSEVNE OG GITTERSTRUKTUR I AMORFT GERMANIUM"

Specialerapport af Hans Hedal, Frank C. Ludvigsen og Finn C. Physant

Vejleder: Niels Boye Olsen

79/84 "MATEMATIK OG ALMENDANNELSE"

Projektrapport af Henrik Coster, Mikael Wennerberg Johansen, Povl Kattler, Birgitte Lydholm og Morten Overgaard Nielsen.

Vejleder: Bernhelm Booss

80/84 "KURSUSMATERIALE TIL MATEMATIK B"

Mogens Brun Heefelt

81/84 "FREKVENSAFHÆNGIG LEDNINGSEVNE I AMORFT GERMANIUM"

Specialerapport af Jørgen Wind Petersen og Jan Christensen

Vejleder: Niels Boye Olsen

82/84 "MATEMATIK- OG FYSIKUNDERVISNINGEN I DET AUTOMATISEREDE SAMFUND"

Rapport fra et seminar afholdt i Hvidovre 25-27 april 1983

Red.: Jens Højgaard Jensen, Bent C. Jørgensen og Mogens Niss

83/84 "ON THE QUANTIFICATION OF SECURITY" nr. 83 er p.t. udgået

PEACE RESEARCH SERIES NO. 1

af Bent Sørensen

84/84 " NOGLE ARTIKLER OM MATEMATIK, FYSIK OG ALMENDANNELSE".

Jens Højgaard Jensen, Mogens Niss m. fl.

85/84 "CENTRIFUGALREGULATORER OG MATEMATIK"

Specialerapport af Per Hedegård Andersen, Carsten Holst-Jensen, Else Marie Pedersen og Erling Møller Pedersen

Vejleder: Stig Andur Pedersen

86/84 "SECURITY IMPLICATIONS OF ALTERNATIVE DEFENSE OPTIONS FOR WESTERN EUROPE"

PEACE RESEARCH SERIES NO. 2

af Bent Sørensen

87/84 "A SIMPLE MODEL OF AC HOPPING CONDUCTIVITY IN DISORDERED SOLIDS"

af Jeppe C. Dyre

88/84 "RISE, FALL AND RESURRECTION OF INFINITESIMALS"

af Detlef Laugwitz

89/84 "FJERNVARMEOPTIMERING"

af Bjarne Lillethorup & Jacob Mørch Pedersen

90/84 "ENERGI I 1.G- en teori for tilrettelæggelse"

af Albert Chr. Paulsen

91/85 "KVANTETEORI FOR GYMNASIET"
1. Lærervejledning

Projektrapport af: Birger Lundgren, Henning Sten Hansen og John Johansson

Vejleder: Torsten Meyer

92/85 "KVANTETEORI FOR GYMNASIET
2. Materiale

Projektrapport af: Birger Lundgren, Henning Sten Hansen og John Johansson

Vejleder: Torsten Meyer

93/85 "THE SEMIOTICS OF QUANTUM-NON-LOCALITY"
af Peder Voetmann Christiansen

94/85 "TREENIGHEDEN BOURBAKI - generalen, matematikeren og ånden"

Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank M. Olsen

Vejleder: Mogens Niss

95/85 "AN ALTERNATIV DEFENSE PLAN FOR WESTERN EUROPE"

Peace research series no. 3
af Bent Sørensen

96/85 "ASPEKTER VED KRAFTVARMEFORSYNING"
af Bjarne Lillethorup

Vejleder: Bent Sørensen

97/85 "ON THE PHYSICS OF A.C. HOPPING CONDUCTIVITY"

Jeppe C. Dyre

98/85 "VALGMULIGHEDER I INFORMATIONSALDEREN"
af Bent Sørensen

99/85 "Der er langt fra Q til R"

Projektrapport af: Niels Jørgensen og Mikael Klintorp
Vejleder: Andur Pedersen

100/85 "TALSYSTEMETS OPBYGNING"
af Mogens Niss